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Dynamic Fixed Income Bond Ladders with Multiple Optimal Decision Making Under Uncertainty

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Dynamic Fixed Income Bond Ladders with Multiple Optimal Decision Making Under Uncertainty

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Outline

Outline

What is a Bond Ladder?

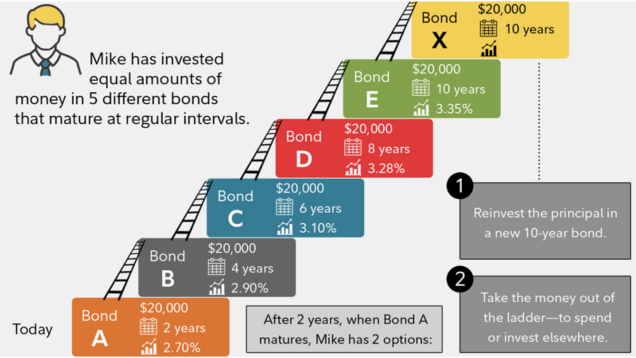


Figure: Source: Blue Haven Capital (2024)

- Built by **staggered bonds**
- Sensitive to **interest rate** $\rightarrow$ Term risk, Convexity risk

Interest rates vs Bond prices

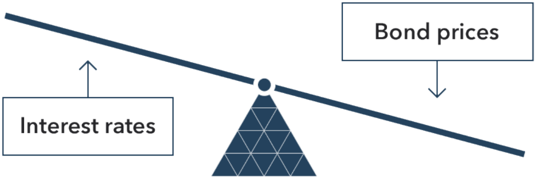


Figure: Source: https://www.ig.com/au/

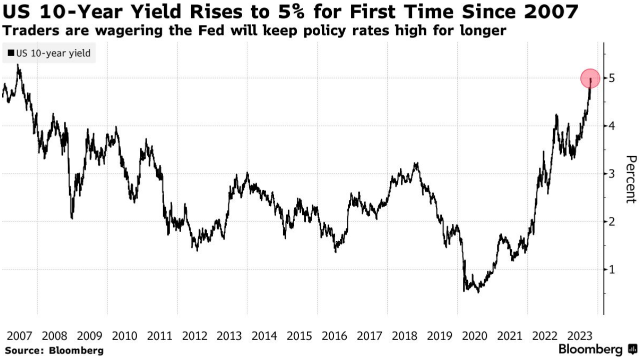


Figure: Source: Bloomberg (2023)

Current interest rate environment

- **Unstable interest** environment in the US
- **Active monetary policy** on interest rates
- Shifting in the **demand and supply of long-and short-term US bonds**
- Significant **impact on bond convexity and duration**

Silicon Valley Bank (SVB) Collapse

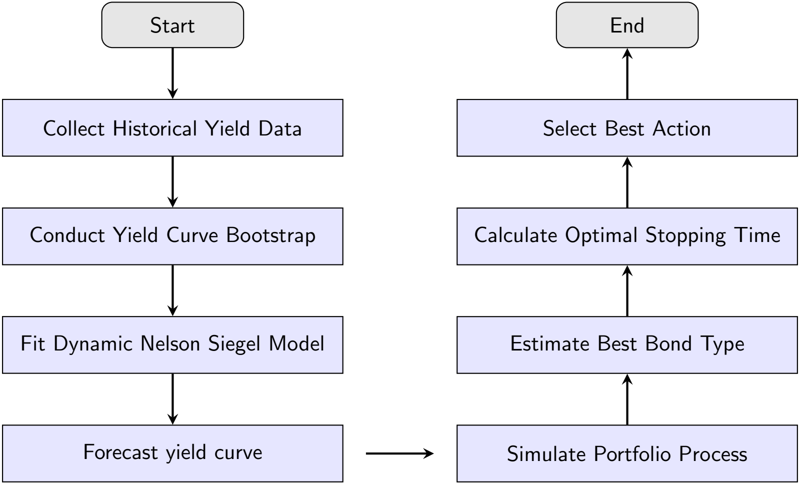
- SVB collapsed in March 2023 due to a **bond strategy mismatch** and rising interest rates, triggering a bank run.
- It heavily invested in **long-term bonds (US Treasuries, MBS)** during low-rate years (2020–2021).
- These were held-to-maturity (HTM), so **losses weren't reflected as rates rose in 2022**.
- **The interest rate hikes reduced bond values**, straining SVB's balance sheet.
- Regulatory capital rules + long-term bond holdings exposed SVB to **liquidity risk and forced loss realization**.

Problem Statement

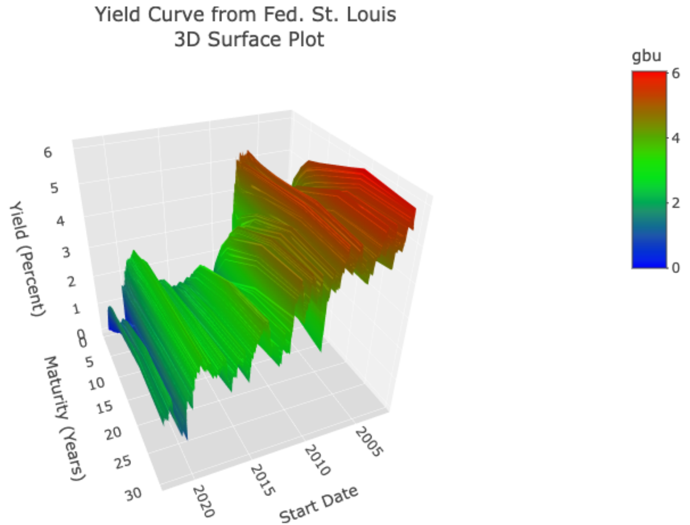
1. Can **dynamic investment strategies** applied to classical bond ladder portfolios improve its performance?
2. How do we develop dynamic **risk-based decision making process** to adjust bond ladder portfolio in real time and **what action** should we consider?

Outline

Methodology



Spot Rate Modeling with the Nelson Siegel Model



Spot Rate Modeling with the Nelson Siegel Model

Dynamic Nelson Siegel Model (DNS):

$$Y_t(\tau_i) = L_t + S_t\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + C_t\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \epsilon_t(\tau_i)$$

In Matrix Form:

$$\mathbf{Y}_t(\boldsymbol{\tau}) = \boldsymbol{\Phi}(\lambda, \boldsymbol{\tau})\mathbf{X}_t + \boldsymbol{\epsilon}_t$$

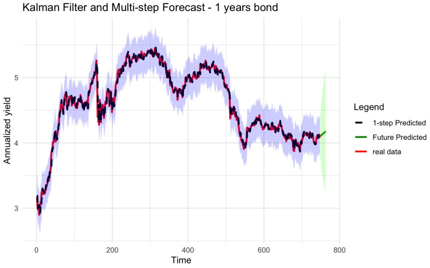
- $\boldsymbol{\tau} \in \mathbb{R}^N$: times to maturity
- $\mathbf{Y}_t(\boldsymbol{\tau}) \in \mathbb{R}^N$: yields to maturity in matrix form
- $\lambda \in \mathbb{R}^+$: decay factor
- $\mathbf{X}_t = [L_t, S_t, C_t]^{\mathrm{T}} \in \mathbb{R}^3$: Level, Slope, Curvature
- $\boldsymbol{\Phi}(\lambda, \boldsymbol{\tau}) \in \mathbb{R}^{N\times 3}$: matrix of basis functions
- $\boldsymbol{\epsilon}_t \overset{i.i.d}{\sim} \mathcal{MVN}(0, \Sigma)$: error term Σ can be assumed to be diagonal $\Sigma = \sigma^2 \cdot \mathbb{I}$ or not

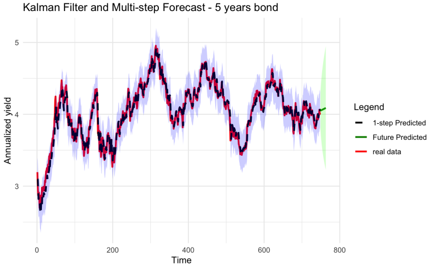
Spot Rate Modeling with the Nelson Siegel Model

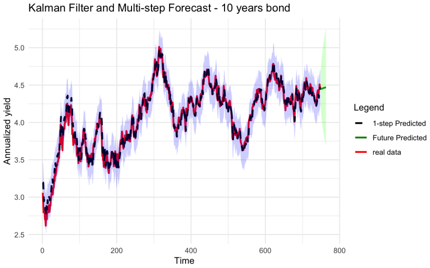
$$\underbrace{\begin{bmatrix} y(\tau_1) \\ y(\tau_2) \\ \vdots \\ y(\tau_n) \end{bmatrix}}_{\mathbf{Y}(\boldsymbol{\tau})} = \underbrace{\begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - e^{-\lambda\tau_n} \end{bmatrix}}_{\boldsymbol{\Phi}(\lambda,\boldsymbol{\tau})} \underbrace{\begin{bmatrix} L \\ S \\ C \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}}_{\boldsymbol{\epsilon}}$$

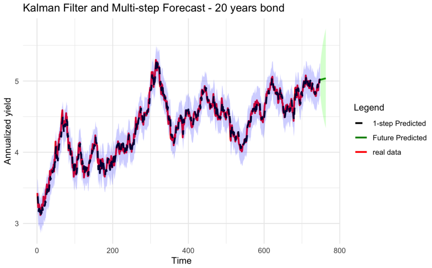
- $\boldsymbol{\tau} \in \mathbb{R}^N$: times to maturity
- $\mathbf{Y}_t(\boldsymbol{\tau}) \in \mathbb{R}^N$: yields to maturity in matrix form
- $\lambda \in \mathbb{R}^+$: decay factor
- $\mathbf{X}_t = [L_t, S_t, C_t]^{\mathrm{T}} \in \mathbb{R}^3$: Level, Slope, Curvature
- $\boldsymbol{\Phi}(\lambda, \boldsymbol{\tau}) \in \mathbb{R}^{N\times 3}$: matrix of basis functions
- $\boldsymbol{\epsilon}_t \overset{i.i.d}{\sim} \mathcal{MVN}(0, \Sigma)$: error term Σ can be assumed to be diagonal $\Sigma = \sigma^2 \cdot \mathbb{I}$ or not
- n: number of different tenors in one single day

Spot Rate Modeling with the Nelson Siegel Model









Data

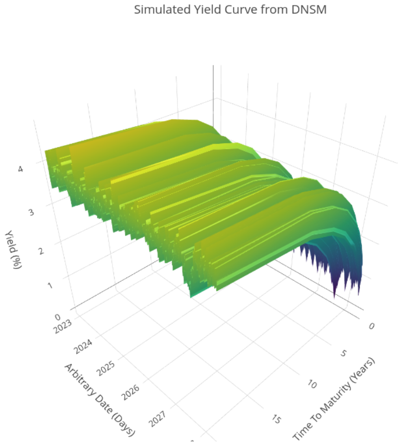


Figure: Bond Yield Curves.

Design of Our Experiment

We define a set of assumptions for our framework.

- **Fixed investment schedule** of one-month periods.
- **Unrestricted borrowing and lending** at an equal risk-free rate r.
- We must make **one action** (other than our static action) per investment period.
- Bonds may bear **coupons**

Investment Actions

- $A_{i,t} = 0$: the state of have **no principle or coupons remaining** to produce yield
- $A_{i,t} = 1$: the state of **continuing to hold** investment position
- $A_{i,t} = 3$: **selling** of any remaining principle and coupons
- $A_{i,t} = 4$: **buying** a bond
- $\boldsymbol{A}_t := \begin{bmatrix} A_{1,t} \\ A_{2,t} \\ \vdots \\ A_{N_t,t} \end{bmatrix}$: a **vector of actions** at time t on bond $1, 2, ..., N_t$
- N_t: the total **number of bonds** after taking an action

Cash Movement h for selling a bond

For selling a bond, the **changes in cash position** for both **deficit** h_1 and **surplus** h_2 are given below.

$$h_{1,t}(\boldsymbol{A}_t) := -\sum_{j=1}^{N_{t-1}} 1_{\{A_{j,t}=3\}} B^c_{t,\tau_j}$$

$$h_{2,t}(\boldsymbol{A}_t) := -\min(0, D_{t-1}e^r + h_{1,t}(\boldsymbol{A}_t))$$

where

- B^c_{t,τ_j} is the cash equivalent to j^{th} bond value at time t
- D_{t-1} is the cash deficit at time $t-1$

Cash Positions

Total Cash Position: The portfolio **total cash position** C_t at time t is defined as

$$C_t := S_t - D_t$$

Deficit Definition: The **deficit** D_t is the level of borrowed capital at time t. The deficit formula is given below.

$$D_t := \begin{cases} 1_{\{C_0 < B_0\}}(B_0 - C_0) \text{ if } t = 0 \\ (D_{t-1}e^r + h_{1,t}(\boldsymbol{A}_t))_+ \text{ if } t > 0 \end{cases}$$

Surplus Definition: The **surplus** S_t is the available cash at time t. The surplus formula is given as follows.

$$S_t := \begin{cases} 1_{\{C_0 > B_0\}}(C_0 - B_0) \text{ if } t = 0 \\ (S_{t-1}e^r + h_{2,t}(\boldsymbol{A}_t))_+ \text{ if } t > 0 \end{cases}$$

Present Value of the k^{th} Bond:

$$B_{t,\tau_k} = \sum_{j=1}^{v_k} \left[c_j e^{-\tau_{k,j} y_t(\tau_{k,j})} 1_{\{\tau_{k,j} > 0\}} \right] \mathcal{G}_1(A_{k,1:t})$$

$$+ F e^{-\tau_k y_t(\tau_k)} 1_{\{\tau_k > 0\}} \mathcal{G}_2(A_{k,1:t})$$

Determining Whether bonds are Still in Effect:

$$\mathcal{G}_1(A_{k,1:t-1}) = \prod_{i=1}^{t} \left| 1_{\{\text{sell (3)}=A_i \text{ or strip (2)}=A_i\}} - 1 \right|$$

$$\mathcal{G}_2(A_{k,1:t}) = \prod_{i=1}^{t} \left| 1_{\{\text{sell (3)}=A_i\}} - 1 \right|$$

Cash Equivalent to the Present Value of the k^{th} Bond

$$B^c_{t,\tau_k} = \sum_{j=1}^{v_k} \left[c_j e^{-\tau_{k,j} y_t(\tau_{k,j})} 1_{\{\tau_{k,j}>0\}} \right] \mathcal{G}^c_1(A_{k,1:t})$$

$$+ F e^{-\tau_k y_t(\tau_k)} 1_{\{\tau_k>0\}} \mathcal{G}^c_2(A_{k,1:t})$$

Determining Whether Cash Equivalents Are Still in Effect:

$$\mathcal{G}^c_1(A_{k,1:t}) = \prod_{i=1}^{t} \left| 1_{\{\text{sell } (3)=A_i \text{ or strip } (2)=A_i \text{ or buy } (4)=A_i\}} \right|$$

$$\mathcal{G}^c_2(A_{k,1:t}) = \prod_{i=1}^{t} \left| 1_{\{\text{sell } (3)=A_i \text{ or buy } (4)=A_i\}} \right|$$

Bond Portfolio Value and Total Portfolio Value

The **bond portfolio value** B_t after taking an investment action at time t is given by,

$$B_t = \sum_{k=1}^{N_t} B_{t,\tau_k}$$

The **total portfolio value** Π_t including both cash and bond bonds after taking an investment action at time t is given by

$$\Pi_t = C_t + B_t$$

Profit/Loss from an action

The **Profit/Loss** W_t from taking an investment action $A_{j,t}$ at time t is given by

$$W_t = \Pi_t\left(\boldsymbol{A}_{1:t-1}, A_{j,t} = a\right) - \Pi_t\left(\boldsymbol{A}_{1:t-1}, A_{j,t} = 1\right) = \Delta C_t^{(a,1)} + \Delta B_t^{(a,1)}$$

where

- $a = 2, 3, 4,$
- $\Delta C_t^{(a,1)}$ is the difference in cash positions from taking the action at time t,
- $\Delta B_t^{(a,1)}$ is the difference in bond values from taking the action at time t.

Accumulating Profit/Loss to time $t + h$

The **accumulating Profit/Loss** $W_{t,t+h}$ from taking an investment action $A_{j,t}$ at time t accumulating at time $t + h$ is given by

$$W_{t,t+h} = \Delta C_t^{(a,1)} e^{rh} + \Delta B_{t,t+h}^{(a,1)}$$

where

- r is the continuously compounding risk-free rate,
- $\Delta C_t^{(a,1)}$ is the difference in cash positions from taking the action at time t,
- $\Delta B_{t,t+h}^{(a,1)}$ is the difference in bond values from taking the action at time t accumulating to time $t + h$.

Multiple Stopping Rule

Gain Function: For a given multiple stopping rule $\boldsymbol{\tau} = (\tau_1, ..., \tau_k)$ the gain function is defined as

$$g(\boldsymbol{\tau}) = W(\tau_1) + ... + W(\tau_k).$$

Value of the Game: Let $\mathcal{S}_m$ be the class of multiple stopping rules $\boldsymbol{\tau} = (\tau_1, ..., \tau_k)$ such that $\tau_1 \geq m$ ($\mathbb{P}$-a.s.). The function

$$v_m = \sup_{\boldsymbol{\tau} \in \mathcal{S}_m} \mathbb{E}[g(\boldsymbol{\tau})]$$

is defined as the m-value of the game and, in particular, if $m = 1$ then v_1 is the value of the game.

Optimal Multiple Stopping Rule: A multiple stopping rule $\boldsymbol{\tau}^* \in \mathcal{S}_m$ is called an optimal multiple stopping rule in $\mathcal{S}_m$ if $\mathbb{E}[W(\boldsymbol{\tau}^*)]$ exists and $\mathbb{E}[W(\boldsymbol{\tau}^*)] = v_m$.

Optimal Multiple Stopping Rule

Let $W(1), \dots, W(T)$ be a sequence of independent r.v.s with distributions $F_1, \dots, F_T$, and a gain function $\sum_{j=1}^{k} W(\tau_j)$. Let $v^{L,l}$ be the value of a game with L ($L \leqslant T$) steps remaining and l ($l \leqslant k$) actions (stops) remaining. If $\mathbb{E}[W(1)], \dots, \mathbb{E}[W(T)]$ exist then the value of the game is given by

$$\begin{aligned}
v^{1,1} &= \mathbb{E}[W(T)], \\
v^{L,1} &= \mathbb{E}\big[\max\{W(T-L+1), v^{L-1,1}\}\big], \quad 1 < L \leq T, \\
v^{L,l+1} &= \mathbb{E}\big[\max\{v^{L-1,l} + W(T-L+1), v^{L-1,l+1}\}\big], \quad l+1 < L \leq T, \\
v^{l,l} &= \mathbb{E}\left[v^{l-1,l-1} + W(T-l+1)\right].
\end{aligned}$$

The **optimal stopping rules** $\tau^* = (\tau_1^*, \dots, \tau_k^*)$ are

$$\begin{aligned}
\tau_1^* &= \min\{m_1 : 1 \leqslant m_1 \leqslant T-k+1, W(m_1) \geqslant v^{T-m_1,k} - v^{T-m_1,k-1}\}, \\
\tau_i^* &= \min\{m_i : \tau_{i-1}^* < m_i \leqslant T-k+i, W(m_i) \geqslant v^{T-m_i,k-i+1} - v^{T-m_i,k-i}\}, \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i = 2, \dots, k-1, \\
\tau_k^* &= \min\{m_k : \tau_{k-1}^* < m_k \leqslant T, W(m_k) \geqslant v^{T-m_k,1}\}
\end{aligned}$$

Optimization Criteria

The version of the multiple optimal stopping rule developed for bond ladders restrict to **one action per investment period**: sell a bond prior to maturity; or strip a bond of coupons; or do nothing.

- Hence only consider **value functions** $v^{L,1}$ per investment period:

$$v^{1,1} = \mathbb{E}[W(T)],$$
$$v^{L,1} = \mathbb{E}\big[\max\{W(T-L+1), v^{L-1,1}\}\big].$$

  where L is then the remaining days until the end of the investment period.

- The iterative procedure developed endeavors to find the:

  1. best bond
  2. optimal stopping-time
  3. and earliest action

  which **maximizes portfolio profit** $W_{t,t+h}$ as of some future date. In this study, this future date corresponds to dates before bond purchases are scheduled.

Step 1 : Acumulating Profit/Loss $W_{s,T}^{(i,j)}$ Calculation

1. Simulate N Monte Carlo samples from the Nelson Siegel model with origin t and forecast horizon $t + h_j$
2. Identify the $j^t h$ bond yield at that time origin and evaluate the change in the portfolio value if that bond has an action applied given an investment action a (Sell/Strip) using:

$$W_{t,t+h_j}^{(i,j)} = \Delta C_s^{(a,1,i,j)} e^{r(h_j)} + \Delta B_{t,t+h_j}^{(a,1,i,j)},$$

where

- $i = 1, 2, 3, ..., N$,
- t is the date for which an action is taken,
- h_j is the waiting time until the next investment schedule.

Step 2 : Calculate the Monte Carlo estimate of the value function for the j^{th} bond action

1. Calculate

$$\widehat{v}_j^{1,1} = \frac{1}{N}\sum_{i=1}^{N} W_j^{(i,j)},$$

$$\widehat{v}_{t,t+h_j}^{t+h_j,1} = \frac{1}{N}\sum_{i=1}^{N} \max\{W_{t,t+h_j}^{(i,j)}, \widehat{v}_j^{t+h_j-1,1}\}$$

where

- $i = 1, 2, 3, ..., N$,
- t is the date for which an action is taken,
- h_j is the waiting time until the next investment schedule.

Step 3 : Select the optimal bond for taking action

1. Solve

$$j^* = \underset{j \in \{1,2,\ldots,N_t\}}{\arg\max} \left\{ \widehat{V}_{t,t+h_1}^{t+h_1,1}, \widehat{V}_{t,t+h_2}^{t+h_2,1}, \ldots, \widehat{V}_{t,t+h_{N_t}}^{t+h_{N_t},1} \right\}$$

where

- $i = 1, 2, 3, \ldots, N$,
- t is the date for which an action is taken,
- h_j is the waiting time until the next investment schedule.

Step 4 : Select the optimal time to take an investment action

The optimal stopping rules $\tau^* = (\tau_1^*, \ldots, \tau_k^*)$ are

$$\begin{aligned}
\tau_1^* &= \min\{m_1 : 1 \leqslant m_1 \leqslant t + h_j - k + 1, W(m_1) \geqslant v^{t+h_j-m_1,k} - v^{t+h_j-m_1,k-1}\}, \\
\tau_i^* &= \min\{m_i : \tau_{i-1}^* < m_i \leqslant t + h_j - k + i, W(m_i) \geqslant v^{t+h_j-m_i,k-i+1} - v^{t+h_j-m_i,k-i}\}, \\
\tau_k^* &= \min\{m_k : \tau_{k-1}^* < m_k \leqslant t + h_j, W(m_k) \geqslant v^{t+h_j-m_k,1}\}
\end{aligned}$$

where $i = 2, \ldots, k-1$.

Multiple Stopping Rule

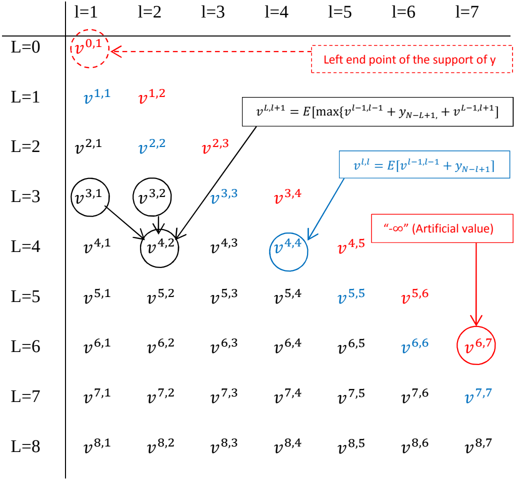


Figure: Value function $v^{L,l}$: L (steps remaining) and l (stops remaining).

A Quick Look at Our Architecture

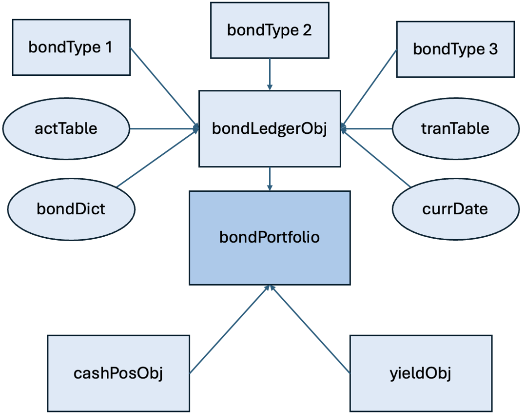


Figure: Portfolio Process Object Oriented Diagram

Simulation Study

We conduct two case studies on synthetic data generated on Nelson Siegel Model Parameters calibrarted to US Treasury Curve.
Assume the following when comparing classical Static non-adaptive Bond Ladder (static) to Dynamic Adpative Bond Ladder (Adaptive)

- start with cash position composed of \$100,000
- purchase two 11 year bonds with 3% quarterly coupons and \$10,000 principal to start and two 10 year bonds at the end of each month with the same coupon rate, period, and principal
- set a risk-free rate of 3% for borrowing and lending
- allow selling and stripping in addition to static and synthetic actions

Note that the dates shown are arbitrary.

Outline

Portfolio Cumulative Return

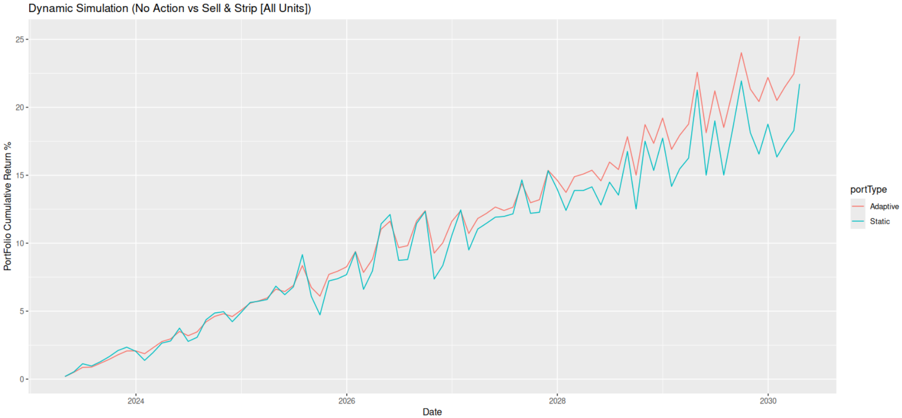


Figure: Cumulative Return (Dynamic vs Classical).

Portfolio Return

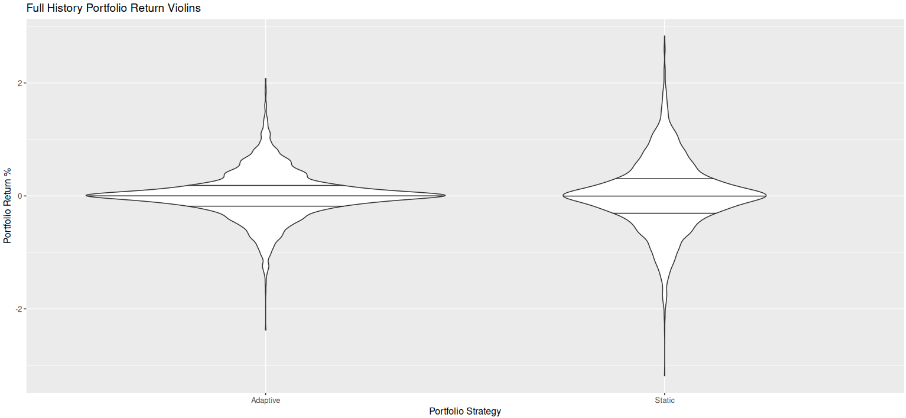


Figure: Full History Portfolio Return Violins (Dynamic (left) vs Classical (right)).

Portfolio Maximum Draw Down

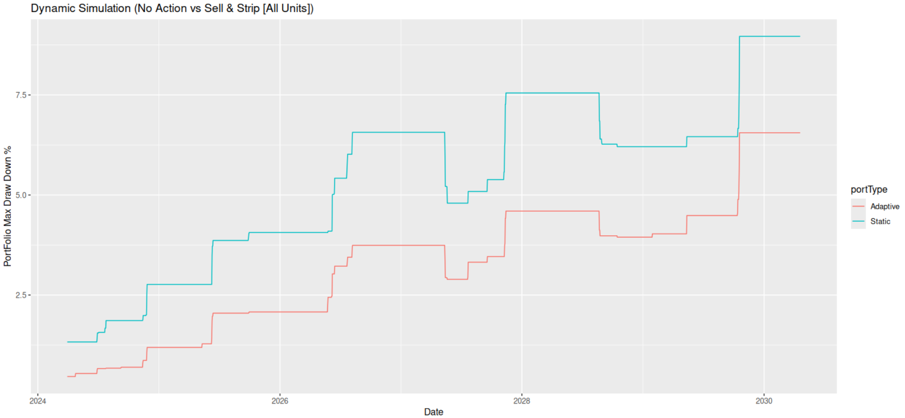


Figure: Portfolio Maximum Draw Down (Dynamic vs Classical).

Portfolio Value at Risk

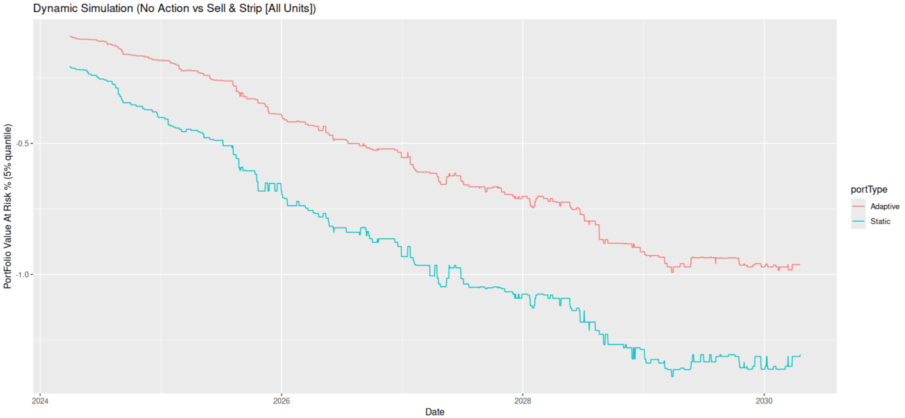


Figure: Portfolio Value at Risk (Dynamic vs Classical).

Outline

Conclusions

- The dynamic bond laddered portfolio outperforms the classical one.
- The dynamic bond laddered portfolio offers more stability and flexibility
- The optimization method improves capital efficiency
- It is a useful tool for both individual and institutional investors

Q&A

Thank You!

Questions are welcome.