Modelling Non-Life Dependencies Using Risk Factor Models

Maximilian Kelm / Pawel Kuczera, SCOR

About the speakers

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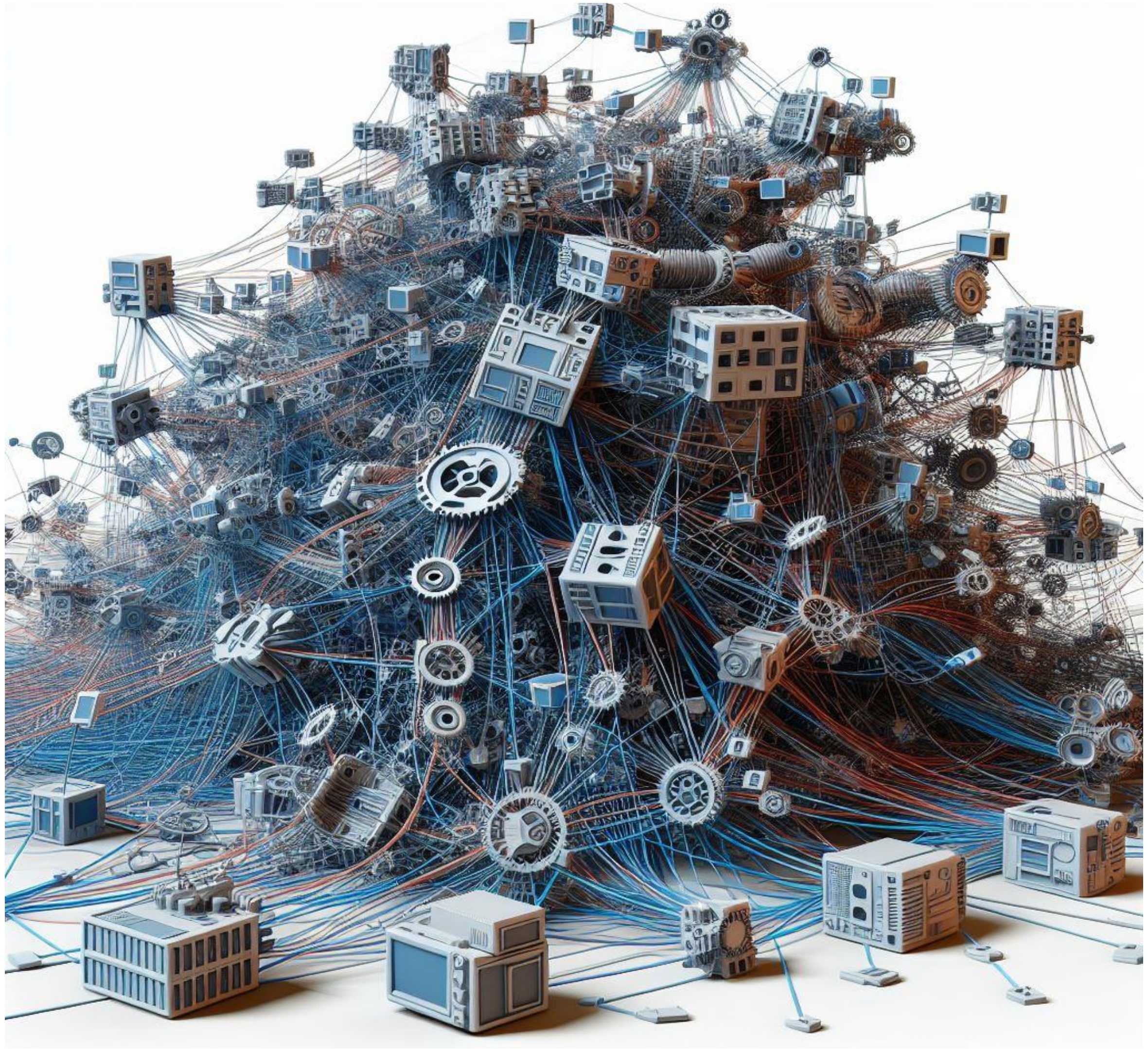
Dependencies are hard to model

A typical dependency structure in an internal model may look like this:



Introduction

Picture credit: **Amaury Laporte**, Stranded Dali Container Ship and Collapsed Baltimore Francis Scott Key Bridge 03, Creative commons license

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What happened?

The bridge collapsed after a cargo ship collided into it.

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How is this typically modelled in an internal model?

There is a copula with a certain parametrization describing the dependency between different portfolios.

Could such a dependency be modelled in a more intuitive way?

Picture credit: **Amaury Laporte**, [Stranded Dali Container Ship and Collapsed Baltimore Francis Scott Key Bridge 03], [Creative commons license]

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Overview of standard dependency modelling approaches

Var-Cov

- ✓ A very simple approach also used in various standardized approaches
- ✓ Formally correct only for normal distributions
- ✓ Limits use cases of the model (no scenario-by-scenario analysis possible)
- ✓ Not a state-of-the-art approach for internal models

Explicit Copula on Loss Distributions

- ✓ Scenario-by-scenario analysis possible
- ✓ Capital allocation use case possible
- ✓ Not always intuitive when explaining the results
- ✓ Difficult to calibrate and always to some extent arbitrary

Bottom-Up Risk Factor Models

- ✓ Scenario-by-scenario analysis possible
- ✓ Bottom-up modelling
- ✓ Complex since, dependencies between all important risk pairs should be considered.
- ✓ Often used for Economic and/or L&H risk factor approaches

complexity

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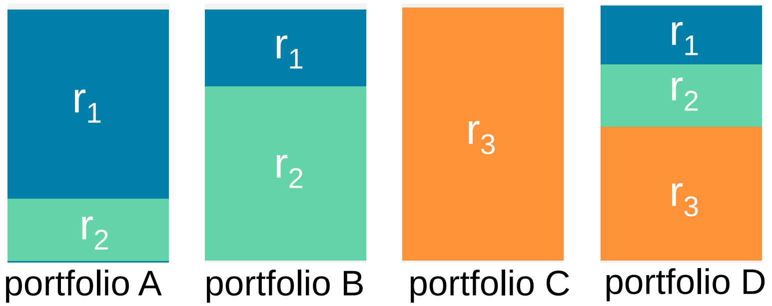
An idea for a risk factor approach in a P&C model

- Some examples:
  - → **Baltimore Bridge** the event could be classified as a **collision** and it could easily cause losses in both USA **property** and **liability** portfolios of a global reinsurer.
  - → **Pandemic**, like COVID19, could cause losses in, e.g., **credit**, **property**, and that across different regions. Note, that a consistent risk factor framework would allow to naturally introduce a dependency between L&H and P&C business.
- Why is bottom-up modelling of P&C dependencies difficult?
  - → The actual distributions of risk factors like, e.g., collision is challenging to estimate,
  - → The link between a stochastic realization of a risk factor and the corresponding losses is also nontrivial.
- **Proposal**: Decompose portfolios into contributions from risk factors. Portfolios exposed to common risk factors would be dependent:

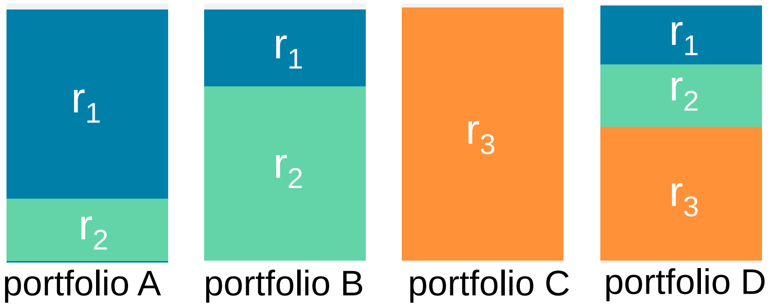


An idea for a risk factor approach in a P&C model

- The idea of a risk factor approach was originally proposed by *Ferriero** for a class of so-called infinitely divisible distributions.
- A distribution F of a random variable (RV) X is *infinitely divisible* if for every positive n there exists a set of n iid RVs. $X_1, X_2, \dots, X_n$ whose sum has the same distribution F.

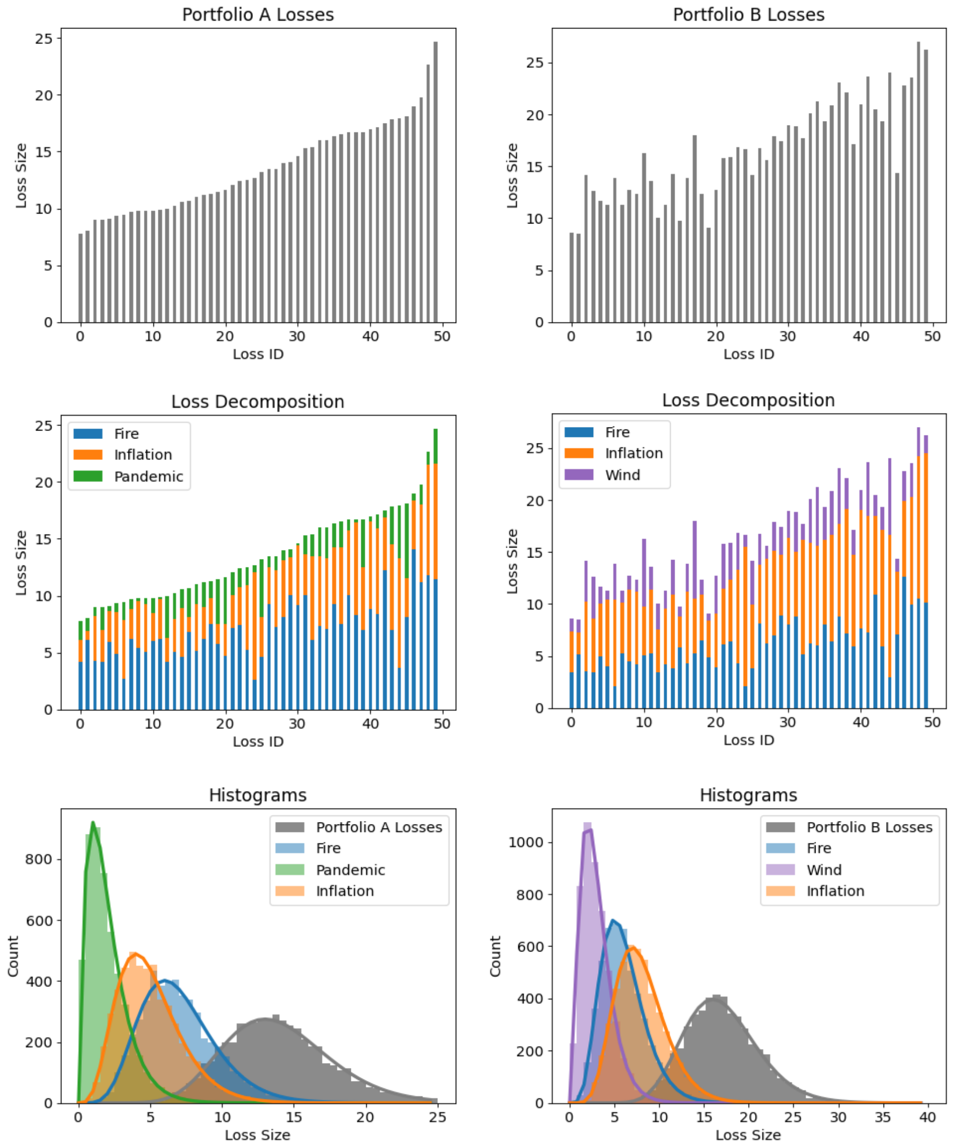
P&C Risk Factor idea in a nutshell

1. Decompose the portfolio loss distributions into contributions from different risk factors relying on the infinite divisibility,
2. Induce dependency between these portfolios by making the contributions from common risk factors comonotonic.



*A. Ferriero. How to build a risk factor model for non-life insurance risk. Journal of Risk, 24(3),2020

P&C risk factor model – a possible approach



- The approach uses the following property of Gamma distribution:

$S_1 \sim \Gamma(k_1, \theta)$, $S_2 \sim \Gamma(k_2, \theta)$ and S_1, S_2 independent,
then: $S_1 + S_2 \sim \Gamma(k_1 + k_2, \theta)$

- For each portfolio fit a Gamma distribution. This results in a $k_{\mathrm{p}}, \theta_{\mathrm{p}}$ pair for each portfolio p.
- Derive the “on average” risk factor weights w_i for each portfolio in an expert judgement process (will be discussed further on).

Risk Factor Decomposition				
	Fire	Inflation	Pandemic	Wind
Portfolio A	50%	36%	14%	0%
Portfolio B	35%	47%	0%	18%

- Set the contribution of a risk factor i in portfolio p a:

$$S_{i,p} \sim \Gamma\left(w_i k_p, \theta_p\right)$$

- Reorder the contributions such that for each risk factor i the contributions $S_{i,p}$ are comonotonic in each portfolio p.

P&C risk factor model – defining the profile of each risk factor

In general, for each risk factor its **accumulation profile** needs to be defined in terms of

Geographical location

- A pandemic like COVID19 will likely cause losses **across the world**
- On the other hand, a collision like the Baltimore Bridge event would be **geographically localized**

Line of business

- A pandemic can create losses e.g. in **Property and Credit at the same time**, a collision could cause losses e.g. in **Property and Liability**
- On the other hand, a smaller insolvency event will likely only affect a **single line** (e.g. Credit)

Business maturity

- A pandemic or collision only affects Premium Risk
- Other risk factors like inflation and estimation risk can affect **Premium Risk and Reserve Risk at the same time**

Determining risk factor profiles can be challenging and may require a split by event severity

Resulting copulas depend on weights and marginals

Example: 2 portfolios A and B with equal marginal distribution $\Gamma(k, \theta)$ and two risk factors each: one common risk factor with weight w, one individual risk factor with weight $1 - w$. Copulas have…

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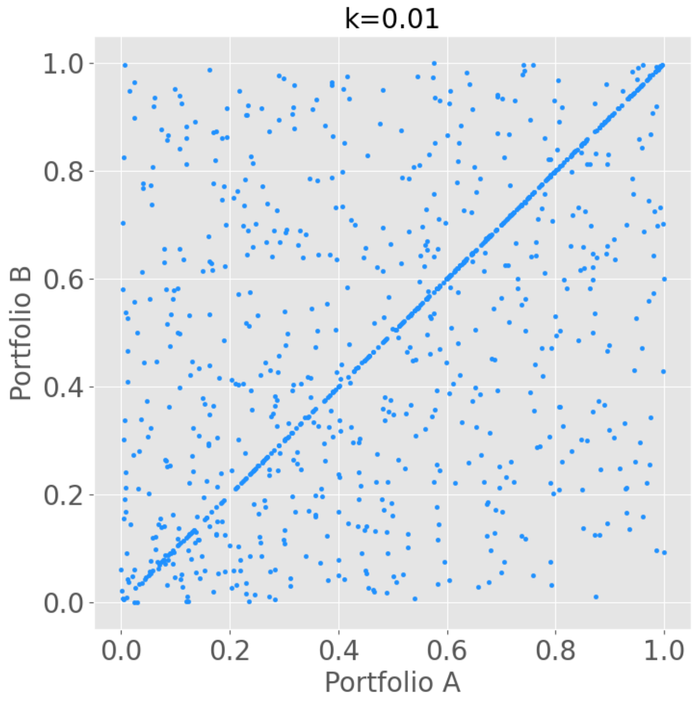

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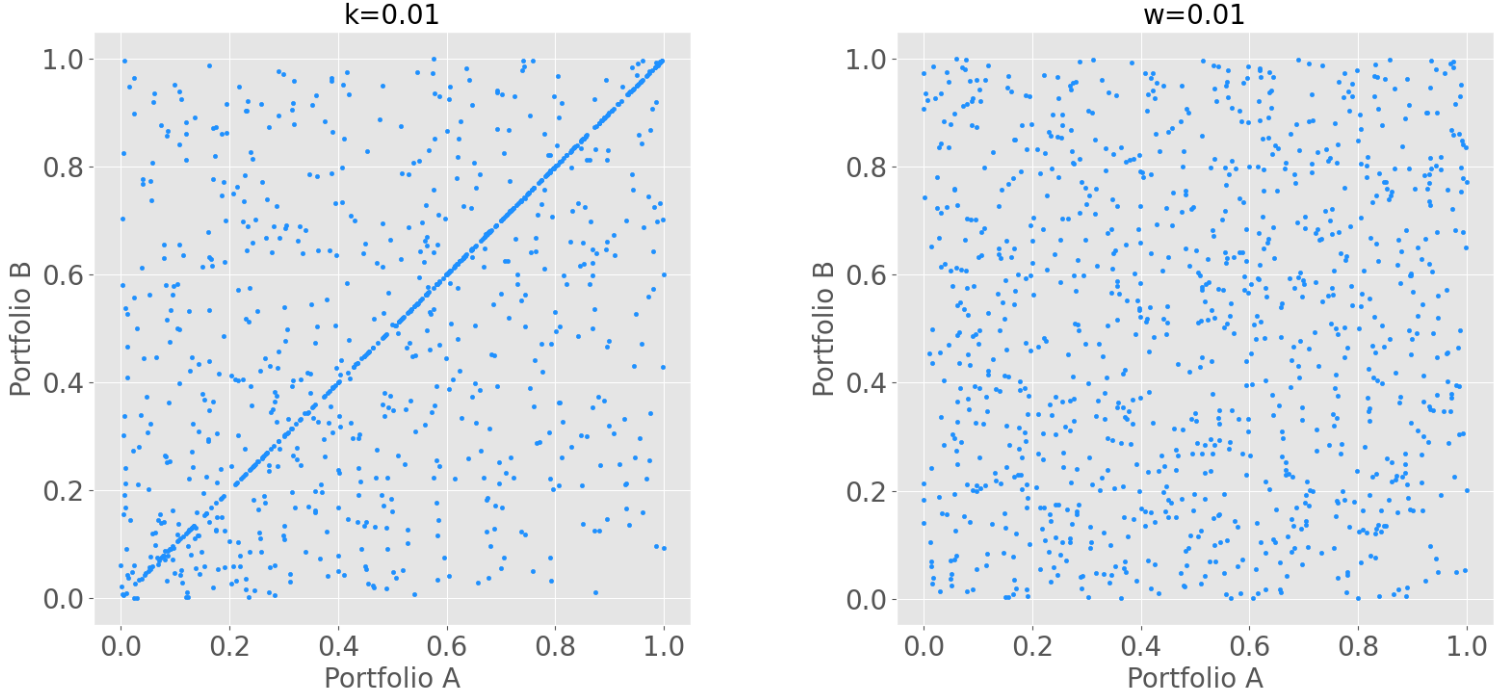

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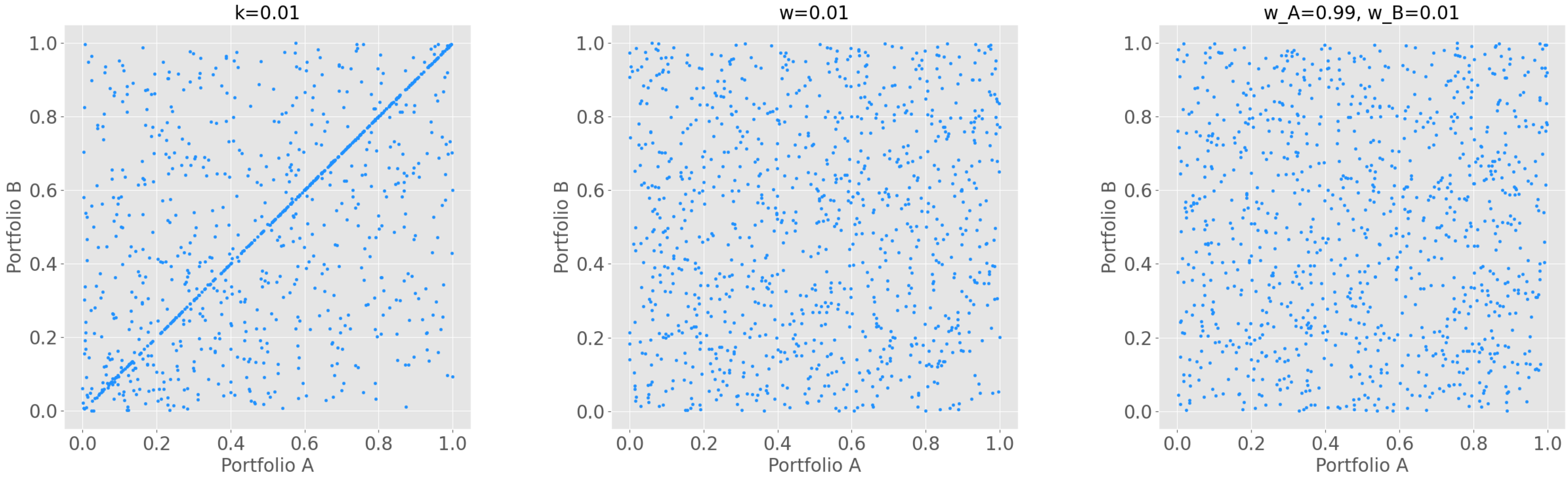
… non-linear shape if w differs between portfolios



P&C risk factor model calibration – several sources of input

We need to **calibrate risk factor weights** for every modeled portfolio – possible sources for calibration include:

Prior information

Any preexisting information that can be used

Observations

Historical claims data containing risk factor information

Expert judgment

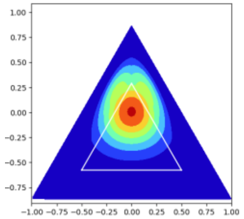
Subject matter experts often have a good idea of risk factor contributions

Prior Information, **Ob**servations and **Ex**pert Judgment (PrObEx) can be combined in a Bayesian approach building on work of Arbenz & Canestraro (2012)*

*Ph. Arbenz, D. Canestraro. Estimating copulas for insurance from scarce observations, expert opinion and prior information: A bayesian approach. ASTIN Bulletin, 42, 05 2012.

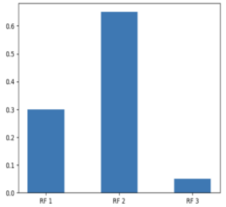
A Bayesian approach can be used to calibrate risk factor weights

Expert judgment



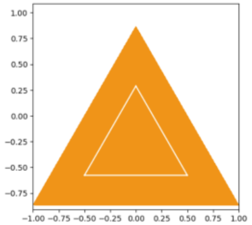
- Inputs: expert estimate and self-reported uncertainty
- Construction uses Dirichlet distribution on the standard simplex
- Likelihood functions of different experts are aggregated by multiplication

Observations



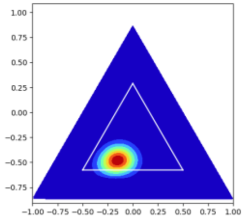
- Historical claims amounts per risk factor can be cast into likelihood function
- Conjugate of the Dirichlet distribution is the multinomial distribution
- Some risk factors may be unobservable in historical data – partial Bayesian update

Prior distribution



- Dirichlet distribution describing previous calibration
- Uniform distribution on simplex in case of uninformed prior

Posterior distribution



- Product of the above likelihood functions
- Point estimate yields final calibration

Summary

- ⇨ Risk factor models can be used to model dependencies between P&C portfolios
- ⇨ No need to change marginal models
- ⇨ More intuitive than more conventional dependency models
- ⇨ Resulting copulas vary with shape of marginals and risk factor profiles
- ⇨ Calibration can be performed using a combination of prior information, observations and expert judgment
- ⇨ Extendable to model cross-risk dependencies between P&C and L&H, Market Risk, etc.

Thank you

Contact Details

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Appendix

Some risk factors are split into global and local version

Whenever necessary, we split the risk factors into a "global" and a "local" version, relating to the event size: some events create more accumulation than others.

- Case 1: Collision, Fire

	Business maturity	Line of business	Geographical location
Local	Specific	Specific	Specific
Global	Specific	Across	Specific

- Case 2: Cyber

	Business maturity	Line of business	Geographical location
Local	Across	Specific	Across
Global	Across	Across	Across

- Case 3: Error & Defect

	Business maturity	Line of business	Geographical location
Local	Across	Specific	Specific
Global	Across	Specific	Across

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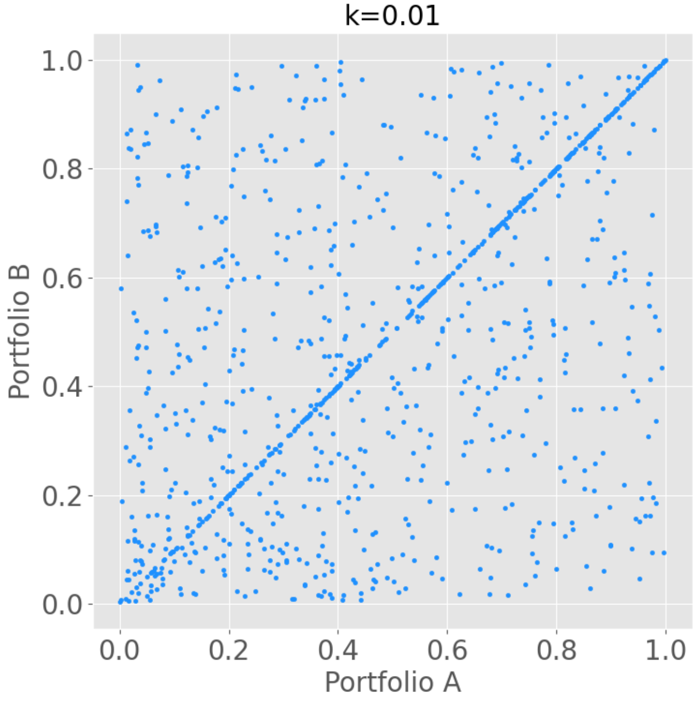

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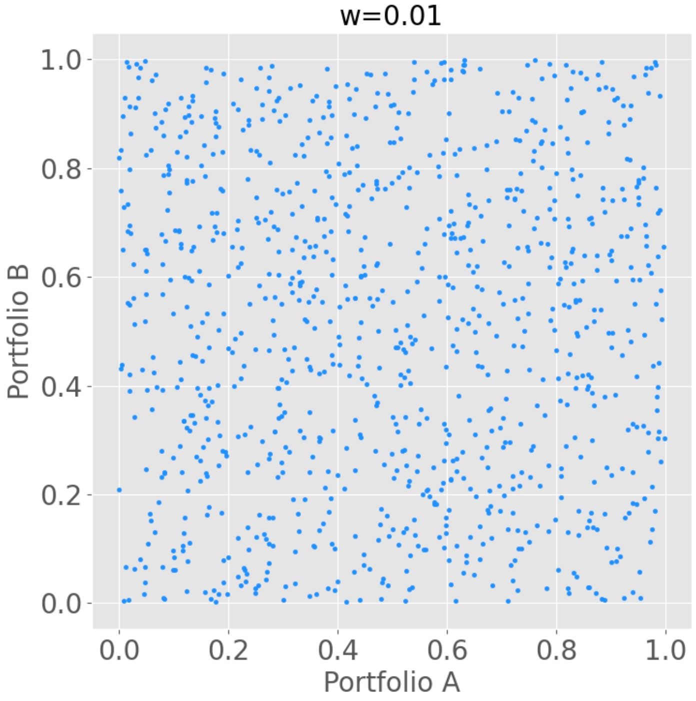

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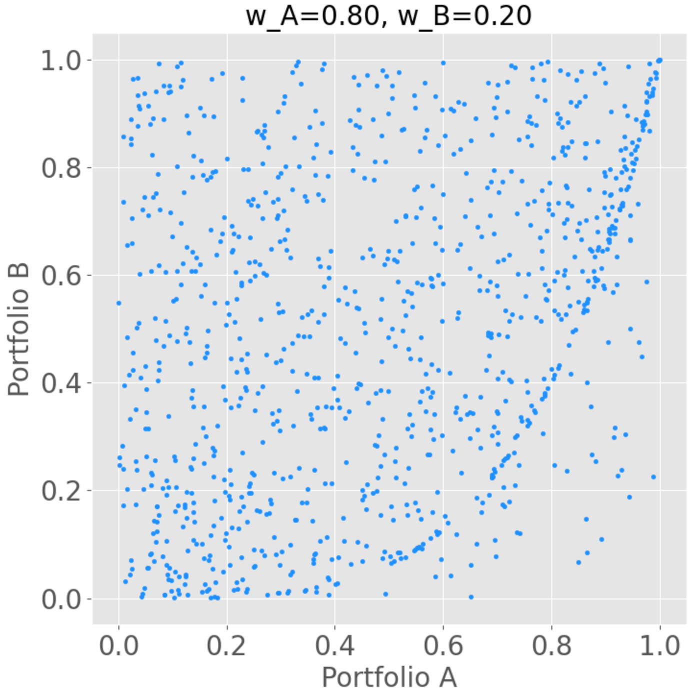

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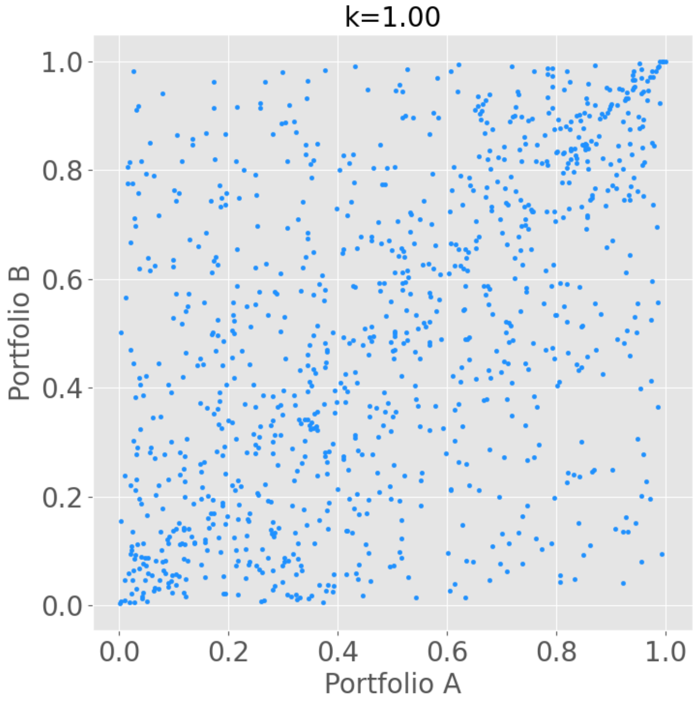

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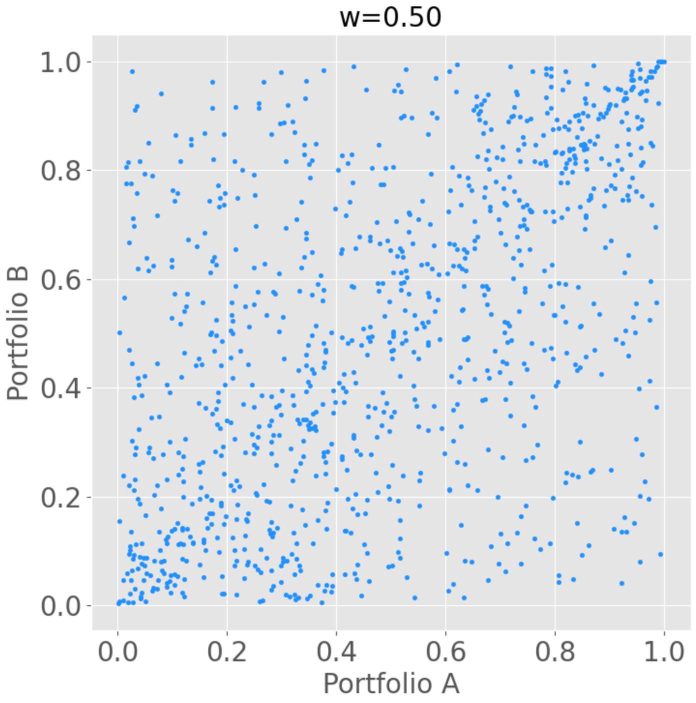

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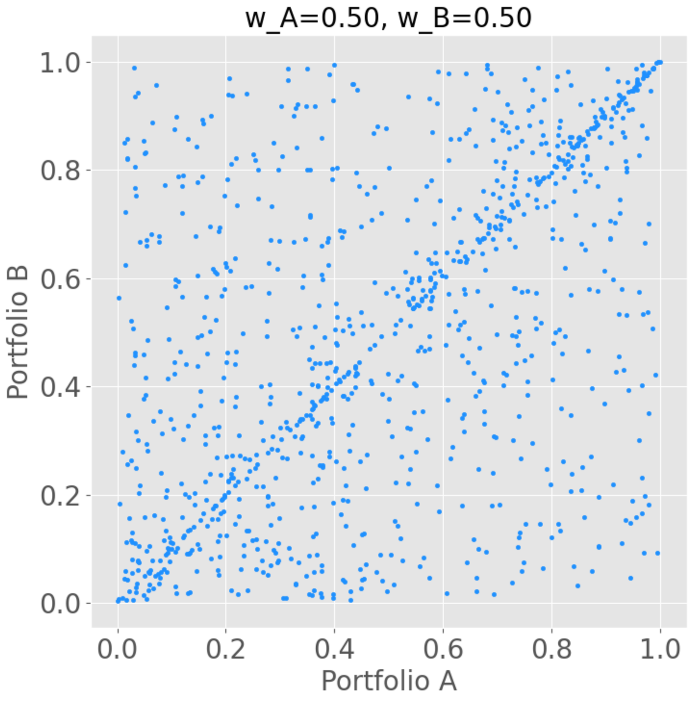

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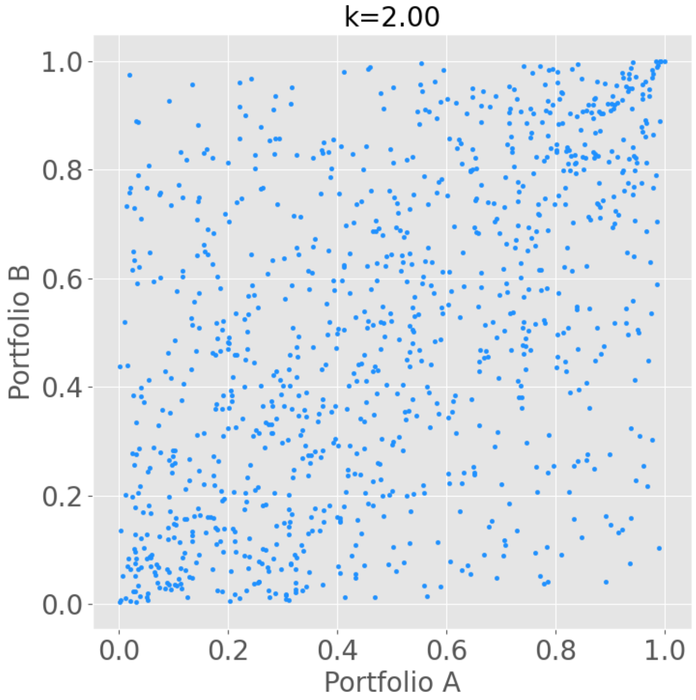

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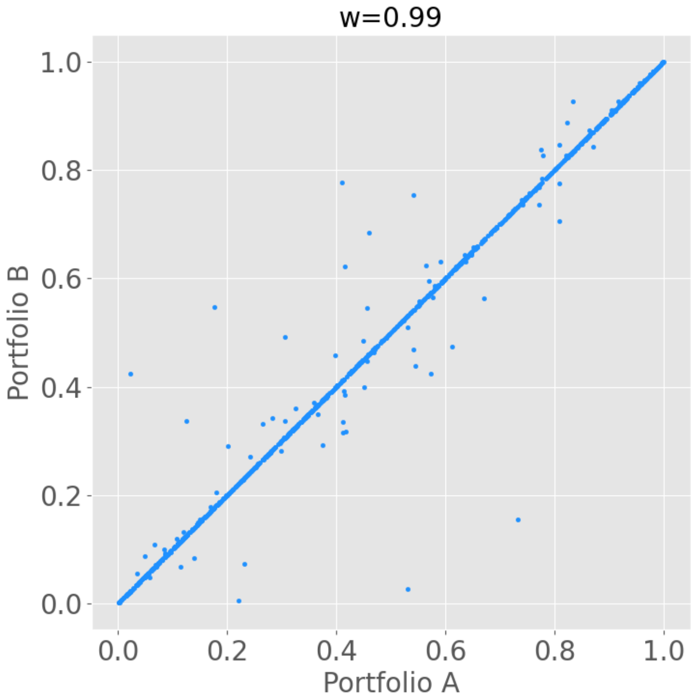

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