A Model for Risk Adjustment (IFRS 17) for Surrender Risk in Life Insurance

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About the speaker

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Background

This work has grown out of a practical problem. I was asked by Swedbank's two life insurance companies how to model surrender risk for the purpose of risk adjustment in IFRS17.

Proposed model: Normally distributed increments of surrender rates.

Well-developed theory (convex ordering of random variables) that could be applied.

Key message: A reasonable risk adjustment can be calculated by approximations, avoiding simulations.

Risk adjustment in IFRS 17

Paragraph 37: ”An entity shall adjust the estimate of the *present value of the future cash flows* to reflect the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk.”

Paragraph 119: ”An entity shall disclose the confidence level used to determine the risk adjustment for non-financial risk. If the entity uses a technique other than the confidence level technique for determining the risk adjustment for non-financial risk, it shall disclose the technique used and the confidence level corresponding to the results of that technique.”

The method for risk adjustment is not specified but there are five requirements it must fulfil.

Cash flows and remain rate

Portfolio of term life contracts

Discounted net cash flows a_t, $t = 1, 2, ..., T$, given a surrender rate of zero. The $\{a_t\}$ are calculated based on actuarial assumptions, including other risks than surrenders.

The present value of future cash flows (”PVFCF”) from the portfolio is

$$\sum_{t=1}^{T} a_t.$$

We will model remain rate, i.e. 1 minus the surrender rate; let its best estimate be R at time 0, $0 \leq R \leq 1$.

Cash flows and remain rate, cont.

Process $\{r_t\}$, meaning that each year t, a proportion of contracts equal to r_t remains in the portfolio. We set $r_0 = R$.

We get a modified cash flow at time t:

$$b_t = a_t \prod_{s=1}^{t} r_s.$$

The total PVFCF for the portfolio becomes

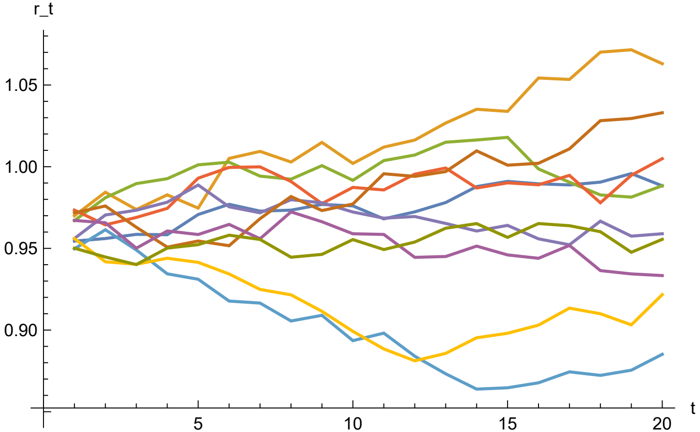
$$S = \sum_{t=1}^{T} b_t.$$

A stochastic model for surrenders

Assume that

$$r_t := r_{t-1} e^{-\sigma^2/2+\sigma X_t}, \qquad t \geq 1,$$

where X_t are i.i.d. standard normal, $\sigma > 0$, and $r_0 = R$.
Obvious that $\mathbb{E}[r_t | r_{t-1}] = r_{t-1}$, i.e. r_t is a martingale.



Real example, four portfolios (transformed data)

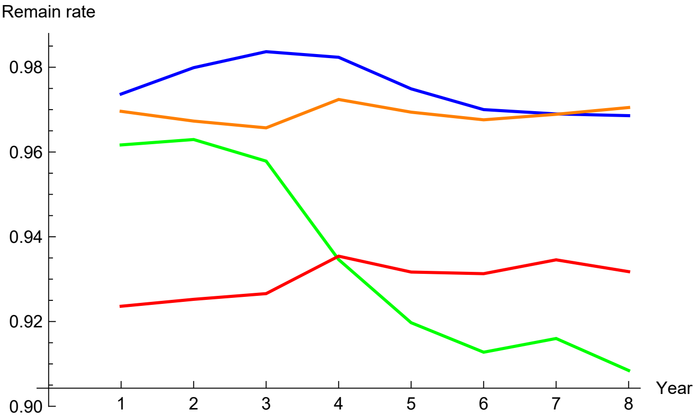


Fig.1

Shapiro-Wilk test on $\{\log r_t - \log r_{t-1}\}$ shows that the hypothesis of normality is not rejected for any of the time series.

Example of cash flow projections ($a_t = 1$ for all t, $R = 0.96$)

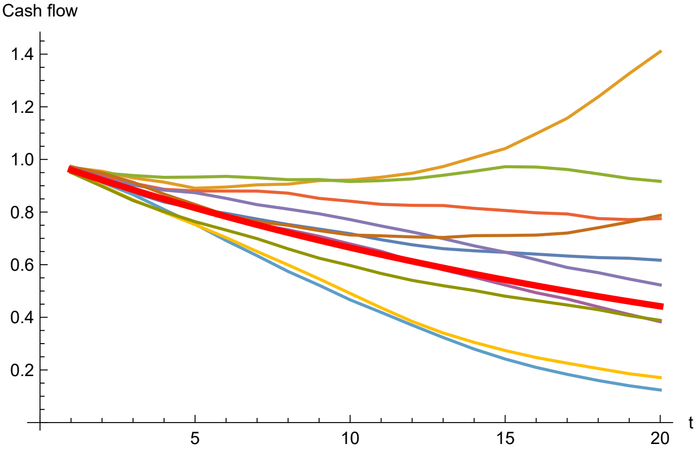


Fig. 3

With $\sigma = 0$ we get the deterministic cash flows 0.96^t (thick red).

A stochastic model for surrender, cont.

We have, using the stochastic iteration,

$$r_s = Re^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)}.$$

Hence

$$\begin{aligned} b_t &= a_t \prod_{s=1}^{t} r_s = a_t \prod_{s=1}^{t} Re^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)} \\ &= a_t R^t e^{-t(1+t)\sigma^2/4+\sigma(tX_1+(t-1)X_2+\ldots+X_t)} \\ &= a_t R^t e^{-t(1+t)\sigma^2/4+\sigma Z_t}, \end{aligned} \tag{1}$$

where $Z_t = \sum_{i=1}^{t}(t-i+1)X_i$. Note that each b_t is lognormal and that $\{b_t\}$ are dependent.

Quantile of PFVCF for individual future years

Each Z_t is a normally distributed variable with mean 0, and their covariance matrix C is:

$$C_{s,t} := \mathrm{Cov}[Z_s, Z_t] = \sum_{i=1}^{t}(t-i+1)(s-i+1) = \frac{t(t+1)(3s-t+1)}{6},$$

where $t \leq s$. In particular, the variance of Z_t is

$$\mathrm{Var}[Z_t] = C_{t,t} = \frac{t(t+1)(2t+1)}{6}.$$

Hence the p-quantile of b_t equals

$$Q_p[b_t] = a_t R^t \exp\left(-t(1+t)\sigma^2/4 + \sigma z_p \sqrt{\frac{t(t+1)(2t+1)}{6}}\right),$$

where $z_p = \Phi^{-1}(p)$. This is a stressed PVFCF for time t and a low quantile (e.g. $p = 0.2$), and a typical risk adjustment is the difference between the deterministic cash flow ($\sigma = 0$) and this quantile.

Risk adjustment for the whole portfolio

We want to make the risk adjustment for the portfolio as a whole, so how do we aggregate?

"Sum of quantiles (of cash flows)":

$$\sum_{t=1}^{T} Q_p[b_t].$$

This is easy, conservative, and basically assumes all the cash flows perfectly dependent.

"Quantile of sums (of cash flows)":

$$Q_p\left[\sum_{t=1}^{T} b_t\right].$$

This is more correct, but requires simulation or approximation.

Convex ordering of random variables

A random variable X is said to precede the random variable Y in the convex order sense, notation $X \leq_{cx} Y$, if

$$\mathbb{E}[v(X)] \leq \mathbb{E}[v(Y)]$$

for all convex functions v.

This is only possible if $\mathbb{E}[X] = \mathbb{E}[Y]$. Also $\mathbb{E}[X] \leq_{cx} X$ for all X (by Jensen's inequality).

Intuitively, the tails of Y go further out from the centre than the tails of X.

Comonotonicity

Let U be uniform. The random vector $Y = (Y_1, \ldots, Y_n)$ is said to be comonotonic if there exist non-decreasing functions $g_1, \ldots, g_n$ defined on $(0,1)$ such that

$$Y \stackrel{d}{=} \{g_1(U), \ldots, g_n(U)\}.$$

The comonotonic counterpart of Y is defined as the random vector $(F_{Y_1}^{-1}(U), \ldots, F_{Y_n}^{-1}(U))$; this random vector has the same marginal distributions as Y.

Note: For comonotonic random variables, quantiles are additive.

Convex bounds for sums of random variables

Theorem: Let $(X_1, \ldots, X_n)$ be any random vector, U uniform on (0,1), and Λ any random variable. Then

$$\sum_{i=1}^{n} E[X_i|\Lambda] \leq_{cx} \sum_{i=1}^{n} X_i \leq_{cx} \sum_{i=1}^{n} F_{X_i}^{-1}(U).$$

The upper bound is the comonotonic counterpart of the sum in the middle. .

The lower bound is very useful, in particular when Λ resembles $S := \sum X_i$ in some way.

Caution: Convex ordering does not always preserve quantiles.

Convex bounds for sums of lognormal variables (Kaas et al.)

Let

$$S = \sum_{t=1}^{T} \alpha_t \exp\{Y_1 + \ldots + Y_t\},$$

where $(Y_1, \ldots, Y_T)$ has a multinormal distribution, and $\alpha_t \geq 0$. Define

$$Y(t) = Y_1 + \ldots + Y_t,$$

$$\sigma_{Y(t)} = \sqrt{\mathrm{Var}[Y(t)]},$$

$$\Lambda = \sum_{s=1}^{T} \beta_s Y_s.$$

Finally let ρ_t be the correlation between Λ and $Y(t)$.

Convex bounds for sums of lognormal variables, cont.

Let

$$S_l := \sum_{t=1}^{T} \mathbb{E}[\alpha_t \exp\{Y(t)\}|\Lambda] = \sum_{t=1}^{T} \alpha_t \exp\left\{\mathbb{E}[Y(t)] + \rho_t \sigma_{Y(t)} \Phi^{-1}(U) + \frac{1}{2}(1-\rho_t^2)\sigma_{Y(t)}^2\right\},$$

and

$$S_u := \sum_{t=1}^{T} F^{-1}_{\alpha_t \exp\{Y(t)\}}(U) = \sum_{t=1}^{T} \alpha_t \exp\{\mathbb{E}[Y(t)] + \sigma_{Y(t)} \Phi^{-1}(U)\},$$

(the sum of quantiles). Then

$$S_l \leq_{cx} S \leq_{cx} S_u.$$

Approximations of quantiles of the total PVFCF

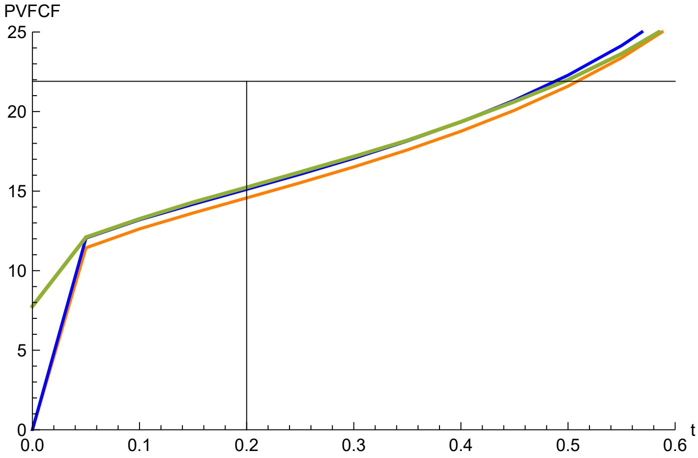


Fig. 6

$T = 60$, $a_t = 1$ for all t, $r = 0.96$, $\sigma = 0.01$. The orange line is S_u, the blue line is S_l, and the green line is S.

Graph of relative risk adjustment as a function of σ

Green lines $T = 60$, black lines $T = 20$. Solid lines $R = 0.96$, dashed lines $R = 0.9$.

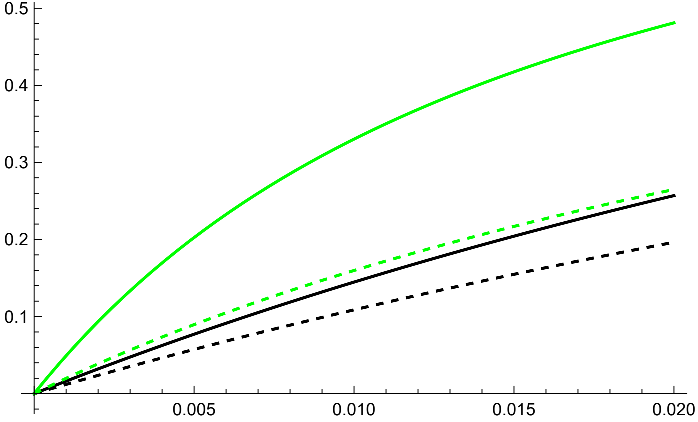


Fig. 6

One may show that the method fulfils the five properties defined in IFRS17.

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Thank You!

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