DAV / DGVFM
Jahrestagung
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DAV
Deutsche Aktuarvereinigung e.V.

DGVFM
Deutsche Gesellschaft für Versicherungs- und Finanzmathematik e.V.

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Burning Cost or Pareto?

Jahrestagung, 29. April 2025

Agenda

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Reinsurer: $$\hat{S} = \sum_{i=1}^{N} \min(C, (X_i - D)^+) =: \sum_{i=1}^{N} \hat{X}_i, \qquad (x^+ := \max(x, 0))$$

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- Purpose of a per risk XL: Protection against large losses.

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We assume that the volumes and the losses have been *indexed* to the cost level of the prospective treaty year q.

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Under these assumptions, the BC estimator is unbiased! The assumptions are plausible if indexation works perfectly and if the v_i are good volumes.

Agenda

1. Burning cost for per risk XLs
2. Pareto based pricing approaches
3. Simulation model for evaluating pricing approaches
4. Observations with underlying Pareto severity
5. Observations with other underlying severity distributions

Pareto distribution

Die Pareto distribution Pareto(t, α) is the most widely used severity distribution for modelling large losses. CDF:

$$F(x) = 1 - \left(\frac{t}{x}\right)^{\alpha}, \quad x \geq t.$$

Proposition:

- Let $X \sim \text{Pareto}(t, \alpha)$. Then $cX \sim \text{Pareto}(ct, \alpha)$ for all $c > 0$.

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Due to these properties, the Pareto distribution is very useful for comparing tails. It is often possible to provide 'market alphas' for various segments.

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Proposition (Maximum likelihood estimation of α):

Let $t > 0$ and $X_1, \ldots, X_n \sim \text{Pareto}(t, \alpha)$. The *maximum likelihood estimator (MLE)* for α is given by

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The bias corrected MLE $\widehat{\alpha}^{*}$ is unbiased and has a lower variance than $\widehat{\alpha}^{\text{ML}}$. Hence, $\widehat{\alpha}^{*}$ is a better estimator than $\widehat{\alpha}^{\text{ML}}$.

M. Rytgaard (1990) Estimation in the Pareto Distribution, ASTIN Bullletin: The Journal of the IAA, 20(2), pp. 201–216

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Proposition (layer mean): Let $X \sim \text{Pareto}(t, \alpha)$. For a layer C xs D with $D \geq t$ we then have:

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Proposition (Pareto extrapolation): Let $\sum_{n=1}^{N} X_n$ be a collective model with $X_n \sim X \sim \text{Pareto}(t, \alpha)$. For layers C_i xs D_i with $D_i \geq t$ we then have

$$\Phi_{C_1 \text{ xs } D_1}^{C_2 \text{ xs } D_2}(\alpha) := \frac{\text{Exp. loss in } C_2 \text{ xs } D_2}{\text{Exp. loss in } C_1 \text{ xs } D_1} = \begin{cases} \frac{(C_2 + D_2)^{1-\alpha} - D_2^{1-\alpha}}{(C_1 + D_1)^{1-\alpha} - D_1^{1-\alpha}} & \text{if } \alpha \neq 1 \\ \frac{\ln(C_2 + D_2) - \ln(D_2)}{\ln(C_1 + D_1) - \ln(D_1)} & \text{if } \alpha = 1 \end{cases}$$

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The expected loss in C xs D (before AAD/AAL) is then estimated by

$$\widehat{\mathrm{E}}_{C \text{ xs } D}^{\mathrm{PP}} := \widehat{\lambda}\, v_q \cdot \mathrm{E}(\min(C; (X_{q,1} - D)^{+})).$$

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- Case $T \in (D, C + D)$: Use Pareto extrapolation for the layer part above T and BC for the layer part below T

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Rules of thumb for the selection of T:

- T = largest loss
- T = 3rd larges loss
- $T = \ldots$

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Apply the pricing approaches for each of the simulated observation periods

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Apply the pricing approaches for each of the simulated observation periods

↓

Calculate the squared deviation between the expected layer loss of the underlying model and the estimators of the pricing models

Simulation model for evaluating pricing approaches

Choose underlying model (‘reality’)

↓

Simulate (for instance) 20,000 observation periods with the underlying model

↓

Apply the pricing approaches for each of the simulated observation periods

↓

Calculate the squared deviation between the expected layer loss of the underlying model and the estimators of the pricing models

↓

Calculate the square roots of the average squared deviations to estimate the standard errors (RMSE) of the pricing methods

Agenda

1. Burning cost for per risk XLs
2. Pareto based pricing approaches
3. Simulation model for evaluating pricing approaches
4. Observations with underlying Pareto severity
5. Observations with other underlying severity distributions

Simulation settings

For simplicity, we use the assumption

$$v_q = \sum_{i=1}^{n} v_i = 1.$$

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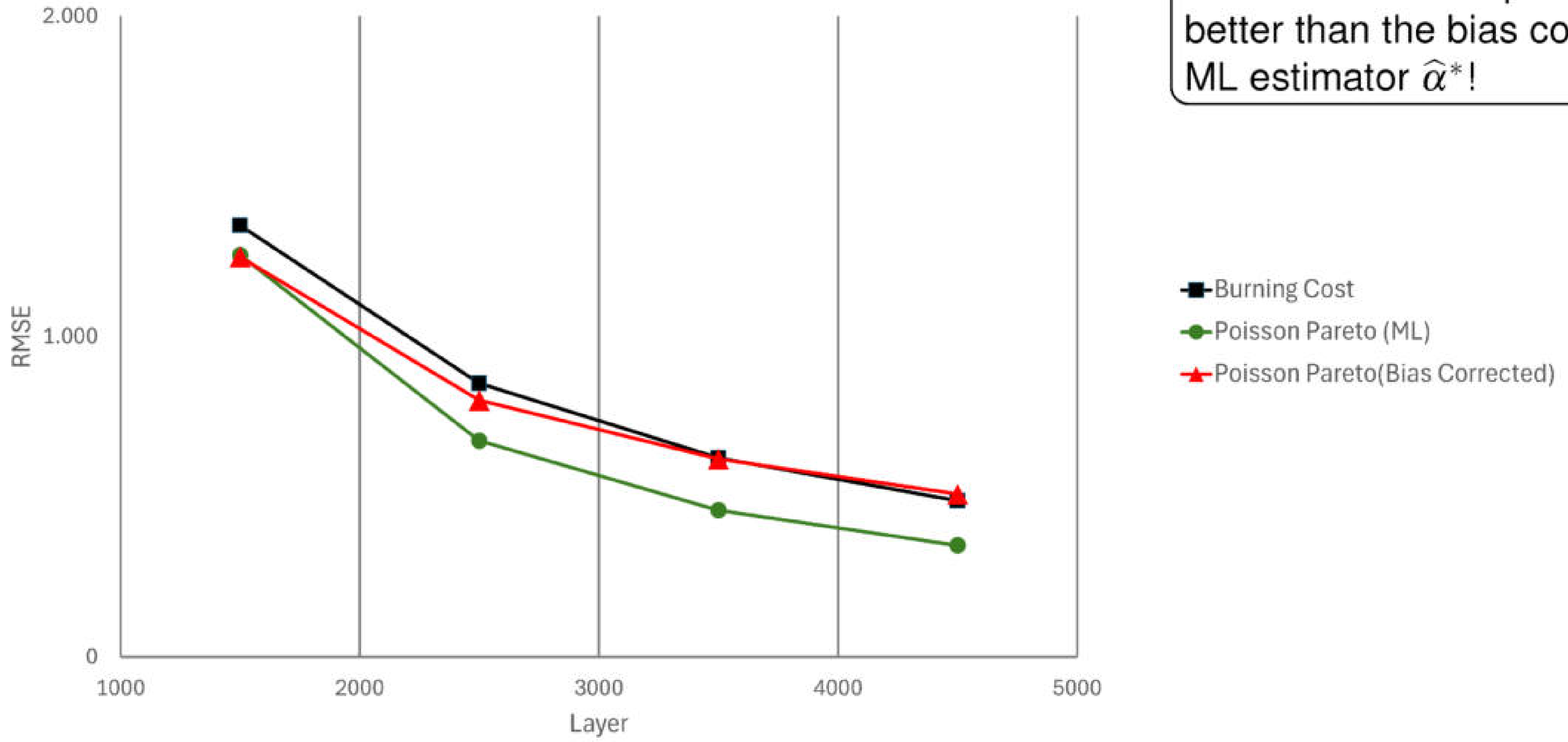
and use the pricing approaches to estimate the expected losses of the following layers:

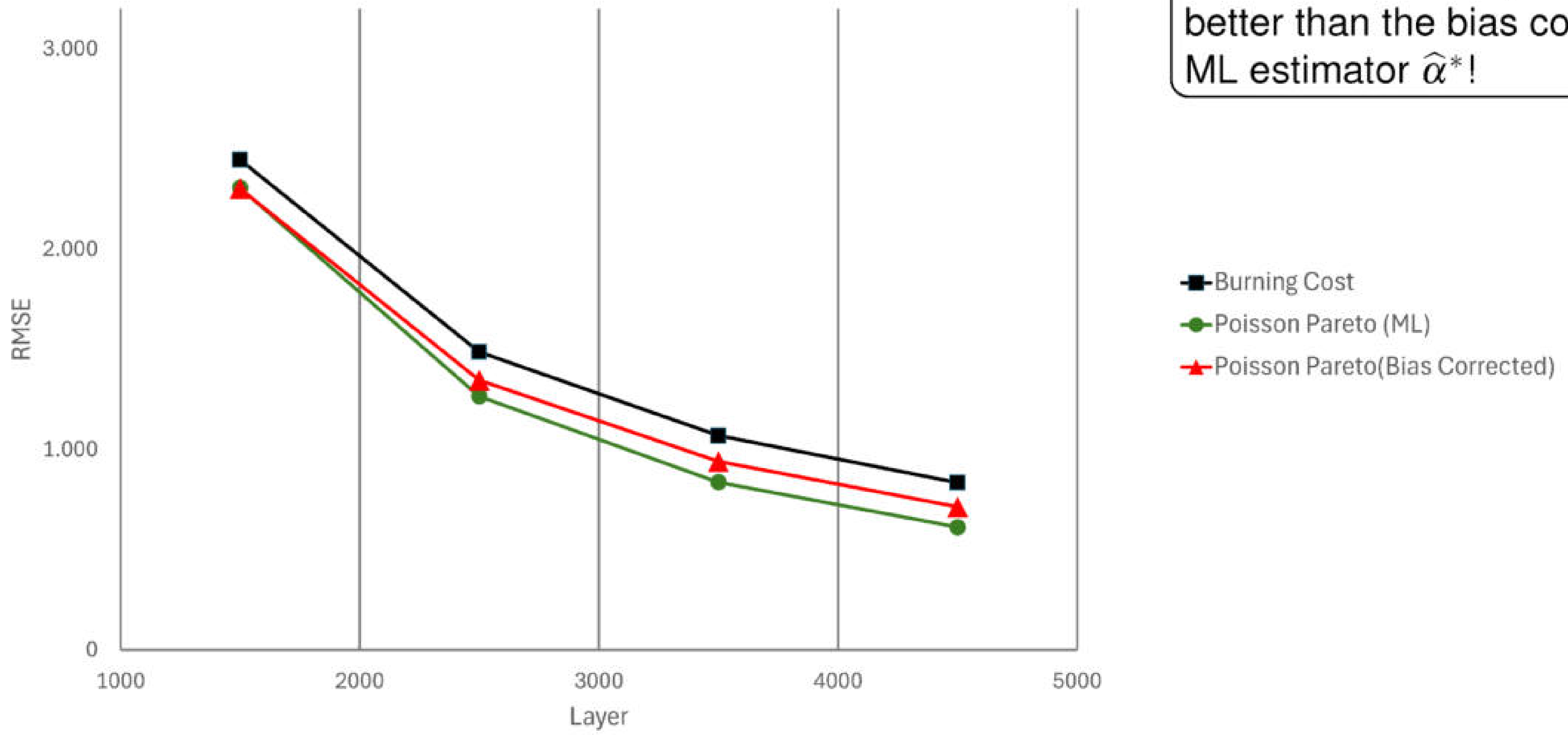
- 1,000 xs 1,000
- 1,000 xs 2,000
- 1,000 xs 3,000
- 1,000 xs 4,000

RMSE with $\lambda = 5$, $\alpha = 2.0$

ML estimator $\widehat{\alpha}^{ML}$ performs better than the bias corrected ML estimator $\widehat{\alpha}^{*}$!



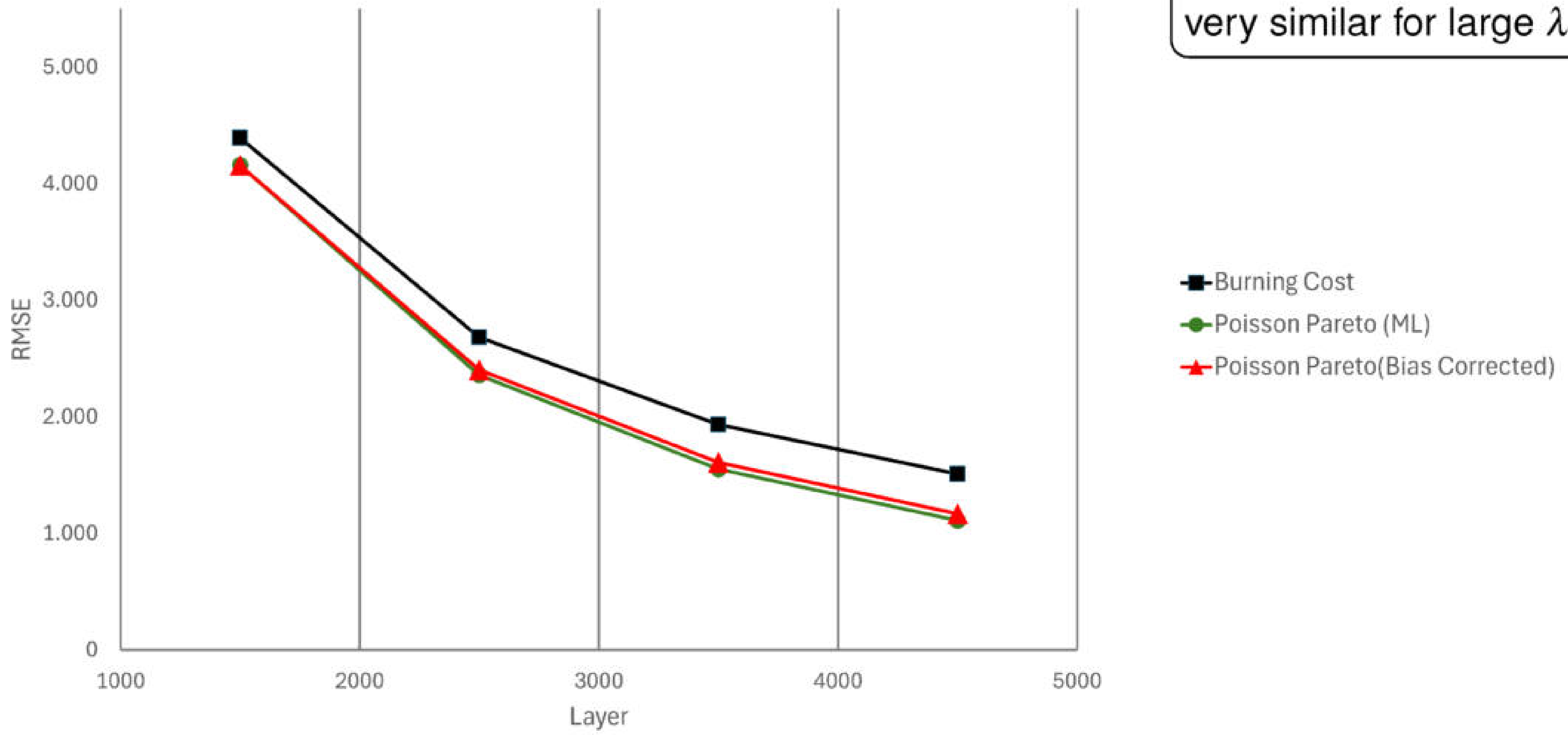
RMSE with $\lambda = 15$, $\alpha = 2.0$



ML estimator $\widehat{\alpha}^{\mathrm{ML}}$ performs better than the bias corrected ML estimator $\widehat{\alpha}^{*}$!

RMSE with $\lambda = 50$, $\alpha = 2.0$

The estimators $\widehat{\alpha}^{\mathrm{ML}}$ and $\widehat{\alpha}^{*}$ are very similar for large λ.



Observation 1

The Pareto based methods seem to perform better than the Burning Cost.

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This is not surprising, as the underlying distribution is Pareto . . .

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The ML estimator $\widehat{\alpha}^{\mathrm{ML}}$ typically performs better than the bias corrected ML estimator $\widehat{\alpha}^{*}$.

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Explanation:

Let $X_\alpha \sim \mathrm{Pareto}(t, \alpha)$ with $t \le D$. Then the function

$$\alpha \mapsto \mu(\alpha) := \mathrm{E}(\min(C, (X_\alpha - D)^+)$$

is convex!

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DAV Deutsche Aktuarvereinigung e.V. DGVFM Deutsche Gesellschaft für Versicherungs- und Finanzmathematik e.V.

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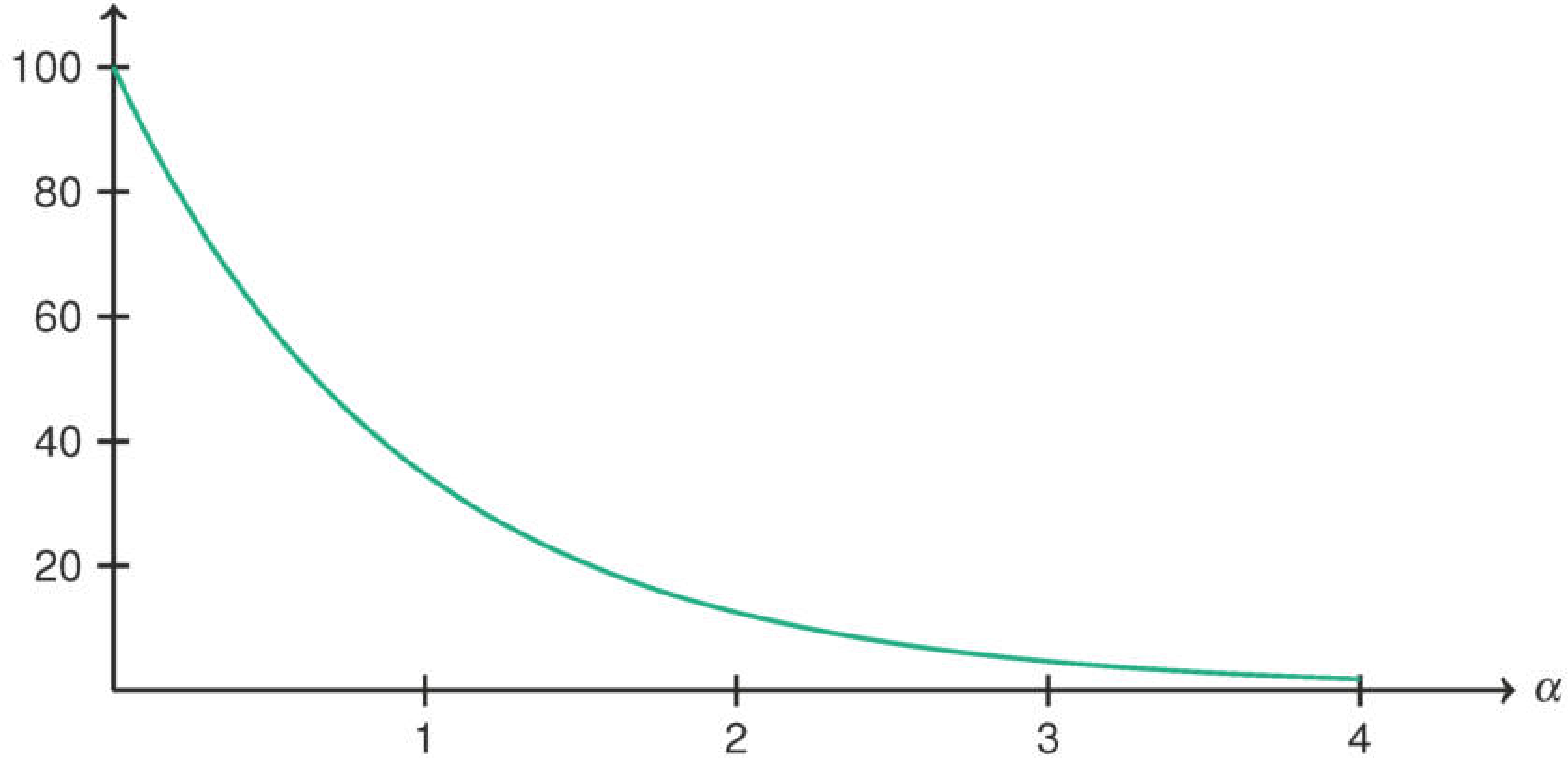
$\Rightarrow$ Estimation of the expected layer loss with $\widehat{\alpha}^*$ has a positive bias!

The ML estimator $\widehat{\alpha}^{\mathrm{ML}}$ results in lower estimates for the expected layer loss, which compensates this positive bias to some extent!

Jensen's Inequality

$\mu(\alpha) = \mathrm{E}(\min(C, (X_\alpha - D)^+)$



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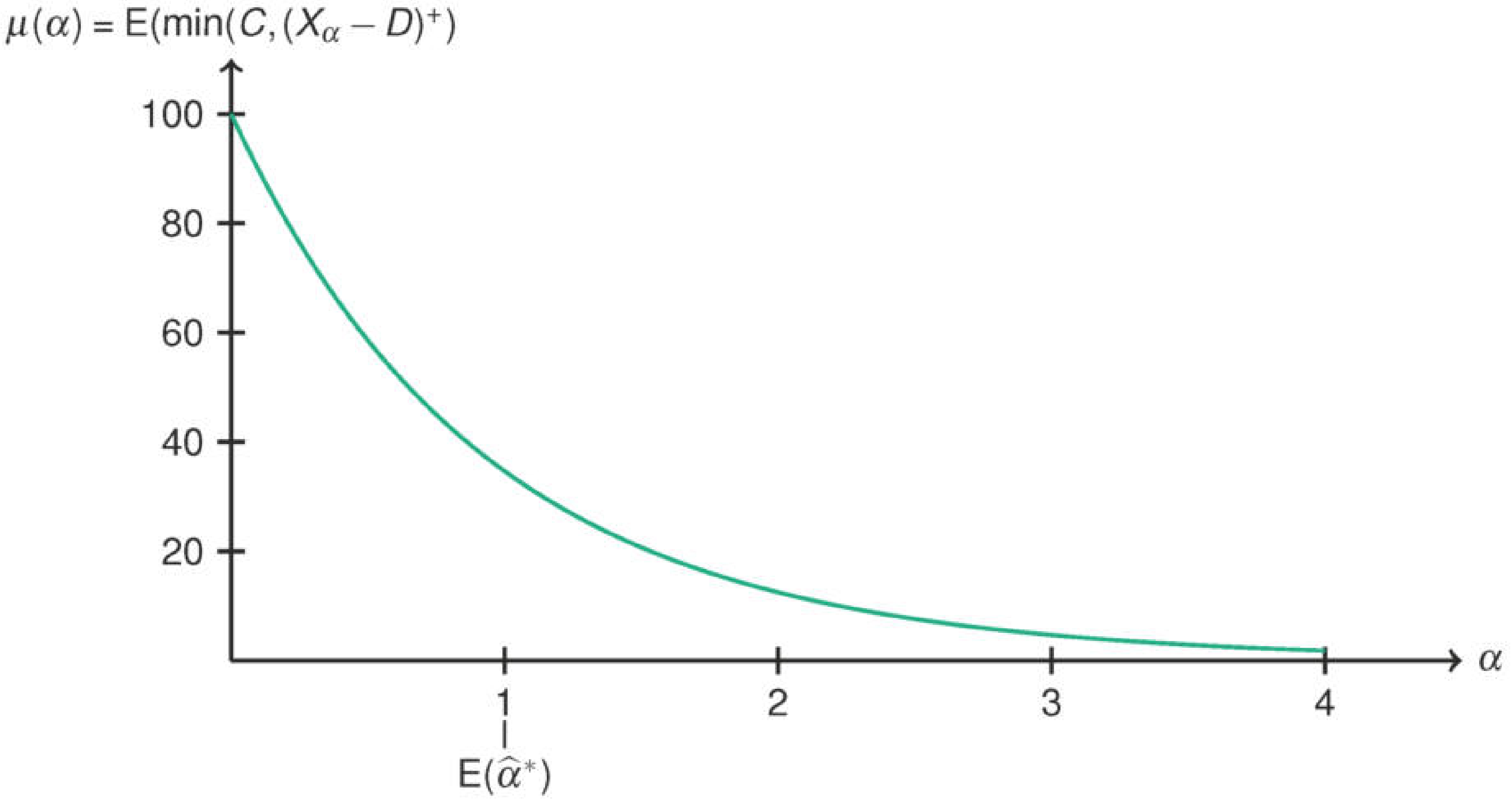

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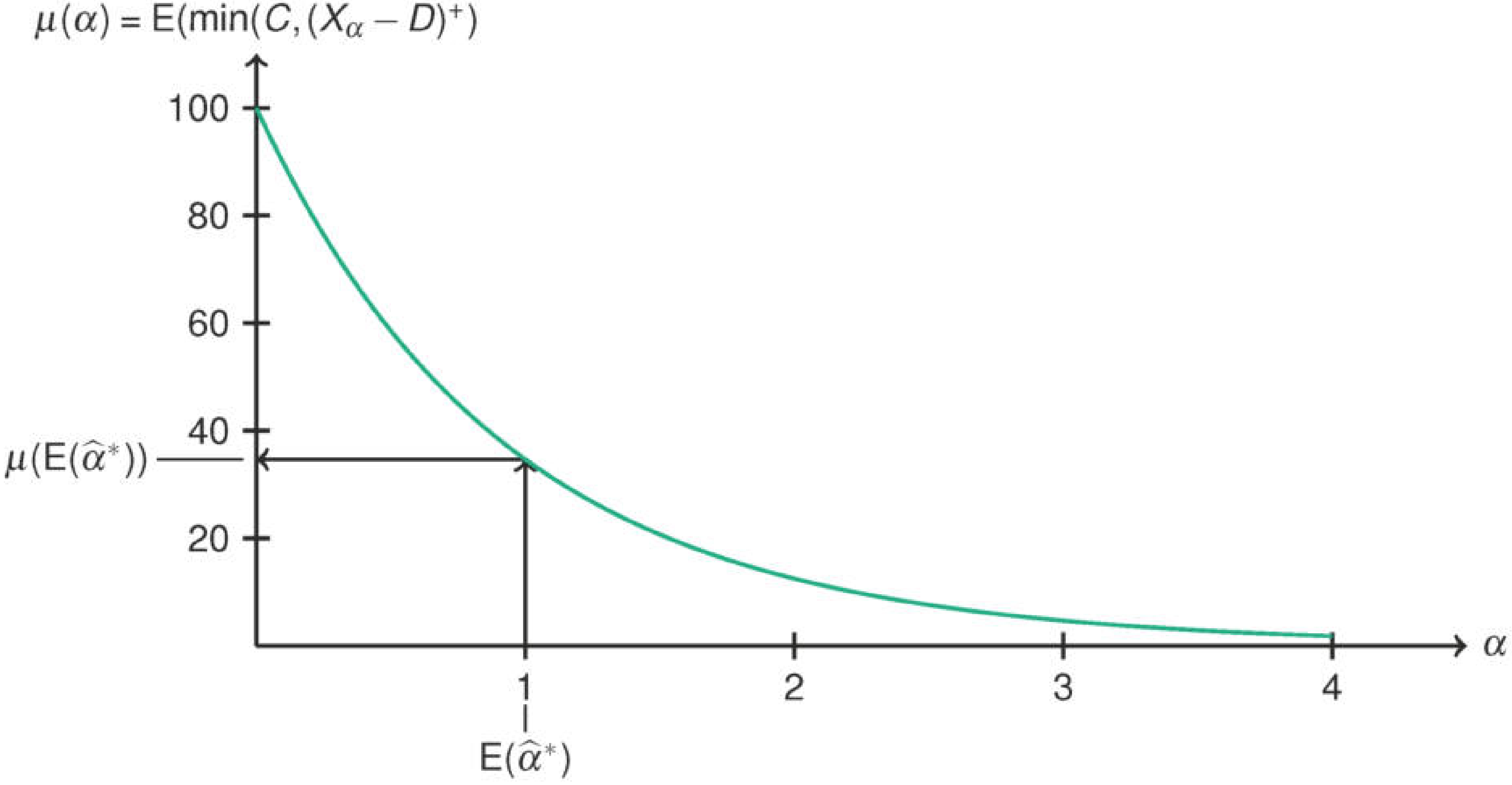

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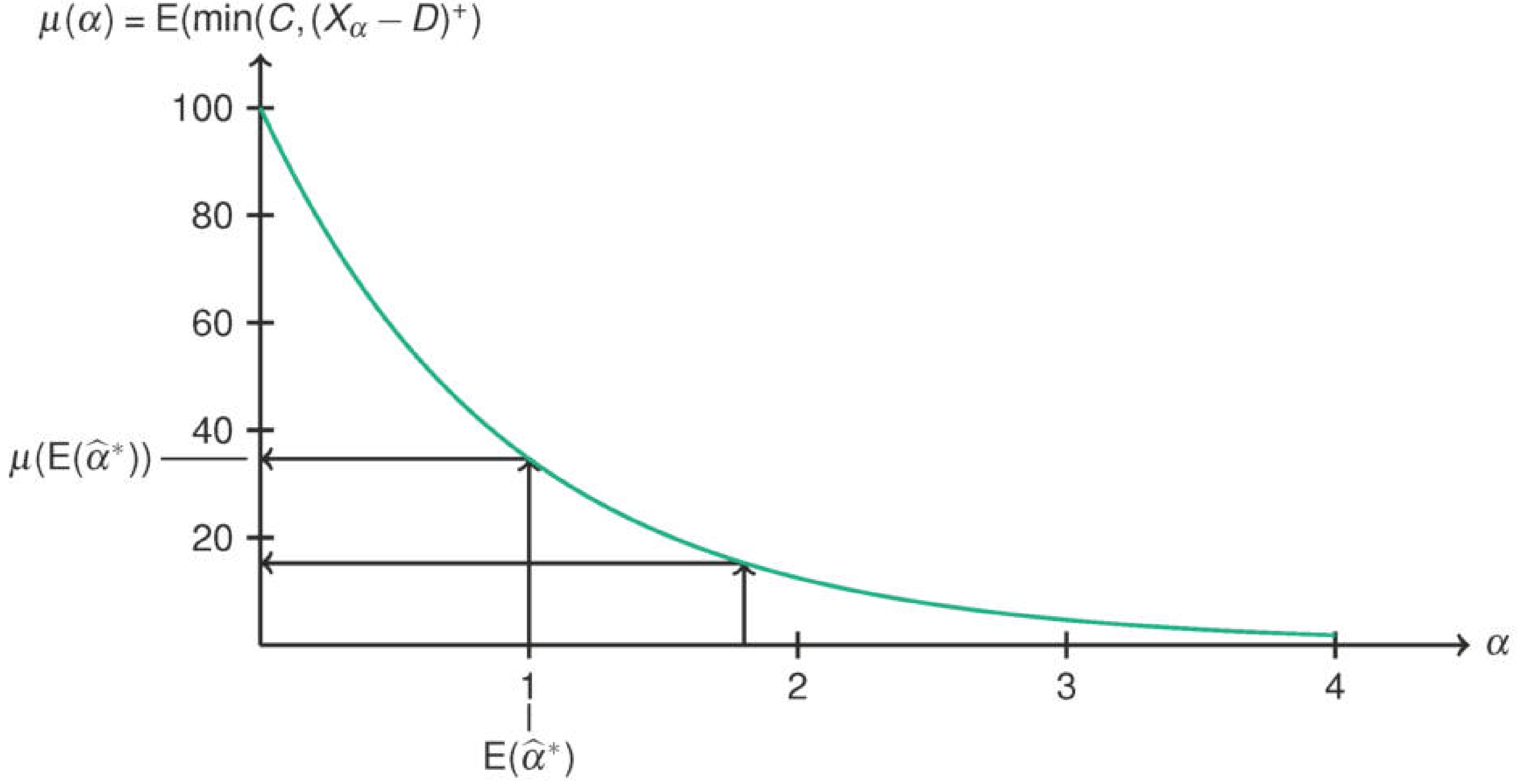

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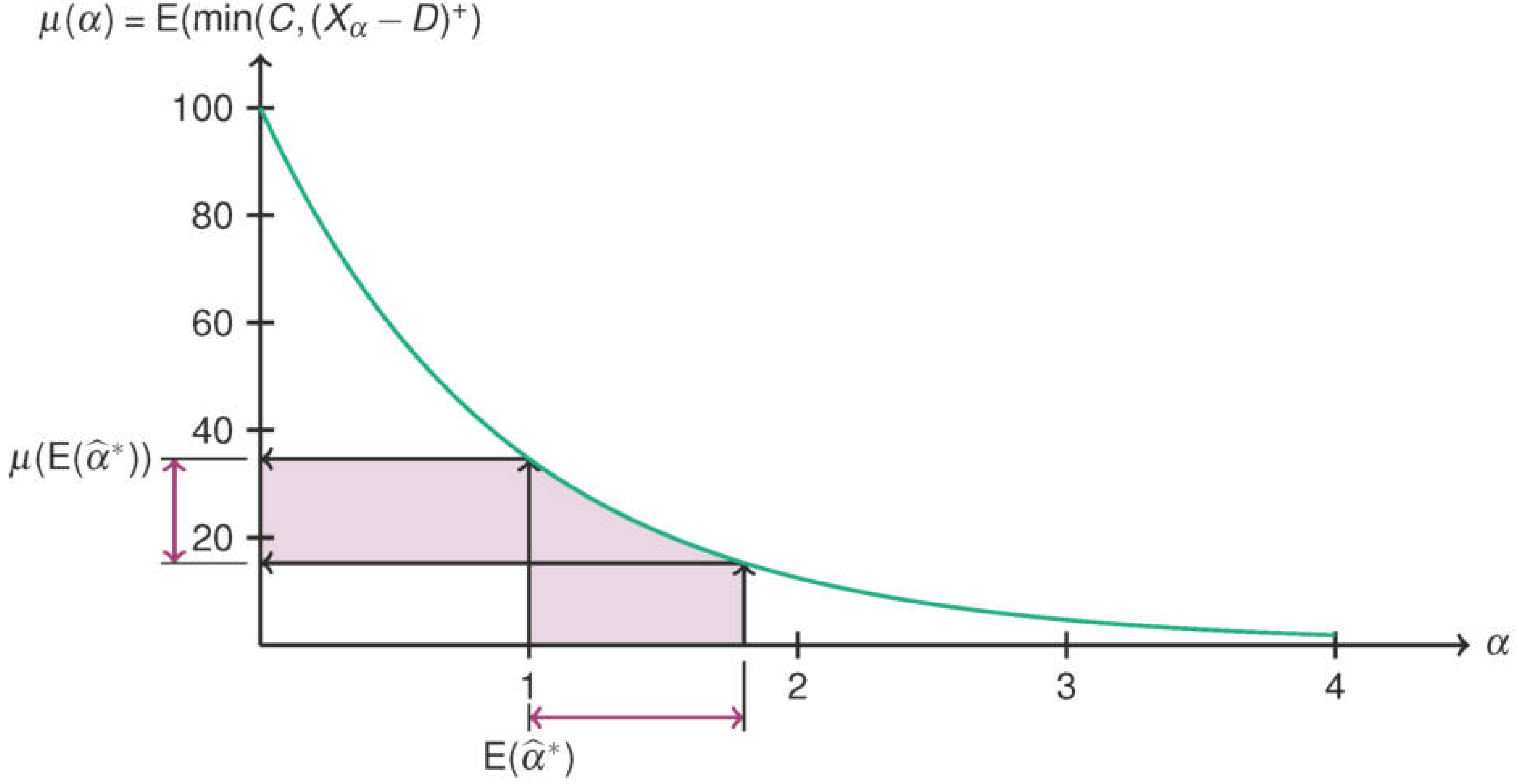

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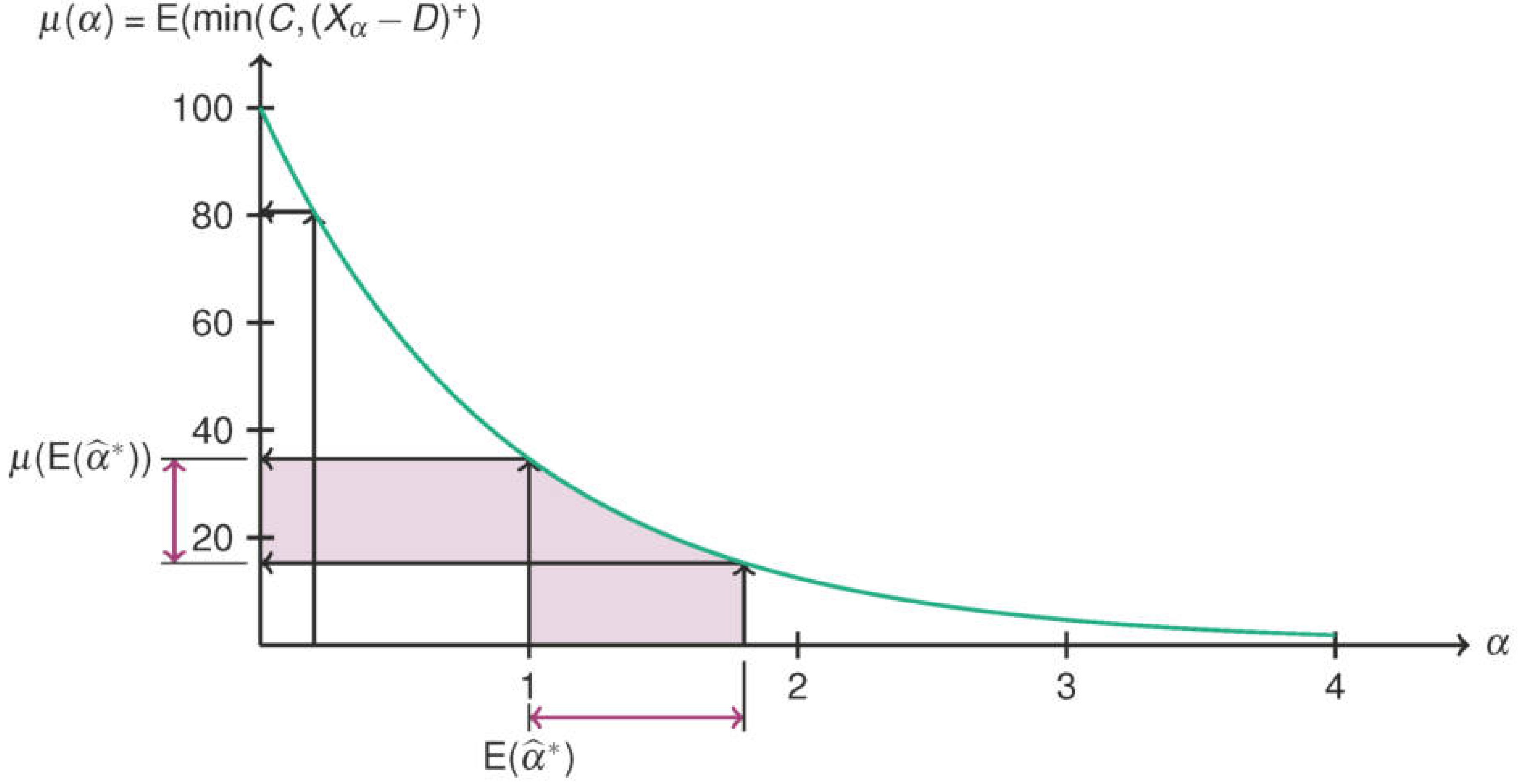

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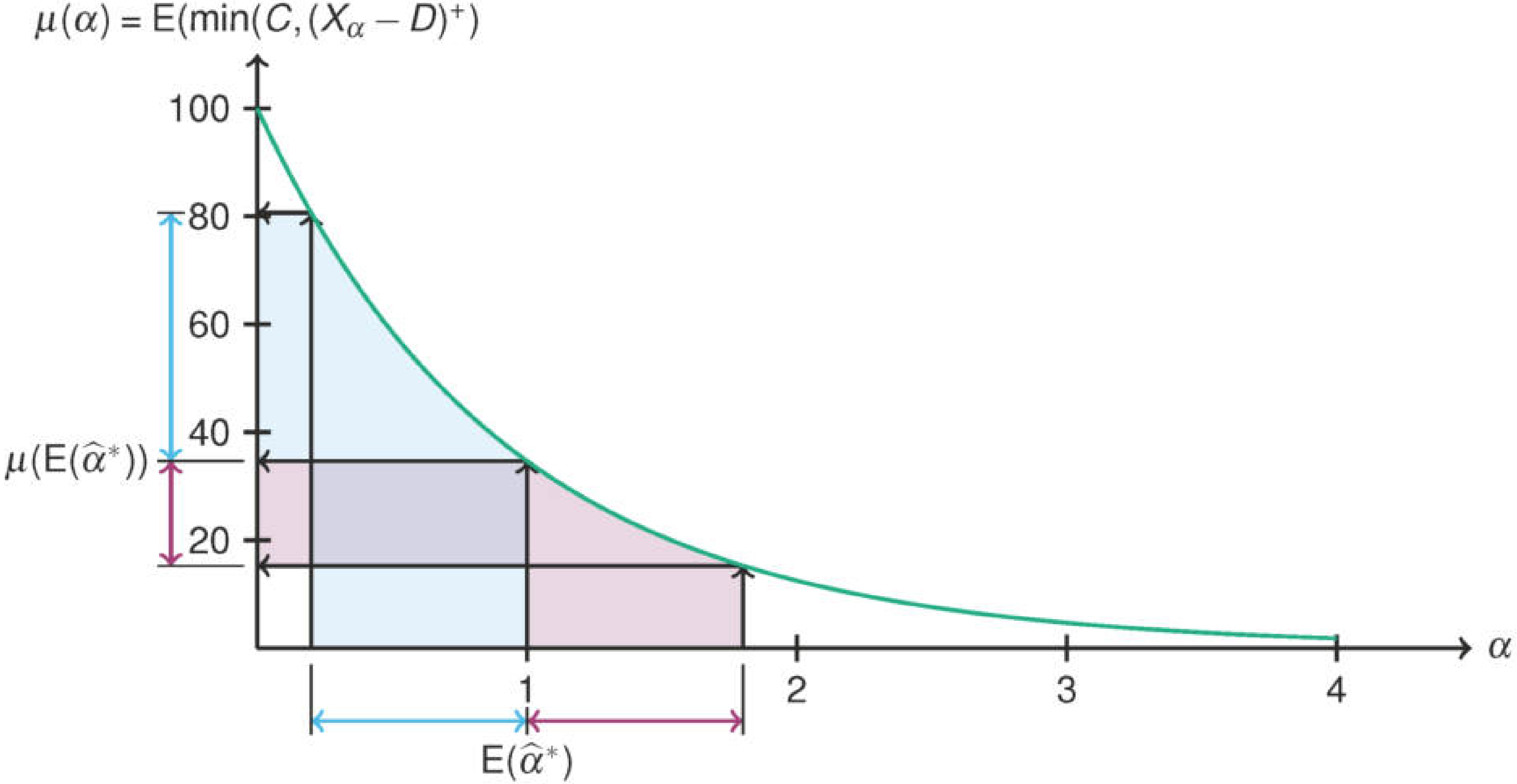

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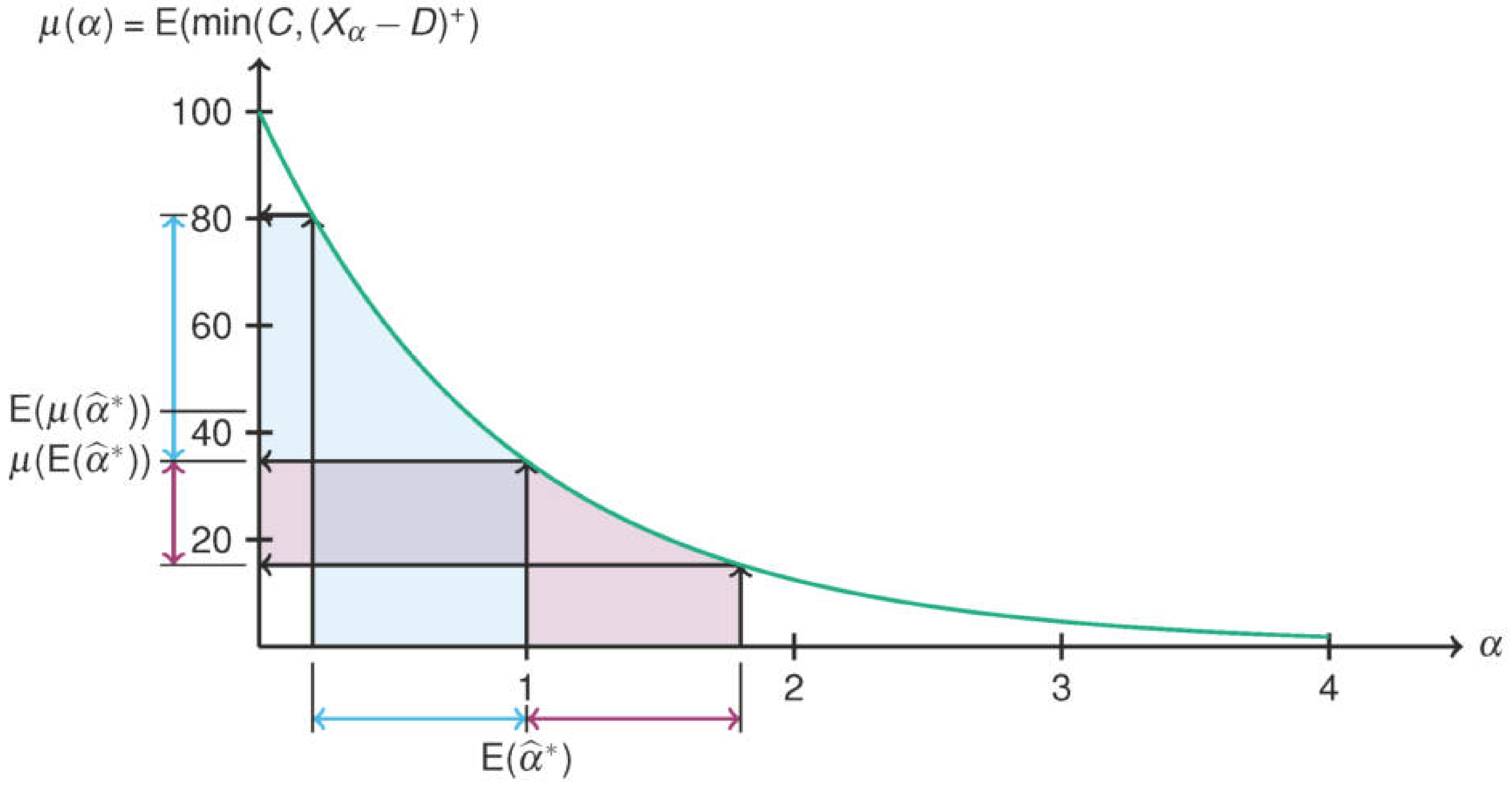
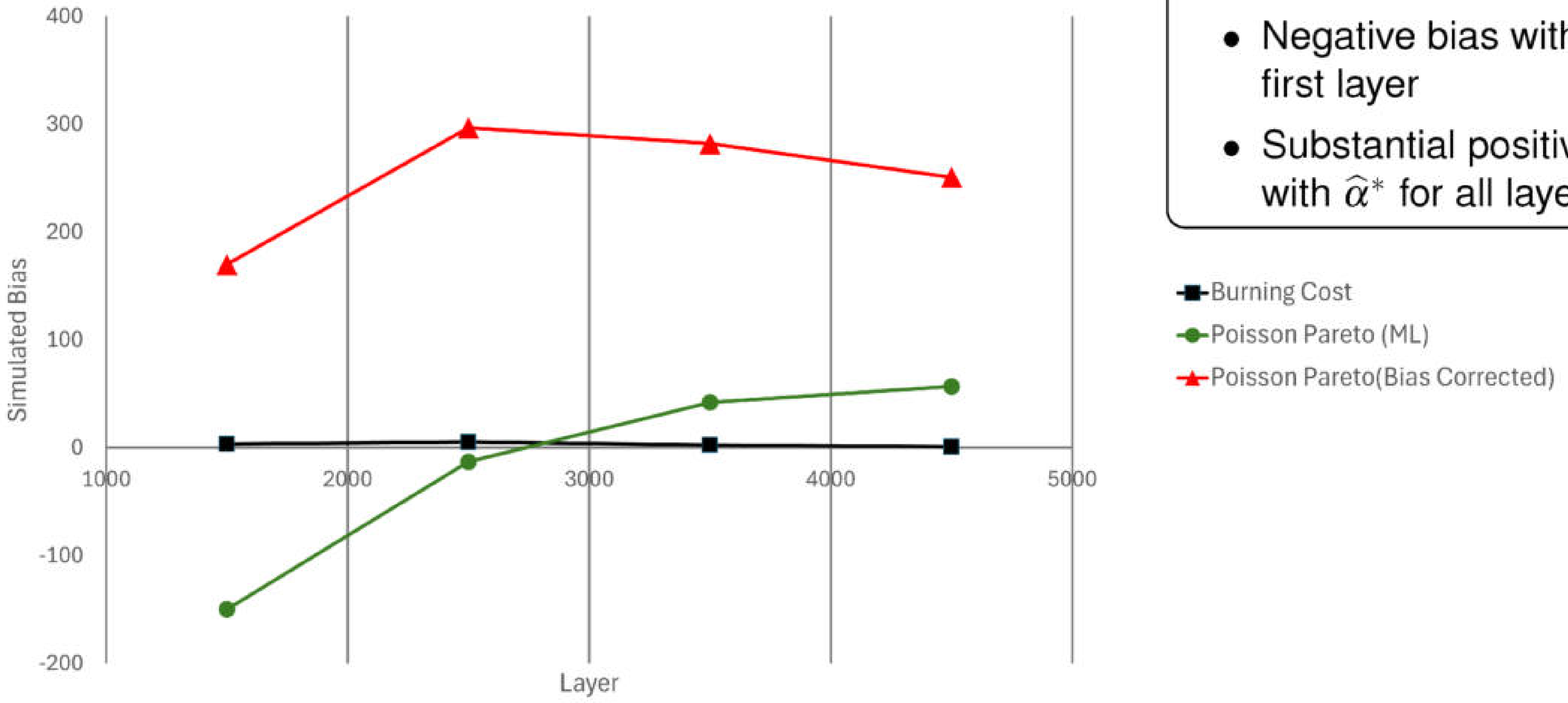
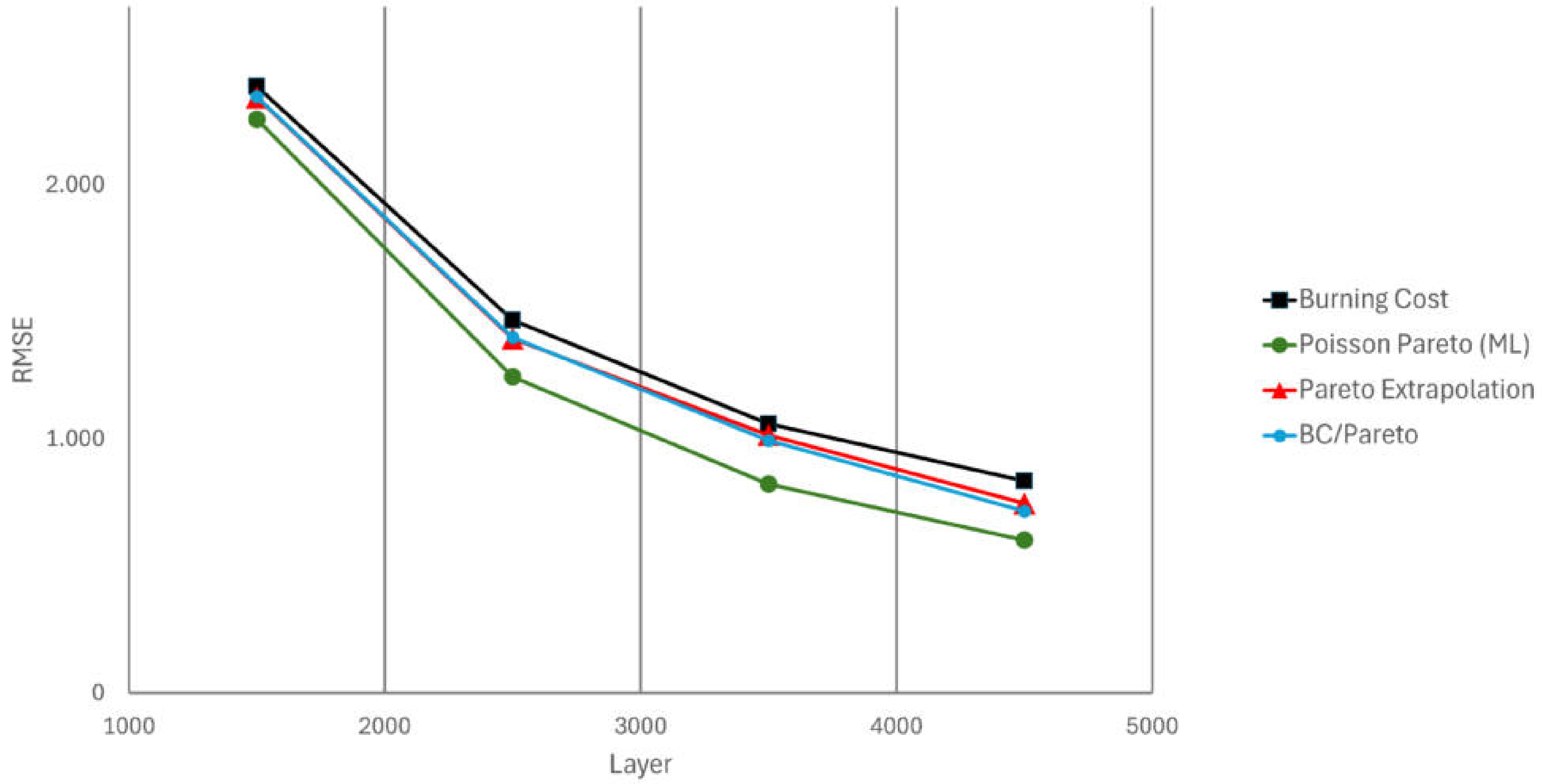


Bias in the case $\lambda = 5$, $\alpha = 2.0$

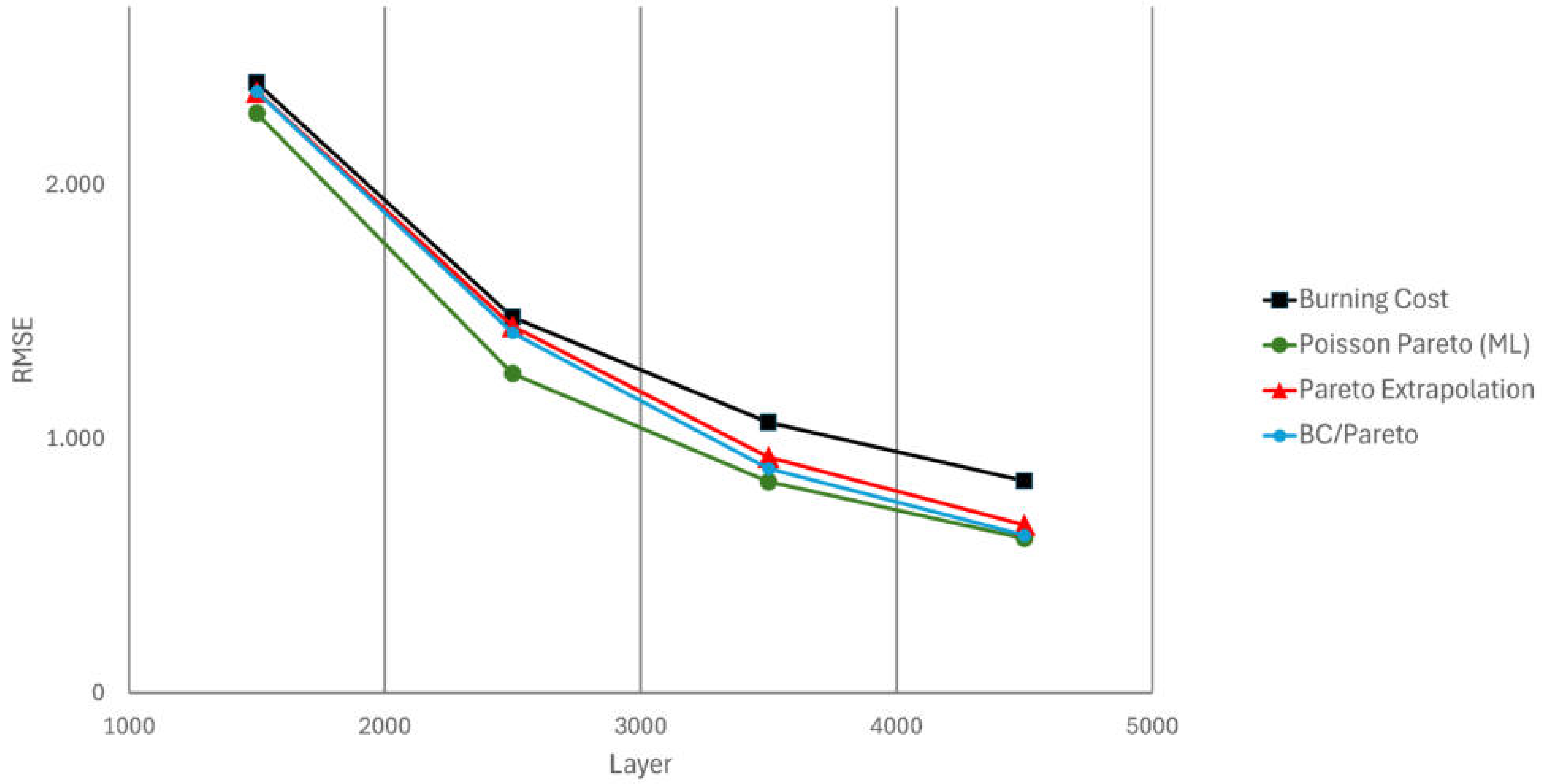


- BC has no bias
- Negative bias with $\widehat{\alpha}^{\mathrm{ML}}$ in first layer
- Substantial positive bias with $\widehat{\alpha}^{*}$ for all layers

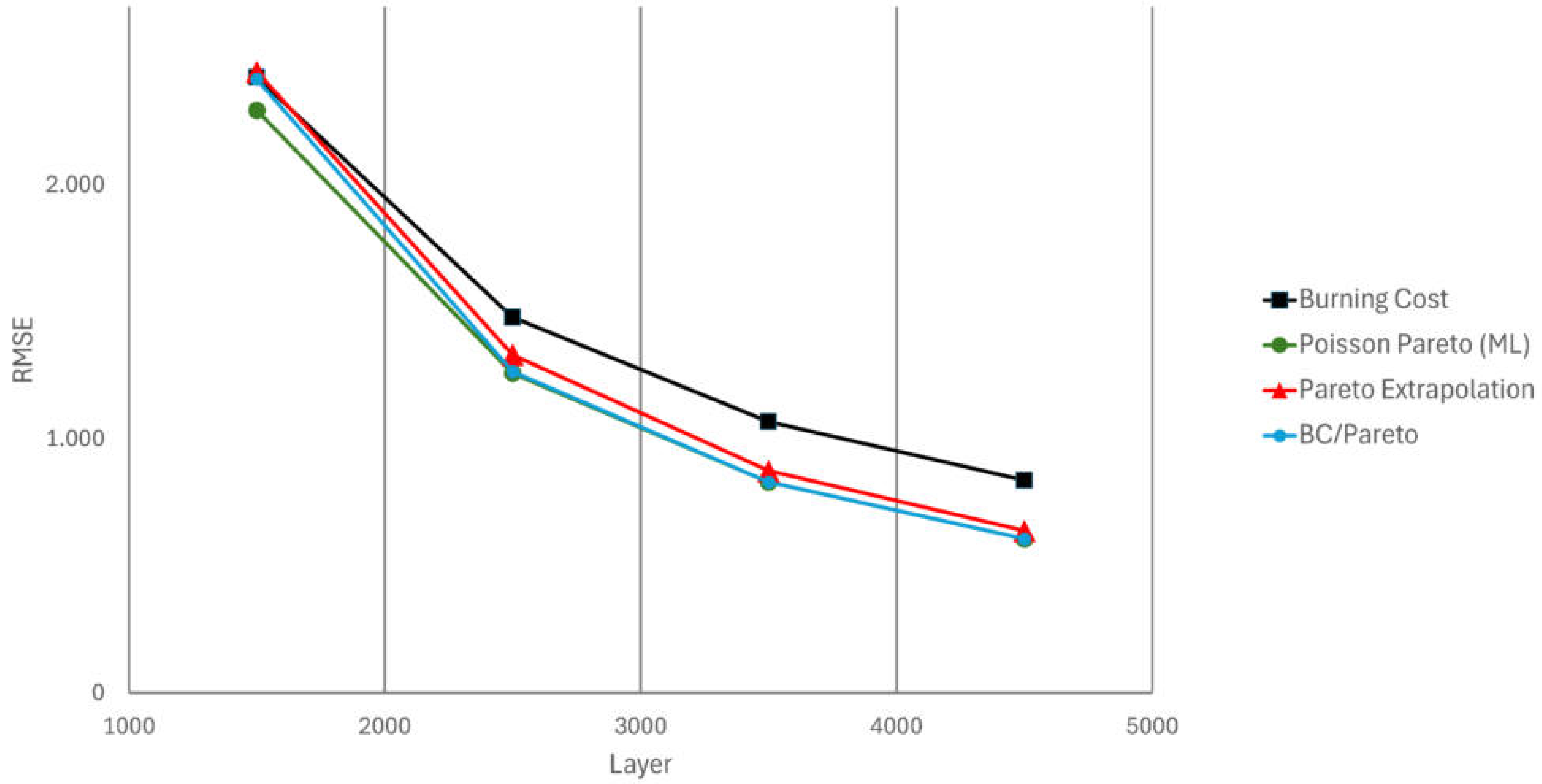
Burning Cost up to 3rd largest loss ($\lambda = 15$, $\alpha = 2.0$)



Burning Cost up to 5th largest loss ($\lambda = 15$, $\alpha = 2.0$)



Burning Cost up to 10th largest loss ($\lambda = 15$, $\alpha = 2.0$)



Observation 3

Methods that use the Burning Cost up to the k-th largest loss seem to have RSMEs between the Burning Cost and the Poisson/Pareto model.

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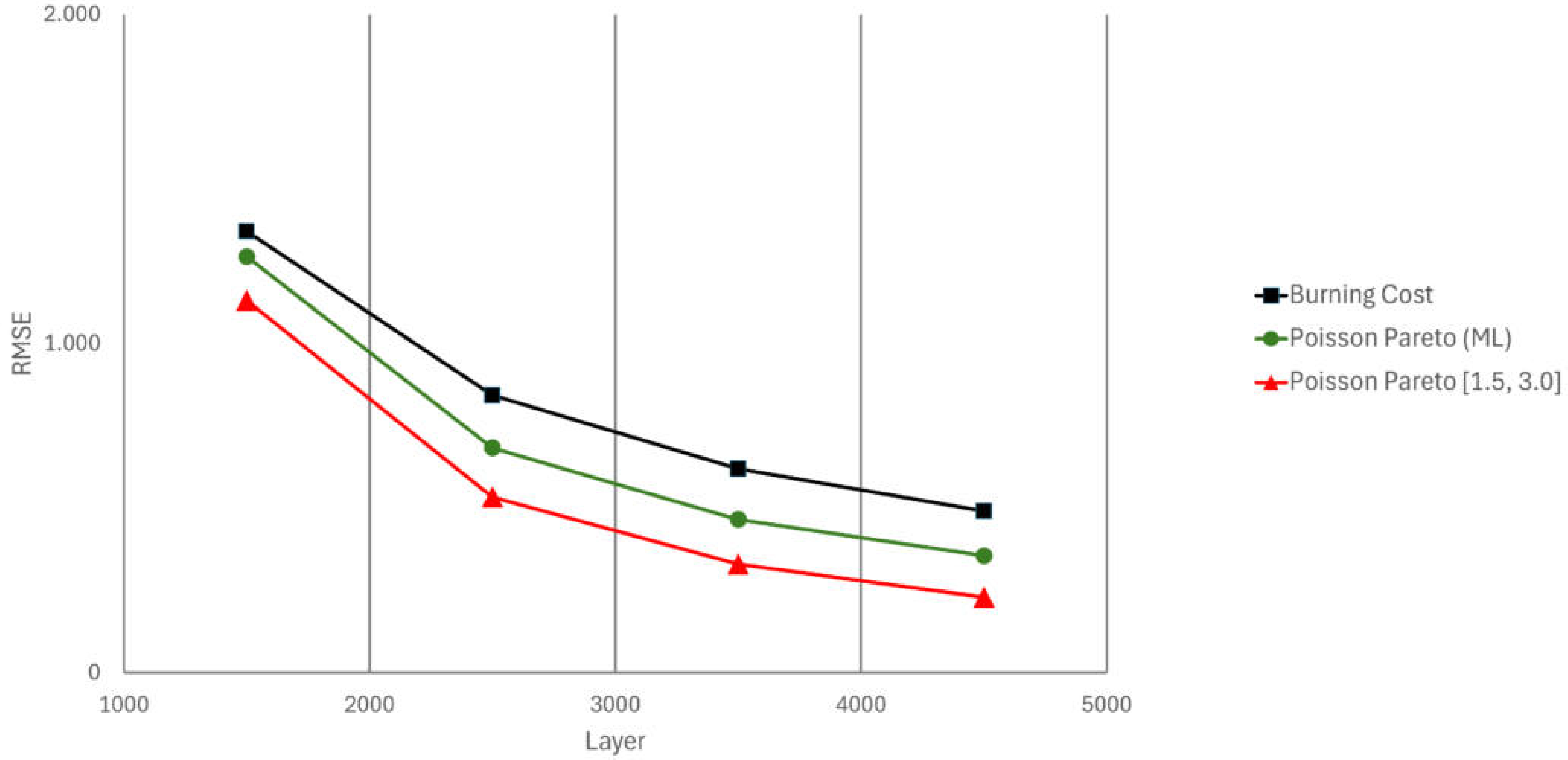
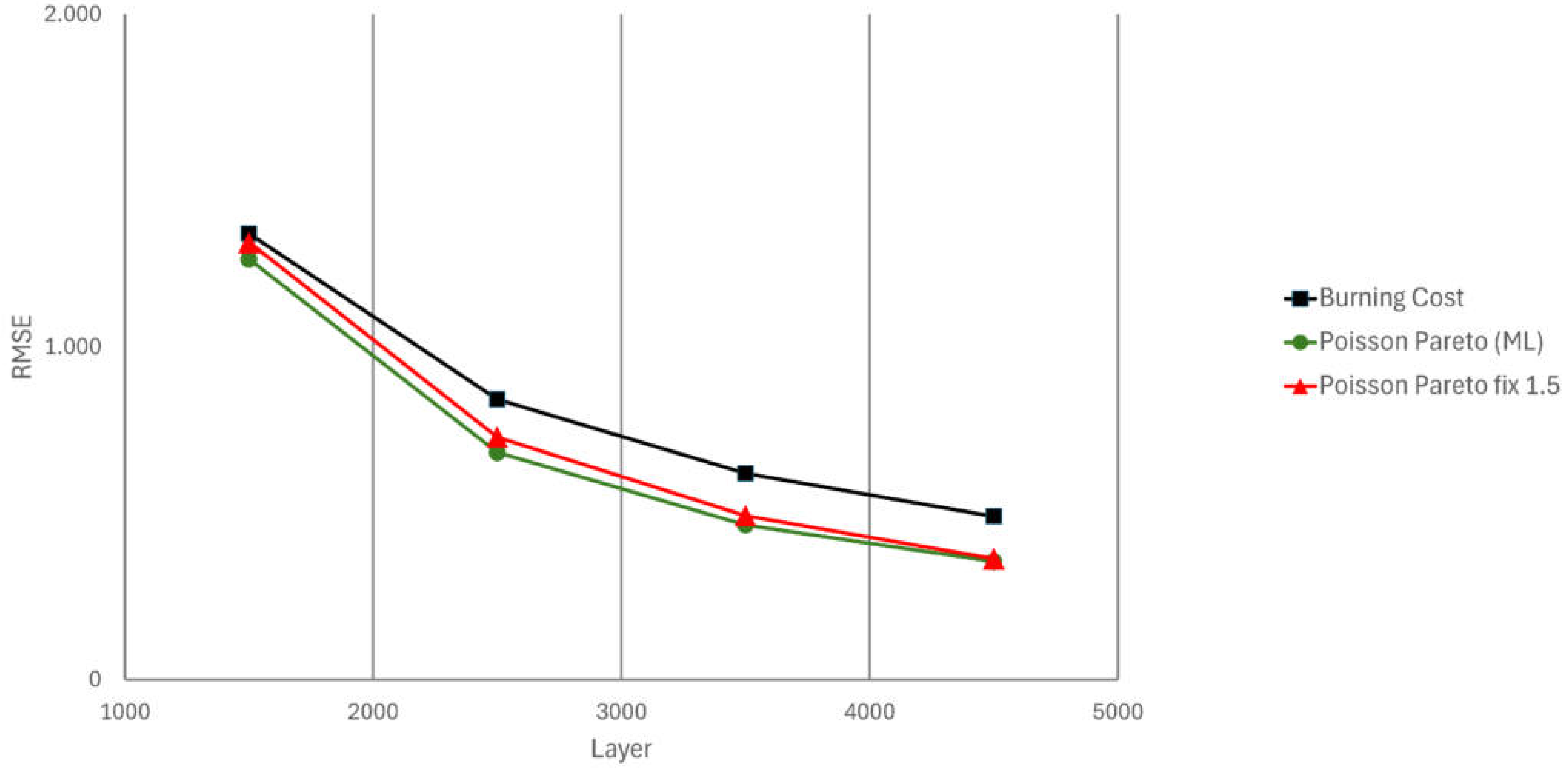
Using the 3rd largest loss is closer to the Burning Cost model, using the 10th largest loss is close to the Poisson/Pareto model.

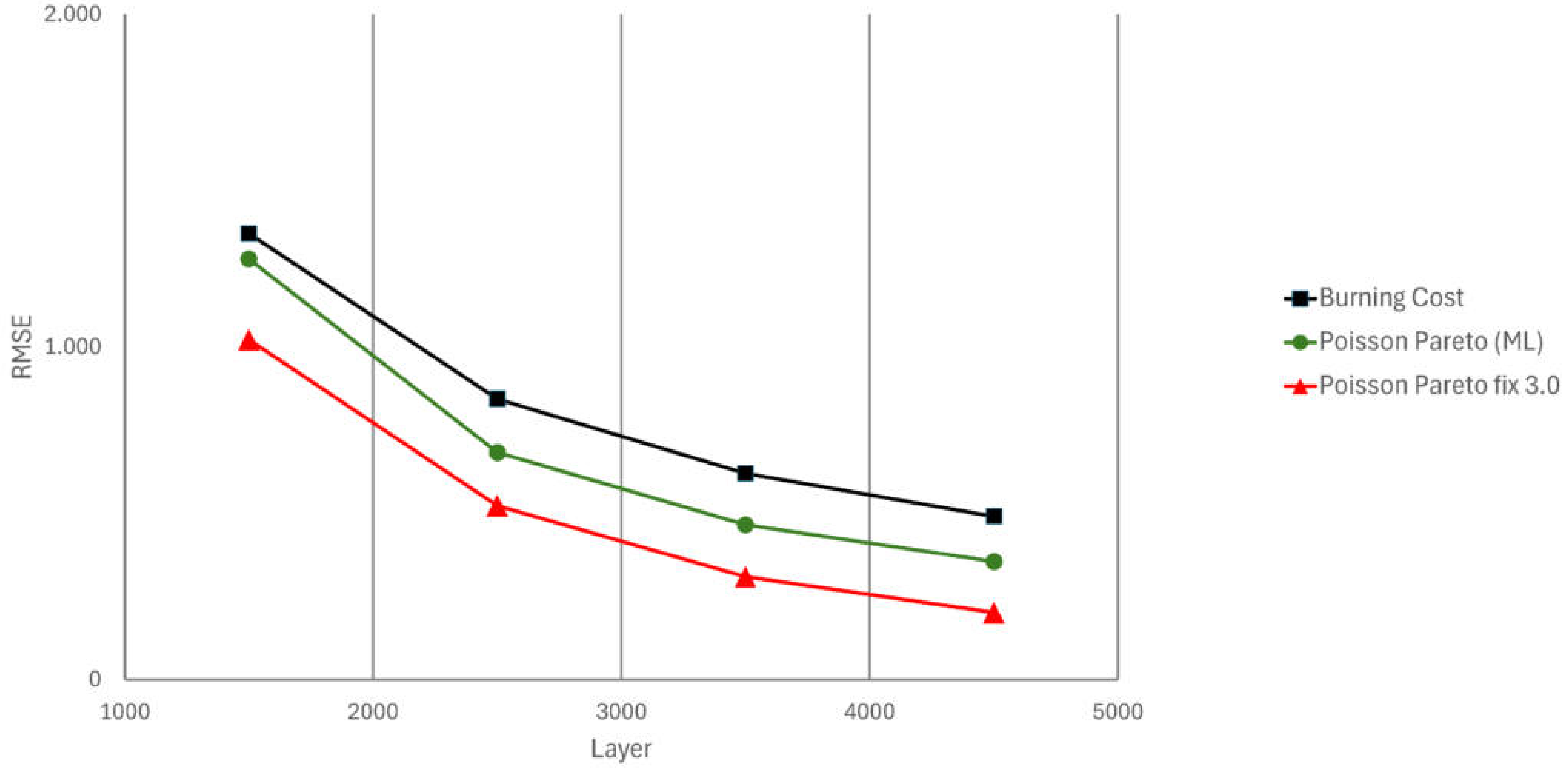
Use boundaries $\alpha_{\min} = 1.5$ and $\alpha_{\max} = 3.0$ for $\widehat{\alpha}^{ML}$ ($\lambda = 5$, $\alpha = 2.0$)



Use fixed *market alpha* $\alpha = 1.5$ ($\lambda = 5$, $\alpha = 2.0$)



Use fixed *market alpha* $\alpha = 3.0$ ($\lambda = 5$, $\alpha = 2.0$)



Observation 4

In case of a small λ the RMSE can be reduced substantially by using market knowledge for the estimation of the Pareto alpha:

- Upper/lower bounds for the estimator
- Market alpha
- Also a credibility approach (Bühlmann Straub) could be employed.

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Note that the RSMEs are smaller, even if the portfolio's alpha deviates substantially from the market mean.

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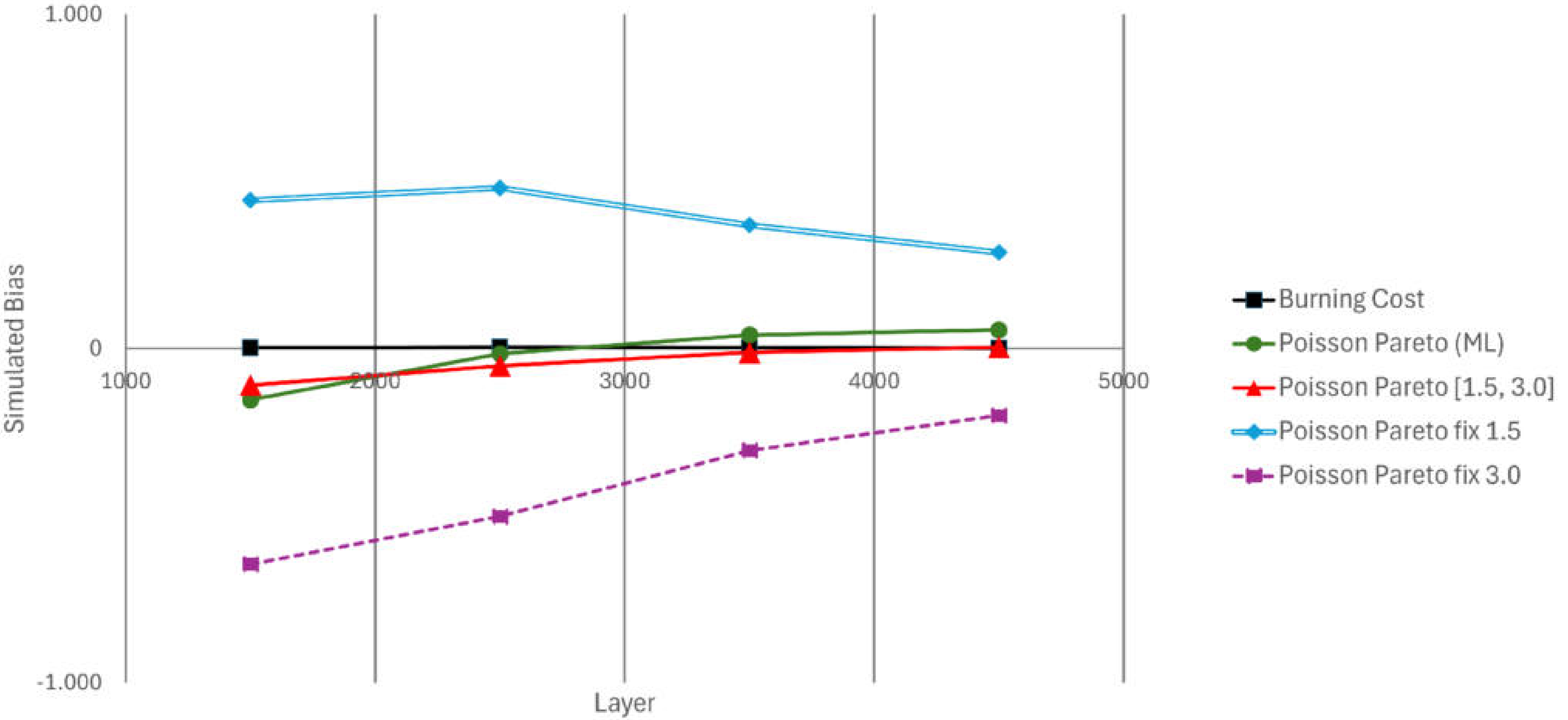
WARNING

Do not look exclusively at the RMSE!

Also care about the bias!

Bias with fixed/restricted alphas ($\lambda = 5$, $\alpha = 2.0$)

WARNING



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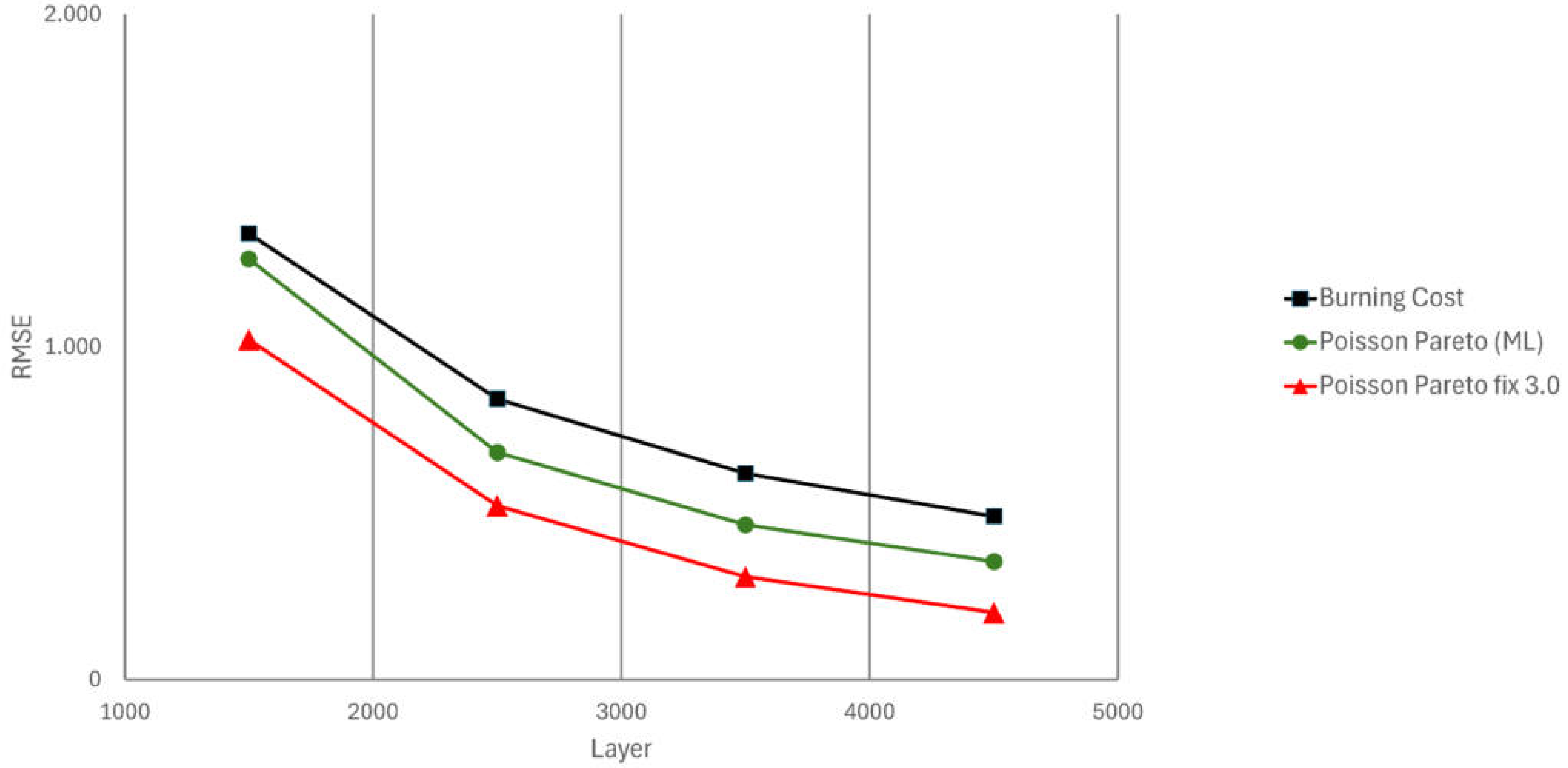

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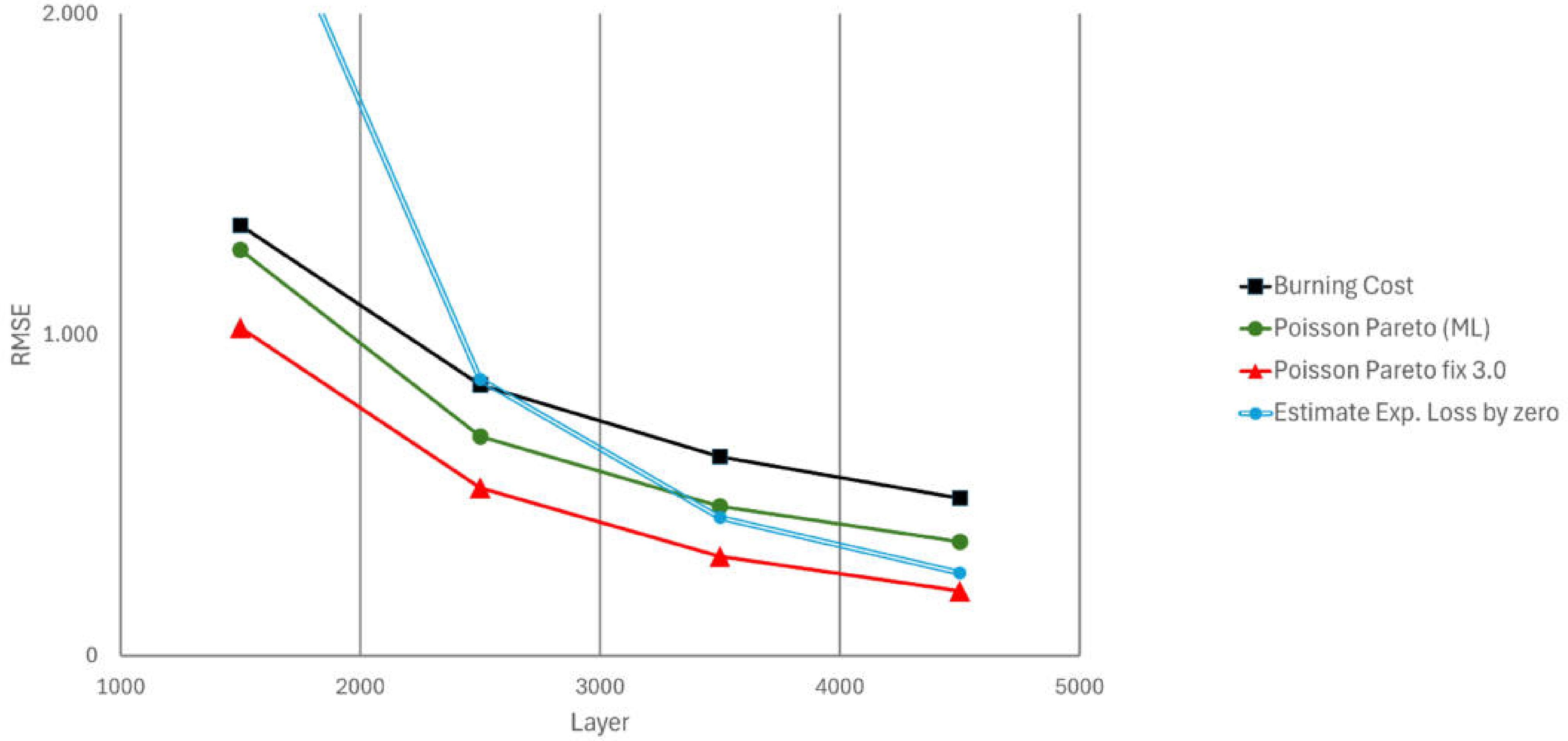

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Observation 5

Case 1: Underlying severity similar to Pareto

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- Thus, methods using BC up to the k-th largest loss should be preferred.

Observation 5

Case 1: Underlying severity similar to Pareto

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- However, we observed a substantial bias in the first layer.
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Case 2: Underlying severity deviates massively from a Pareto distribution

- Pareto based pricing methods can have a substantial bias.

Observation 5

Case 1: Underlying severity similar to Pareto

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- However, we observed a substantial bias in the first layer.
- Thus, methods using BC up to the k-th largest loss should be preferred.

Case 2: Underlying severity deviates massively from a Pareto distribution

- Pareto based pricing methods can have a substantial bias.
- The observations with underlying Pareto severity do not apply in this case.

Vielen Dank für Ihre Aufmerksamkeit.

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