



Stochastic assessment of special rate annuities in a value-based perspective

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Short Bio

- Daniela Tabakova

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Member of Core Faculty at MIB Trieste School of Management and delivers lectures in Insurance and Risk Management area for the Master in insurance risk management program. Research interests include *multi-state insurance models*, *mortality trends*, *longevity risk*, and quantitative risk assessment for life insurance

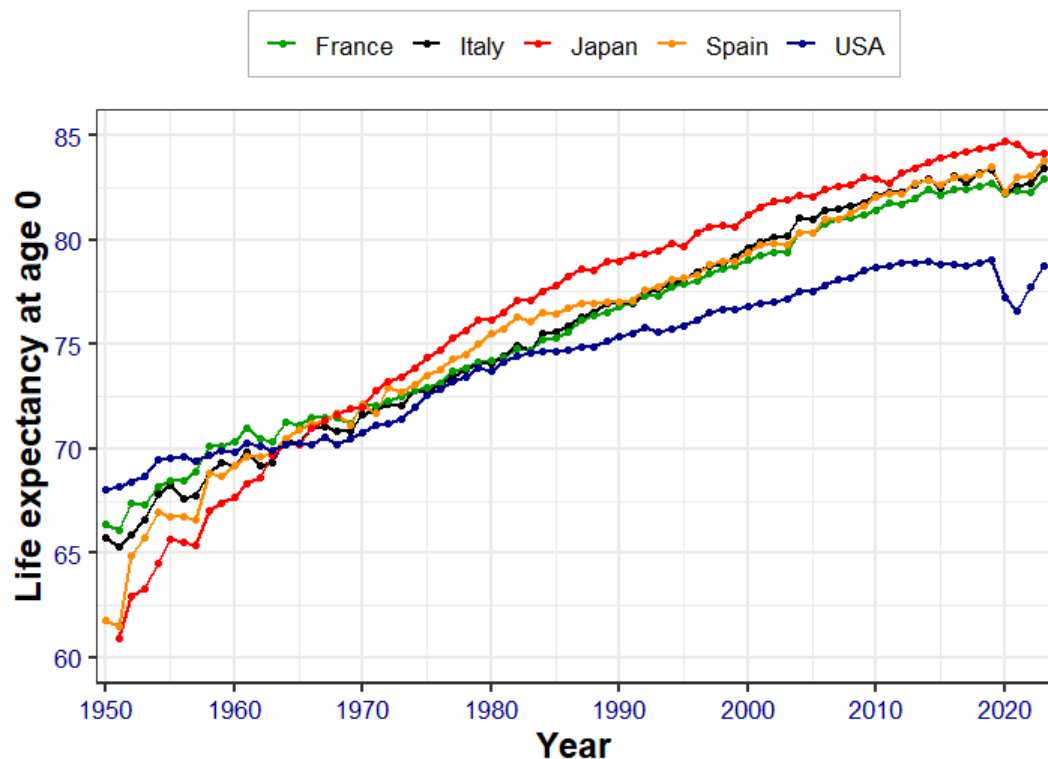


- MIB Trieste School of Management

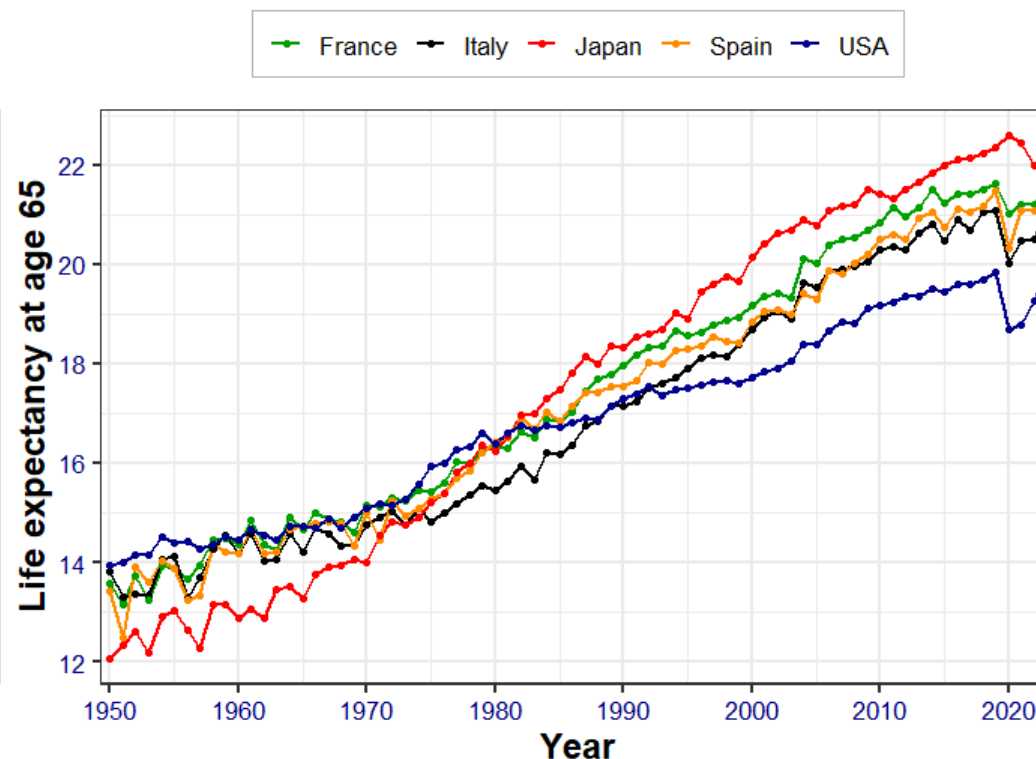
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Longevity Dynamics



Source: Human Mortality Database (HMD)



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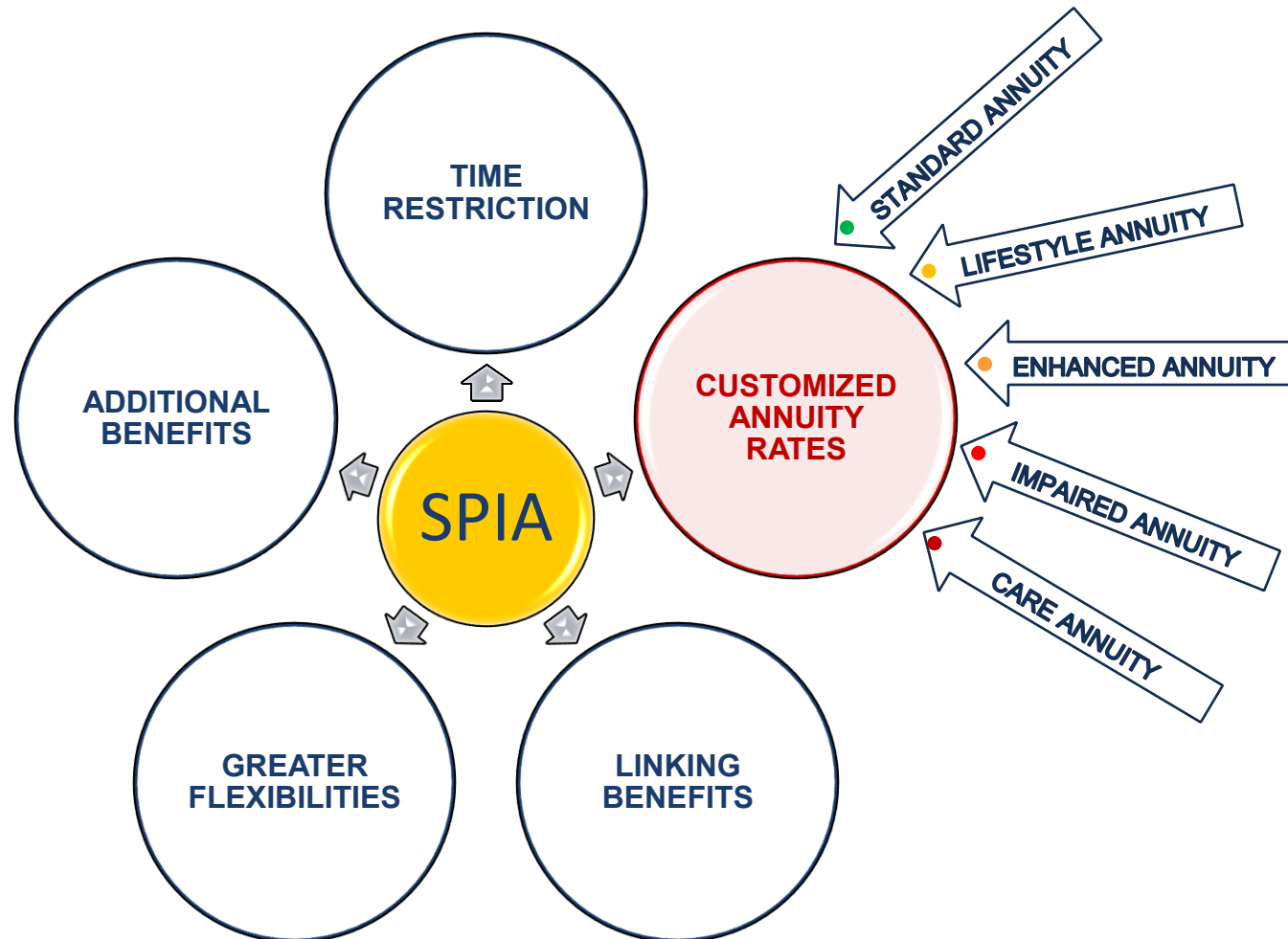
Longevity improvements increase the cost and uncertainty of lifetime guarantees, requiring safety loadings and contributing to thin annuity markets.

Introduction and Motivation

- Weaknesses of traditional life annuity products



Innovations in life insurance annuities



Taxonomy as used in the UK market based on
(Ridsdale 2012), (Pitacco 2021)

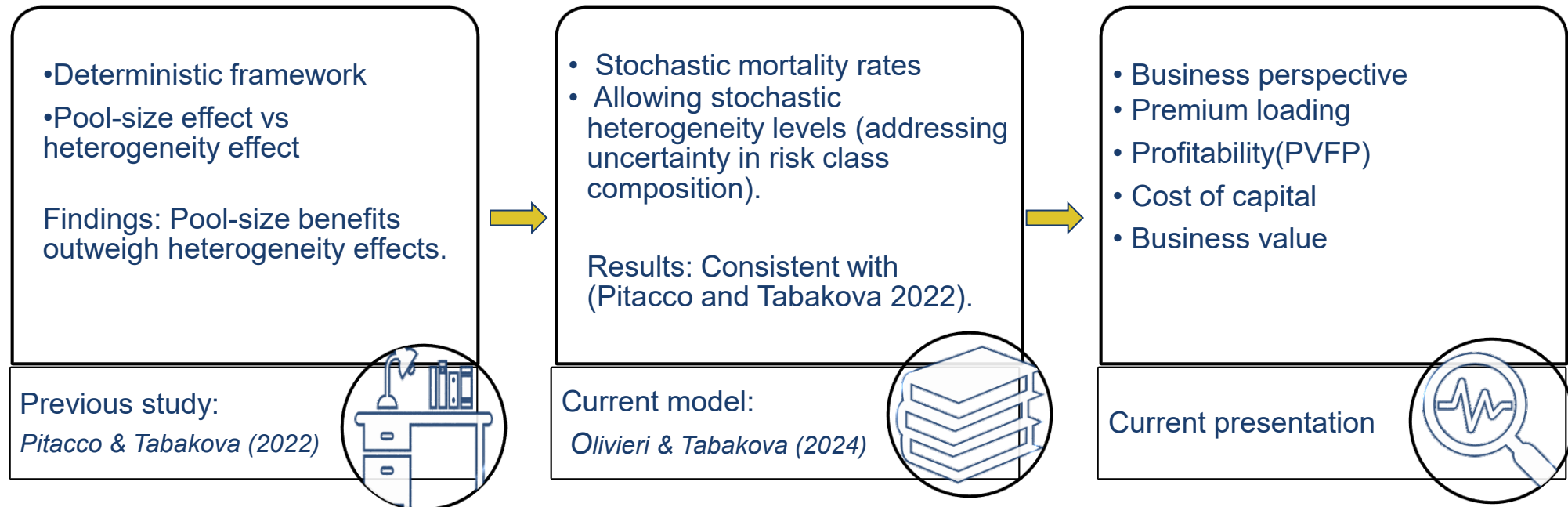
- STANDARD
Very good health and lifestyle
Lower mortality
- LIFESTYLE
Lifestyle-related risk factors
Slightly shorter life expectancy than standard
- ENHANCED
Personal history of moderate medical conditions,
Higher mortality rates
- IMPAIRED
Serious health conditions
Substantially higher mortality
- CARE
Long-term care needs
Highest mortality

Special-rate life annuities

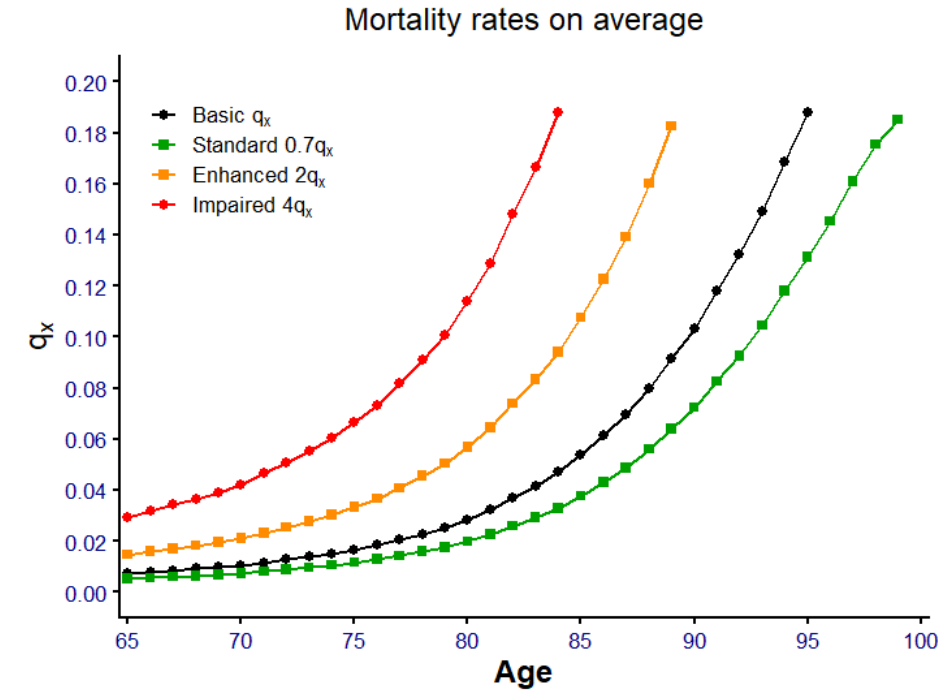
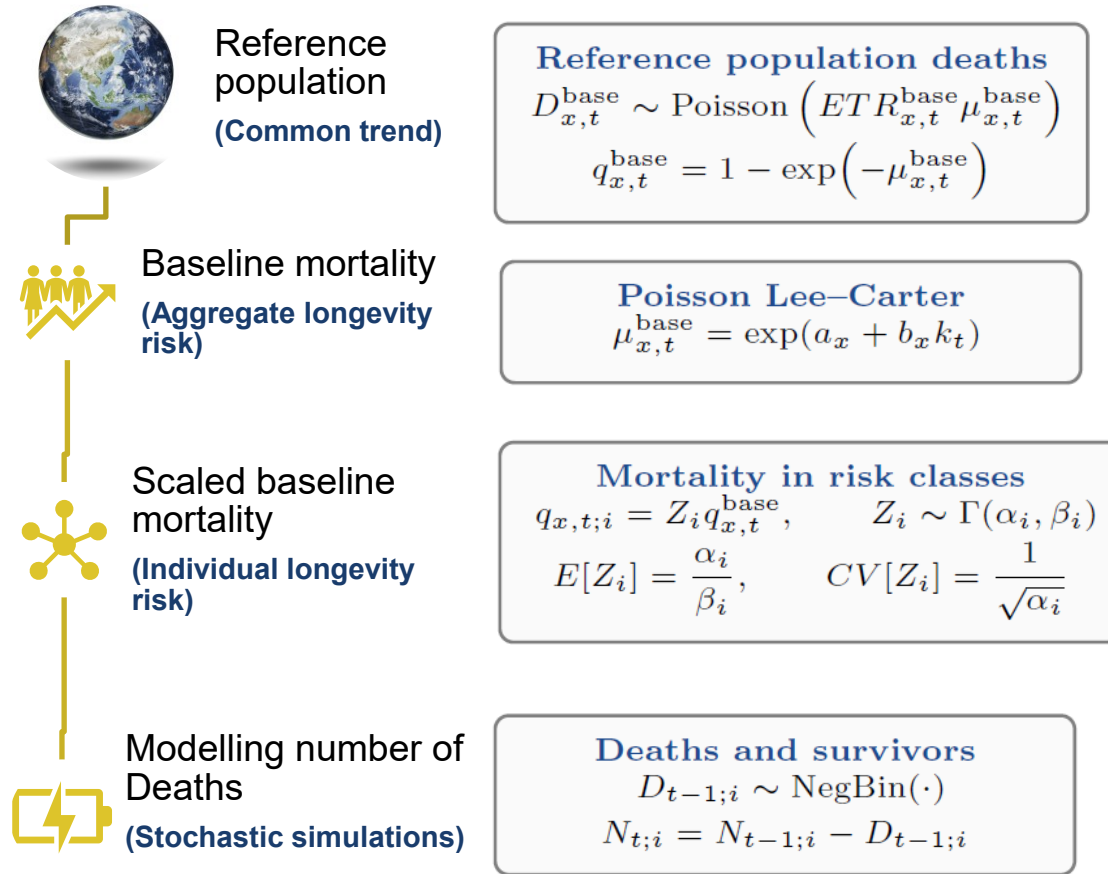
Broaden the annuity market through risk-adjusted pricing while improving the alignment between premiums and longevity costs.

- Increase the pool size (+) → Stronger pooling effect
- Higher degree of heterogeneity of the pool (-) → Weaker pooling effect

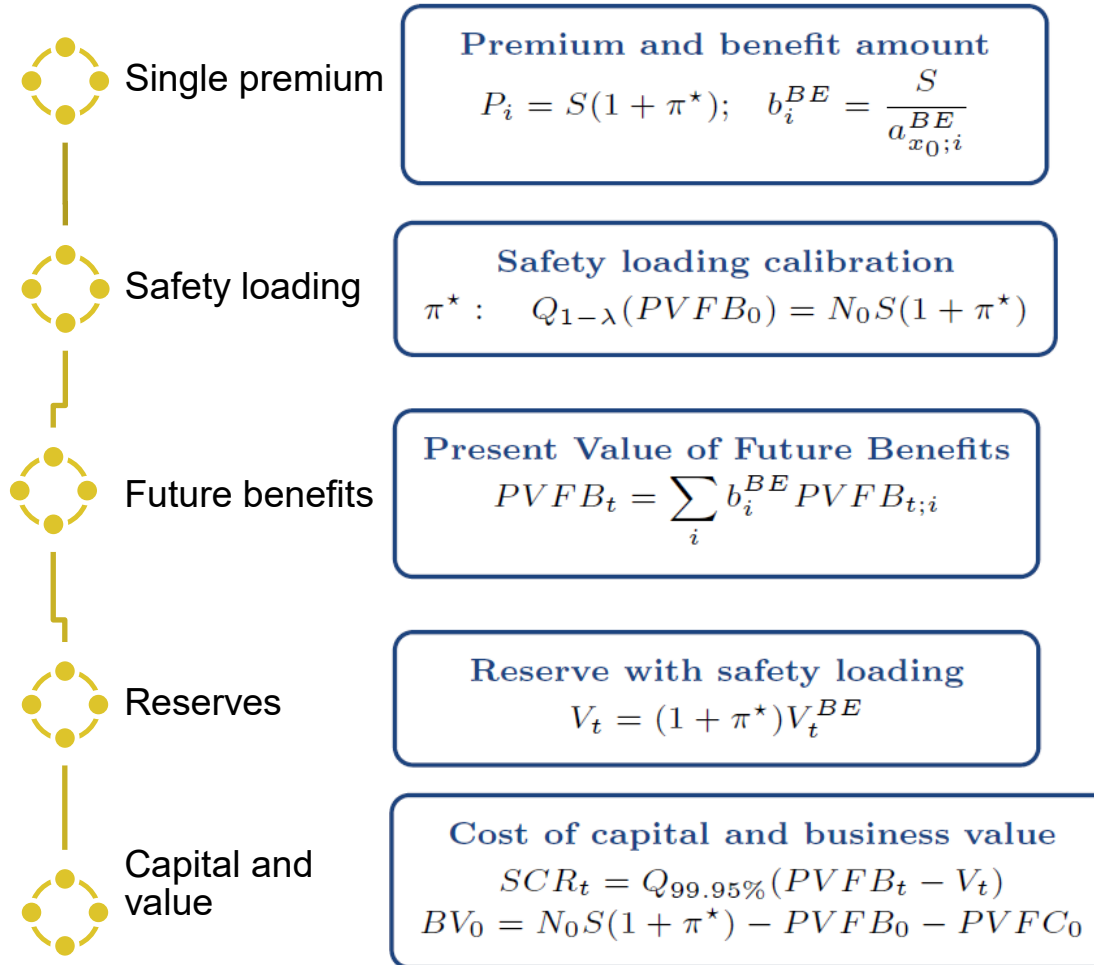
Research question: What is the net impact of larger size vs. higher heterogeneity on the risk profile of special-rate annuities in a value-based perspective?



Mortality modeling framework



Portfolio valuation framework



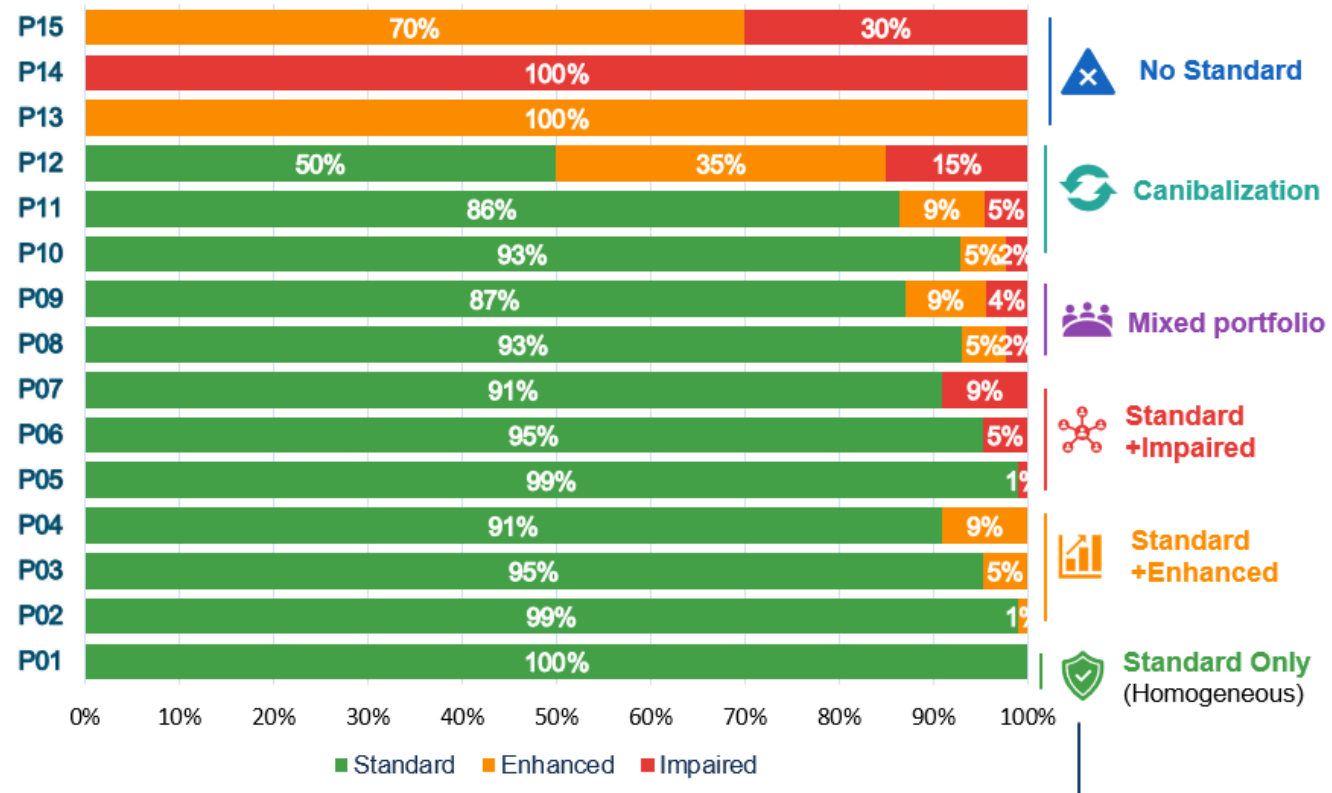
Portfolio Characteristics

Risk classes	Benefit Amount	Annuity factor
Standard	54.49	19.98
Enhanced	70.85	15.11
Impaired	93.49	11.7

Modelling assumptions

- ✓ Same age at issue (same cohort)
- ✓ Closed portfolio (no new entrants)
- ✓ Random number of survivors within each risk class
- ✓ Benefits depend on the risk class and are identical for all annuitants within the same class
- ✓ Single premium paid at inception, including the safety loading

Portfolio compositions



Impact of Portfolio composition

Label	Size	Standard	Enhanced	Impaired	Premium loading ↑	Capital Required at time 0 (% of PVFB ₀ (BE)) ↑	Cost of Capital (% of PVFB ₀ (BE)) ↑	Business value (% of PVFB ₀ (BE)) ↑
P01	10 000	10 000	0	0	2.20%	2.04%	0.97%	1.19%
P02	10 100	10 000	100	0	2.22%	2.06%	0.97%	1.20%
P03	10 500	10 000	500	0	2.27%	2.11%	0.98%	1.24%
P04	11 000	10 000	1 000	0	2.33%	2.16%	0.99%	1.29%
P05	10 100	10 000	0	100	2.22%	2.08%	0.97%	1.21%
P06	10 500	10 000	0	500	2.32%	2.16%	0.97%	1.29%
P07	11 000	10 000	0	1 000	2.42%	2.27%	0.98%	1.38%
P08	10 750	10 000	500	250	2.32%	2.17%	0.98%	1.28%
P09	11 500	10 000	1 000	500	2.43%	2.28%	1.00%	1.37%
P10	10 500	9 750	500	250	2.32%	2.17%	0.98%	1.29%
P11	11 000	9 500	1 000	500	2.44%	2.29%	1.00%	1.38%
P12	10 000	5 000	3 500	1 500	3.09%	2.96%	1.12%	1.88%
P13	10 000	0	10 000	0	3.73%	3.62%	1.38%	2.25%
P14	10 000	0	0	10 000	5.16%	5.07%	1.52%	3.47%
P15	10 000	0	7 000	3 000	4.04%	3.97%	1.38%	2.54%

Note: Values are expressed as a percentage of PVFB₀ (Best-estimate)
Acceptable level of risk: $\lambda = 10\%$

Observations

-  **Premium loading**
Increases as portfolio heterogeneity grows, with the strongest impact in unbalanced portfolios.
-  **Capital required**
Similar to premium loading.
Rise with portfolio heterogeneity and peak in portfolios with a high share of impaired annuities.
-  **Business value**
Increases despite higher capital requirements.
-  **Most significant effect**
Observed in portfolios with a high concentration of impaired annuitants.

Sensitivity analysis

Effect of reducing $\lambda=10\%$ to $\lambda=5\%$													
Label	Size	Premium loading $\lambda=10\%$	Premium loading $\lambda=5\%$	Δ Premium loading	Capital Required at time 0 (% of PVFB ₀ (BE)) ↑ $\lambda=10\%$	Capital Required at time 0 (% of PVFB ₀ (BE))	Δ Capital Required at time 0	Cost of Capital (% of PVFB ₀ (BE)) $\lambda=10\%$	Cost of Capital (% of PVFB ₀ (BE)) $\lambda=5\%$	Δ Cost of Capital	Business value (% of PVFB ₀ (BE)) $\lambda=10\%$	Business value (% of PVFB ₀ (BE)) $\lambda=5\%$	Δ Business value
P01	10 000	2.20%	2.79%	+0.59%	2.04%	1.45%	-0.59%	0.97%	0.81%	-0.15%	1.19%	1.93%	+0.74%
P02	10 100	2.22%	2.81%	+0.59%	2.06%	1.47%	-0.59%	0.97%	0.82%	-0.15%	1.20%	1.95%	+0.75%
P03	10 500	2.27%	2.87%	+0.61%	2.11%	1.50%	-0.61%	0.98%	0.82%	-0.16%	1.24%	2.00%	+0.76%
P04	11 000	2.33%	2.95%	+0.62%	2.16%	1.55%	-0.62%	0.99%	0.83%	-0.16%	1.29%	2.06%	+0.77%
P05	10 100	2.22%	2.81%	+0.59%	2.08%	1.48%	-0.59%	0.97%	0.82%	-0.15%	1.21%	1.95%	+0.74%
P06	10 500	2.32%	2.93%	+0.62%	2.16%	1.55%	-0.62%	0.97%	0.82%	-0.16%	1.29%	2.06%	+0.77%
P07	11 000	2.42%	3.07%	+0.65%	2.27%	1.62%	-0.65%	0.98%	0.82%	-0.16%	1.38%	2.19%	+0.81%
P08	10 750	2.32%	2.94%	+0.62%	2.17%	1.55%	-0.62%	0.98%	0.82%	-0.16%	1.28%	2.06%	+0.78%
P09	11 500	2.43%	3.07%	+0.65%	2.28%	1.63%	-0.65%	1.00%	0.83%	-0.16%	1.37%	2.18%	+0.81%
P10	10 500	2.32%	2.94%	+0.62%	2.17%	1.55%	-0.62%	0.98%	0.83%	-0.16%	1.29%	2.06%	+0.77%
P11	11 000	2.44%	3.09%	+0.65%	2.29%	1.64%	-0.65%	1.00%	0.84%	-0.16%	1.38%	2.19%	+0.81%
P12	10 000	3.09%	3.92%	+0.83%	2.96%	2.13%	-0.83%	1.12%	0.94%	-0.19%	1.88%	2.90%	+1.02%
P13	10 000	3.73%	4.74%	+1.01%	3.62%	2.61%	-1.01%	1.38%	1.17%	-0.21%	2.25%	3.46%	+1.21%
P14	10 000	5.16%	6.58%	+1.42%	5.07%	3.65%	-1.42%	1.52%	1.28%	-0.24%	3.47%	5.12%	+1.66%
P15	10 000	4.04%	5.14%	+1.10%	3.97%	2.87%	-1.10%	1.38%	1.16%	-0.21%	2.54%	3.85%	+1.31%

Note: Values are expressed as a percentage of PVFB₀ (Best-estimate)

Sensitivity analysis



Level of Heterogeneity													
Label	Size	Premium loading (Base)	Premium loading (Sens.)	Δ Premium loading	Capital Required at time 0 (% of PVFB ₀ (BE)) (Base)	Capital Required at time 0 (% of PVFB ₀ (BE)) (Sens.)	Δ Capital Required at time 0	Cost of Capital (% of PVFB ₀ (BE)) (Base)	Cost of Capital (% of PVFB ₀ (BE)) (Sens.)	Δ Cost of Capital	Business value (% of PVFB ₀ (BE)) (Base)	Business value (% of PVFB ₀ (BE)) (Sens.)	Δ Business value
P01	10 000	2.20%	2.19%	-0.01%	2.04%	2.03%	0.00%	0.97%	0.96%	-0.01%	1.19%	1.19%	0.002%
P02	10 100	2.22%	2.21%	-0.01%	2.06%	2.05%	-0.01%	0.97%	0.96%	-0.01%	1.20%	1.21%	0.007%
P03	10 500	2.27%	2.26%	-0.01%	2.11%	2.11%	0.00%	0.98%	0.97%	-0.01%	1.24%	1.24%	0.000%
P04	11 000	2.33%	2.32%	-0.01%	2.16%	2.17%	0.01%	0.99%	0.98%	-0.01%	1.29%	1.29%	-0.004%
P05	10 100	2.22%	2.22%	-0.01%	2.08%	2.06%	-0.01%	0.97%	0.96%	-0.01%	1.21%	1.21%	0.007%
P06	10 500	2.32%	2.30%	-0.01%	2.16%	2.16%	0.00%	0.97%	0.97%	-0.01%	1.29%	1.29%	-0.001%
P07	11 000	2.42%	2.41%	-0.01%	2.27%	2.27%	0.00%	0.98%	0.97%	-0.01%	1.38%	1.38%	0.000%
P08	10 750	2.32%	2.31%	-0.01%	2.17%	2.16%	-0.01%	0.98%	0.97%	-0.01%	1.28%	1.29%	0.004%
P09	11 500	2.43%	2.42%	-0.01%	2.28%	2.27%	-0.01%	1.00%	0.99%	-0.01%	1.37%	1.37%	0.002%
P10	10 500	2.32%	2.31%	-0.01%	2.17%	2.16%	-0.01%	0.98%	0.97%	-0.01%	1.29%	1.29%	-0.001%
P11	11 000	2.44%	2.43%	-0.01%	2.29%	2.29%	-0.01%	1.00%	0.99%	-0.01%	1.38%	1.38%	0.000%
P12	10 000	3.09%	3.08%	-0.01%	2.96%	2.96%	0.00%	1.12%	1.12%	0.00%	1.88%	1.87%	-0.009%
P13	10 000	3.73%	3.70%	-0.04%	3.62%	3.57%	-0.04%	1.38%	1.35%	-0.02%	2.25%	2.23%	-0.014%
P14	10 000	5.16%	4.95%	-0.21%	5.07%	4.86%	-0.21%	1.52%	1.45%	-0.06%	3.47%	3.32%	-0.147%
P15	10 000	4.04%	4.01%	-0.04%	3.97%	3.92%	-0.05%	1.38%	1.36%	-0.02%	2.54%	2.52%	-0.018%

Risk Class	α	CV[Z ₁]	α	CV[Z ₁]
Standard	400	5.00%	1000	3.20%
Enhanced	100	10.00%	150	8.20%
Impaired	40	15.80%	80	11.20%

Note: Values are expressed as a percentage of PVFB₀ (Best-estimate)

Conclusions

- Premiums tailored on the individual risk profile;
- Greater portfolio heterogeneity:
 - Higher safety loading, moderated by risk-pooling effects;
 - Higher capital requirements
- Business value increases. Higher safety loadings enhance value creation despite the greater capital requirements
- Portfolios composed exclusively of standard annuities generate the lowest business value in a relative term.

Essential References

- Olivieri, A., & Pitacco, E. (2016). Frailty and risk classification for life annuity portfolios. *Risks*, 4(4), 39.
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- Pitacco, E. (2021). *Life Annuities*. Risk Books.
- Pitacco, E., & Tabakova, D. Y. (2022). Special-rate life annuities: Analysis of portfolio risk profiles. *Risks*, 10(3), 65.

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Thank you for your kind attention!

