São Paulo 2025

IAA AAI SECTIONS

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Analysis of the Mortality Trends using the Lee-Crater model in Japan during Pandemic

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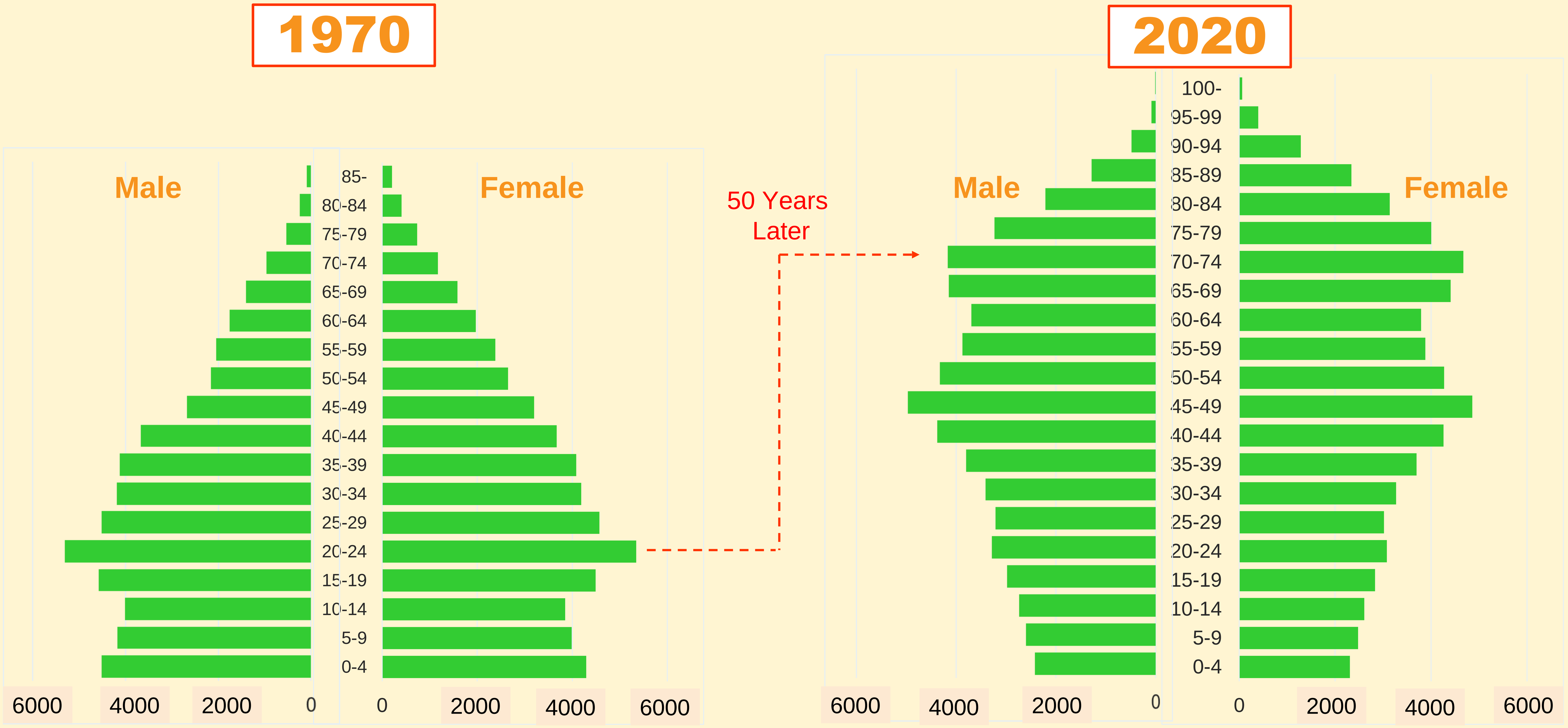
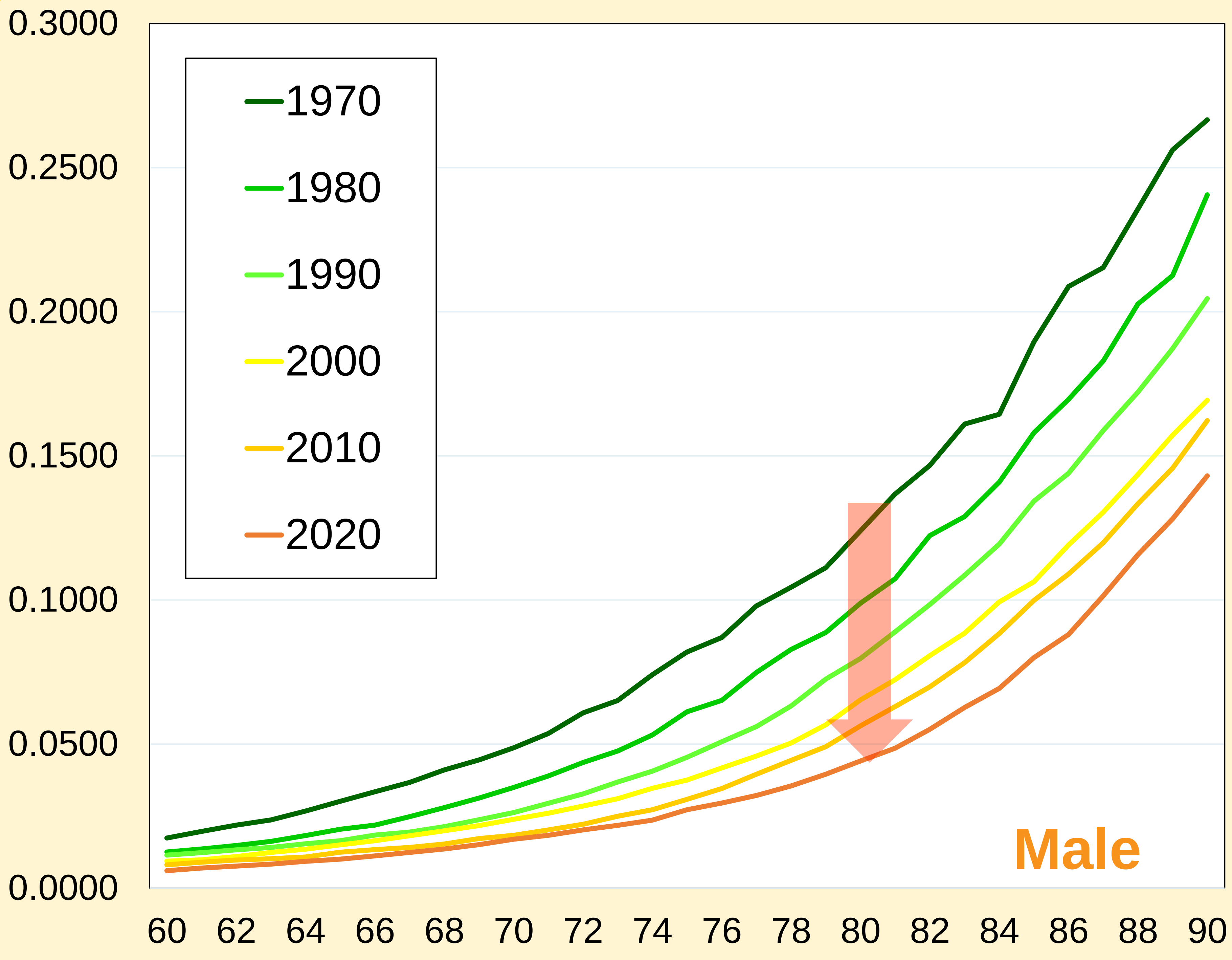
Ⅰ. Population Dynamics of Japan

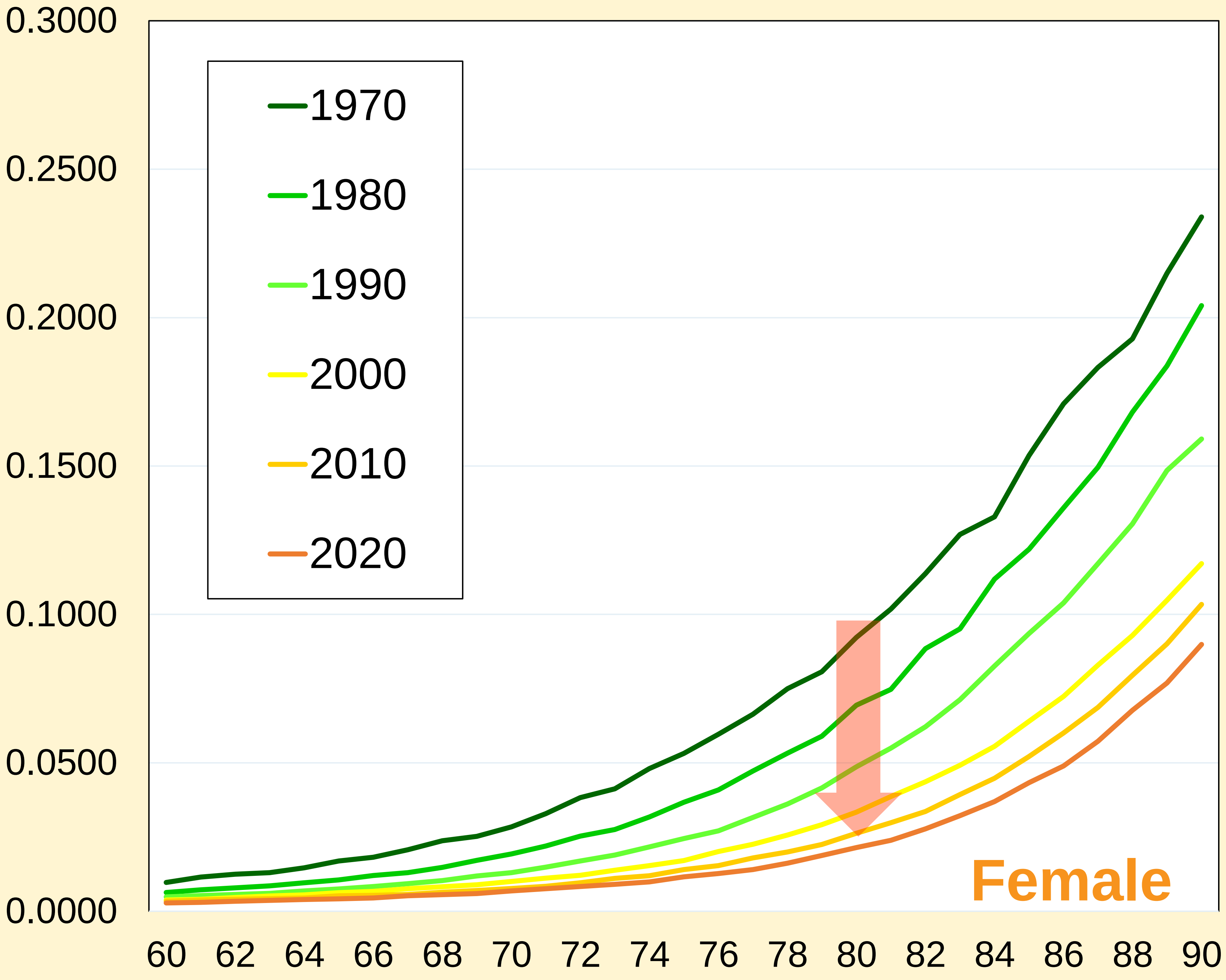
Trends in Population by Sex in Japan



Source: Ministry of Health, Labor and Wealth

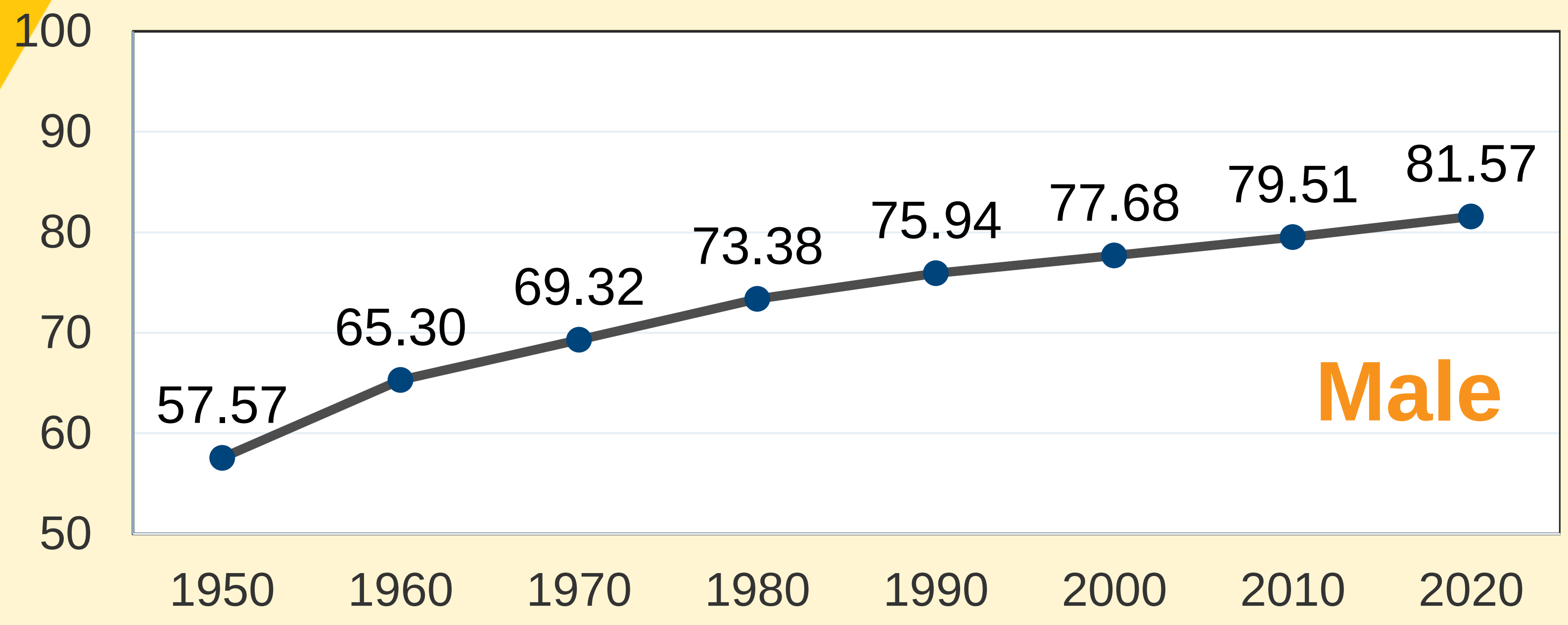
Trends in Mortality Rates for Aged 60 and over in Japan

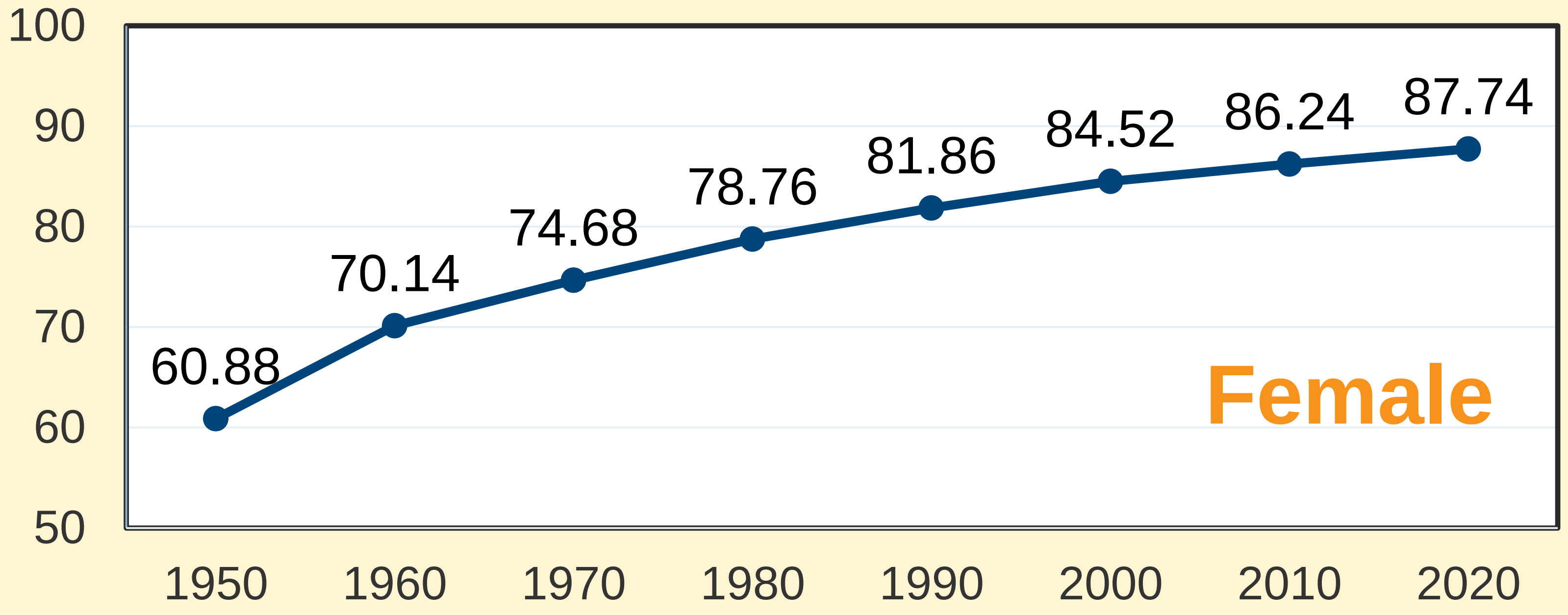




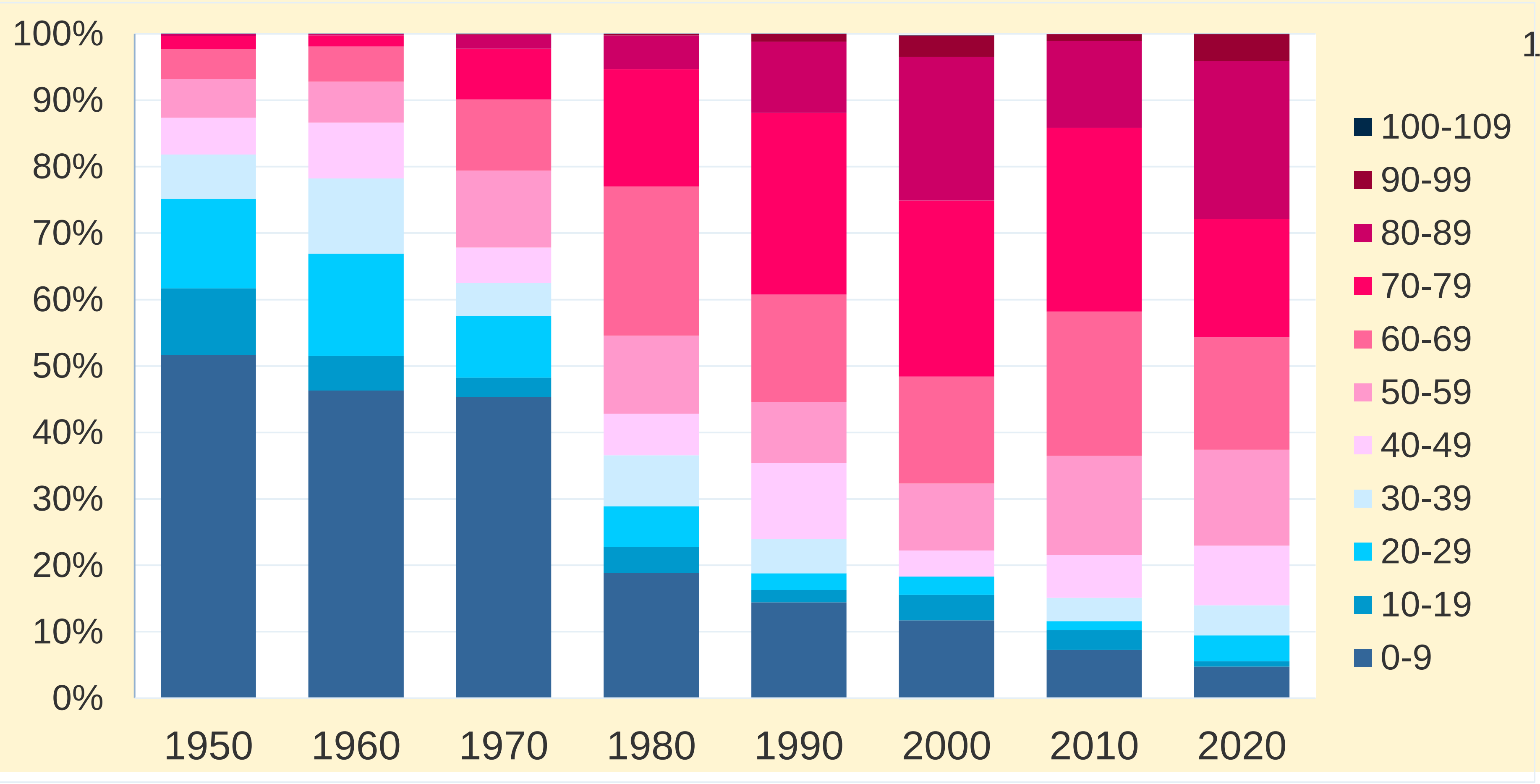
Source: Ministry of Health, Labor and Wealth

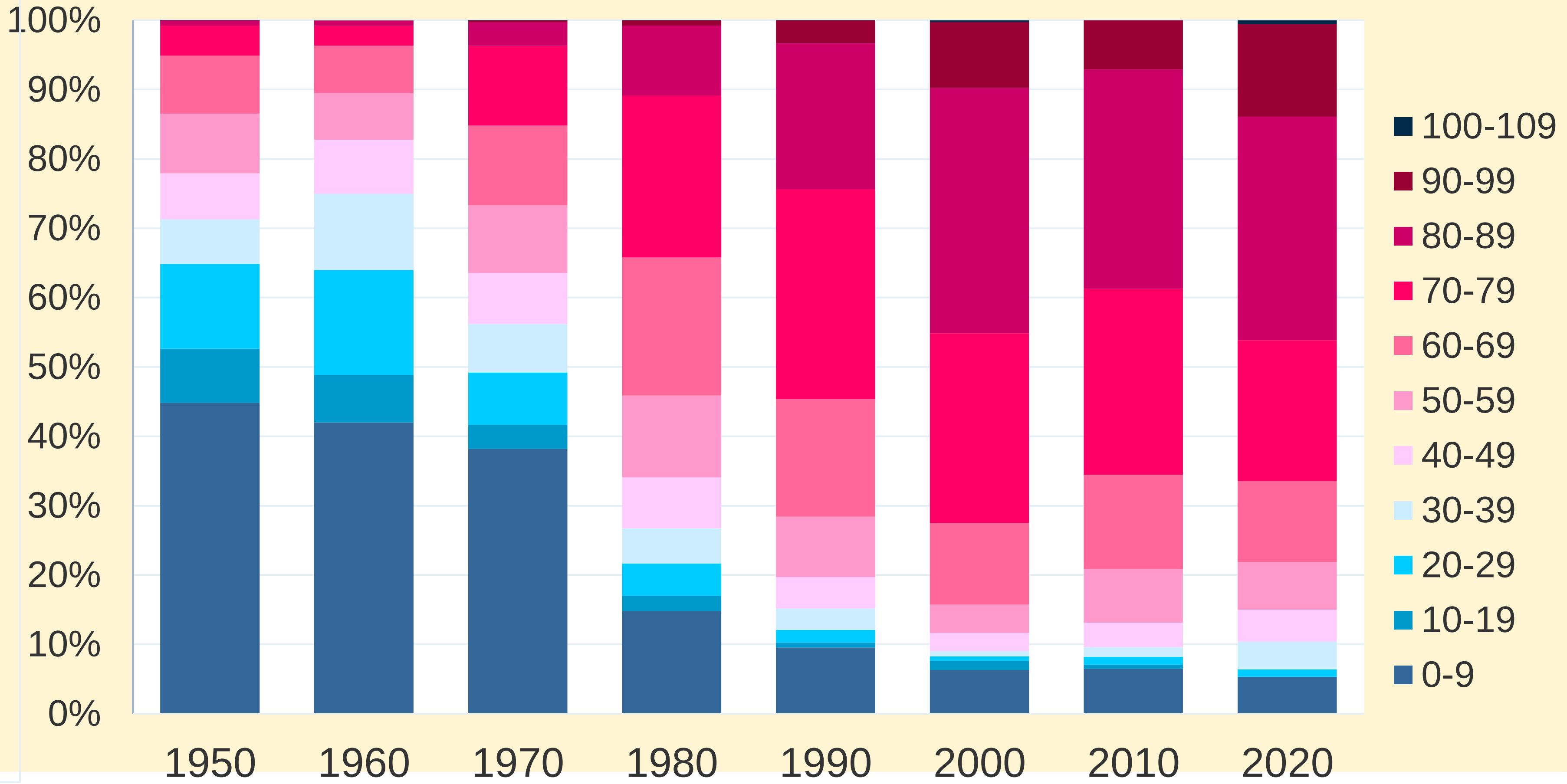
Trends in Life Expectancy in Japan





Contribution of Age-Specific Mortality Improvement to Life Expectancy

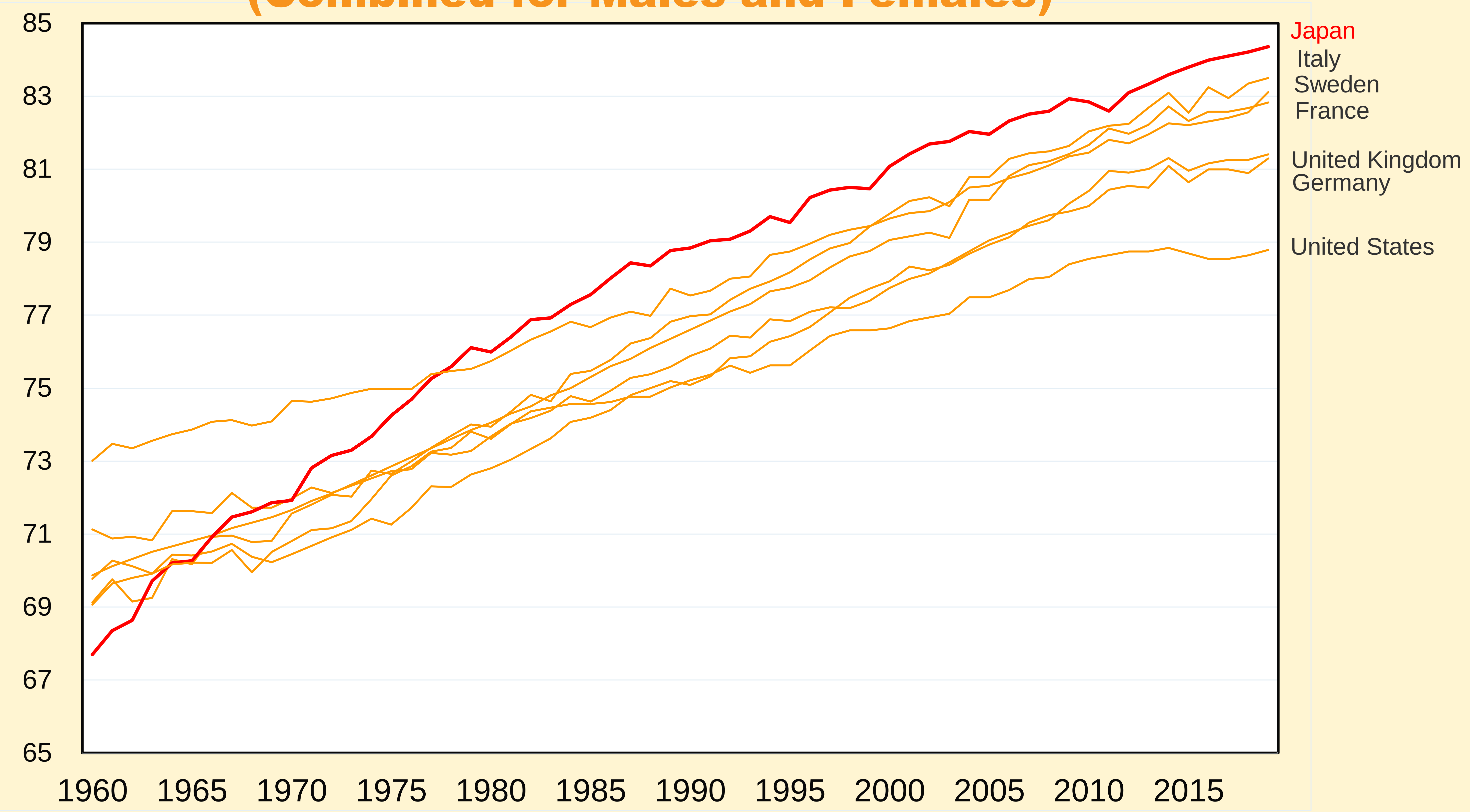




Source: Ministry of Health, Labor and Wealth

International Comparison of Life Expectancy

(Combined for Males and Females)



Source: World Bank

Uses of Mortality Tables in Japanese Life Insurance Companies

1. Insurance Product Pricing

- Mortality rates used for pricing insurance products can be freely set by insurance companies.
- Actuaries consider future mortality trends when reflecting these rates in pricing.

2. Regulatory Valuation Reserves

- Mortality tables used for accounting valuation reserves are standardized across all life insurance companies.
- These tables are created by the Institute of Actuaries of Japan and approved by the Financial Services Agency.
- The mortality tables are revised approximately every 10 years.

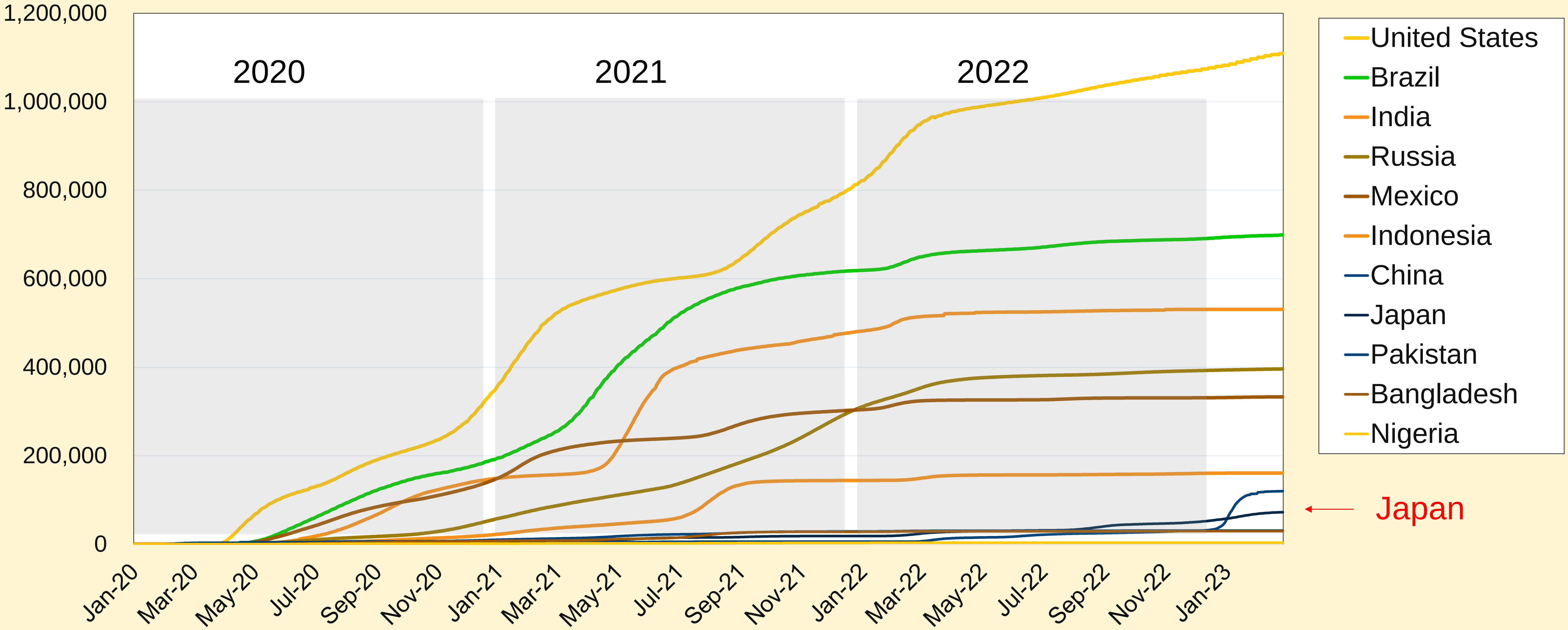
Impact of Mortality Rate Updates

- Japan's significant improvement in mortality rates compared to other countries greatly influences the level of valuation reserves when mortality rates are updated.

Ⅱ. Outbreak of COVID-19

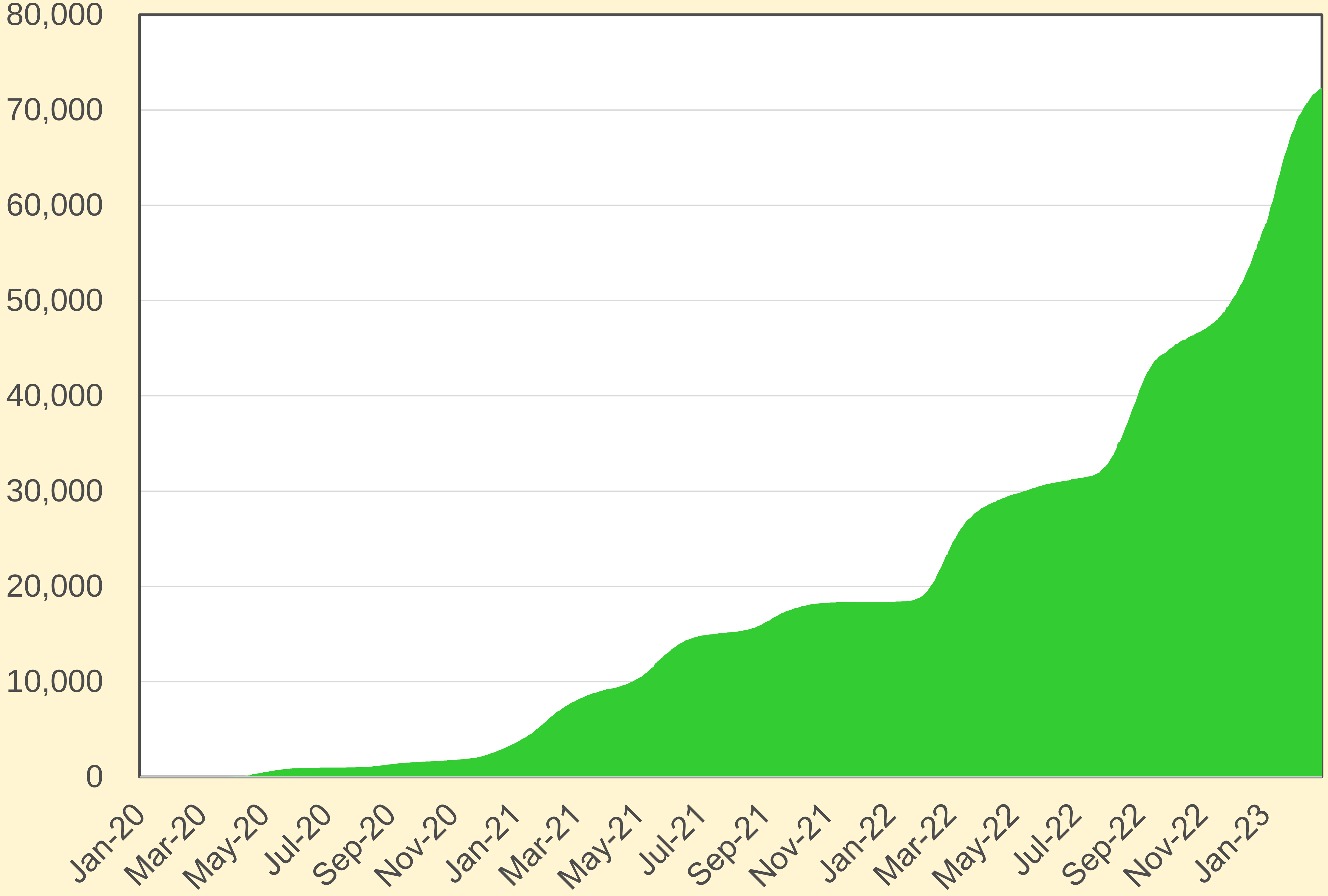
São Paulo 2025

Cumulative number of deaths associated with COVID-19



Source: OurWorldinData.org/coronavirus

Cumulative number of COVID-19 deaths in Japan



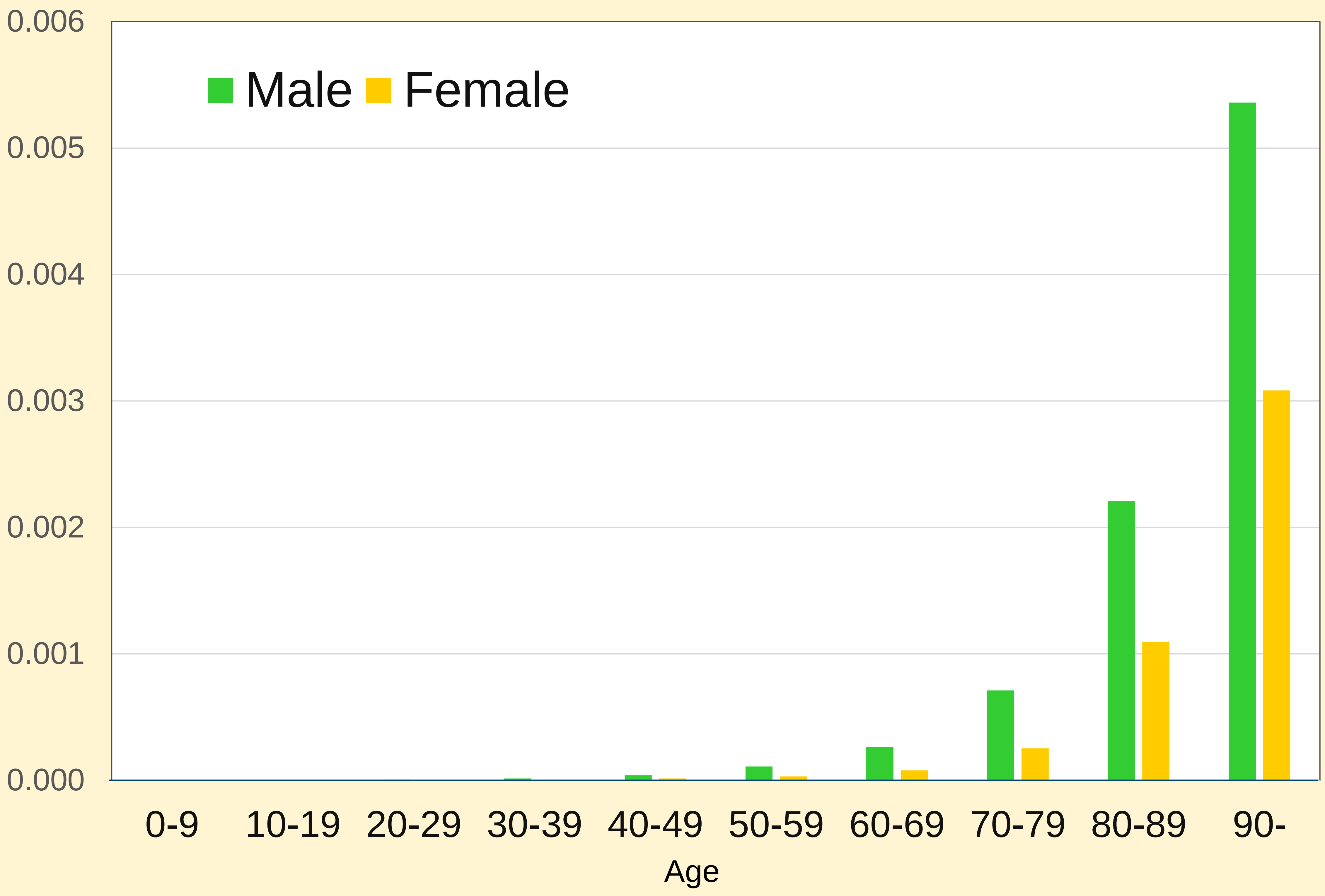
Death rates associated with COVID-19

(per 1,000)

	2020	2021	2022
Japan	0.027	0.131	0.311
United States	1.040	1.380	0.778
United Kingdom	1.114	1.106	0.754
France	0.943	0.839	0.550

Source: OurWorldinData.org/coronavirus

COVID-19 death rates by age in Japan



Source: Ministry of Health, Labor and Wealth

Ⅲ. Using the Lee-Crater model

Lee-Carter Model

$$ln(m_{x,t}) = a_x + b_x\kappa_t + \varepsilon_{x,t}$$

Constraint $\sum_{x=0}^{\omega} b_x = 1, \quad \sum_{t=1}^{T} \kappa_t = 0,$

Minimise $S = \sum_{x=0}^{\omega}\sum_{t=1}^{T}\left[ln(m_{x,t}) - a_x - b_x\kappa_t\right]^2 = \sum_{x,t} \varepsilon_{x,t}^2 ,$

$m_{x,t}$; the observed death rate at age x in year t

a_x ; the average mortality for the observation period

b_x ; the pattern of deviation from the age profile as the k_t varies

κ_t ; the change in overall mortality

$\varepsilon_{x,t}$; the residual term at age x and time t

Specific Steps:

1. Data Preparation:
- Collect age-specific and time-specific mortality rate data.
- Use mortality rate data by gender and age from 1996 to 2021.

2. Application of the Lee-Carter Model:
- Apply the Lee-Carter model and set initial values for each parameter.
- Calculate the difference between observed mortality rates and those predicted by the model.

3. Parameter Optimization:
- Update the parameters to minimize the residuals.

Minimise $$S = \sum_{x=0}^{\omega}\sum_{t=1}^{T}\left[ln\left(m_{x,t}\right) - a_x - b_x\kappa_t\right]^2$$

$$\frac{\partial S}{\partial a_x} = 0 \Rightarrow a_x = \frac{1}{T}\sum_{t=1}^{T}\ln\left(m_{x,t}\right) \Rightarrow (Z)_{x,t} := ln\left(m_{x,t}\right) - a_x \Rightarrow$$

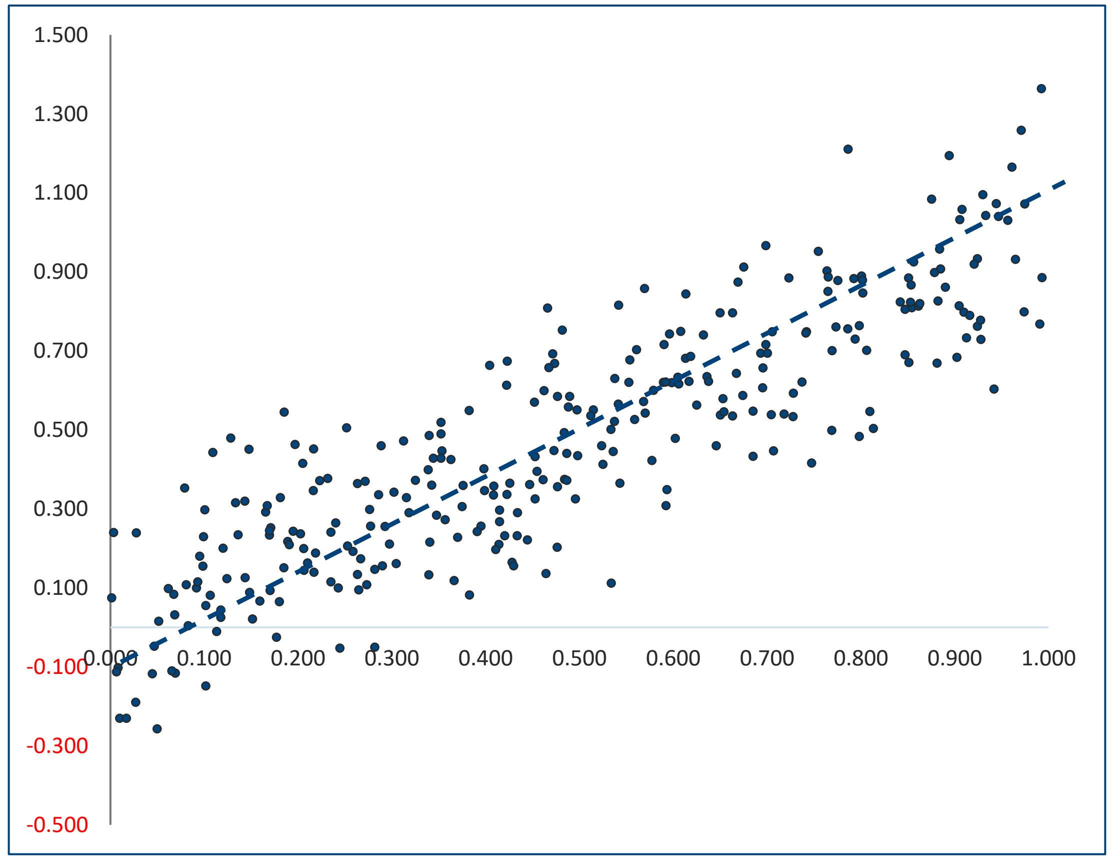
Minimise $$S = \sum_{x=0}^{\omega}\sum_{t=1}^{T}\left((Z)_{x,t} - b_x\,\kappa_t\right)^2$$

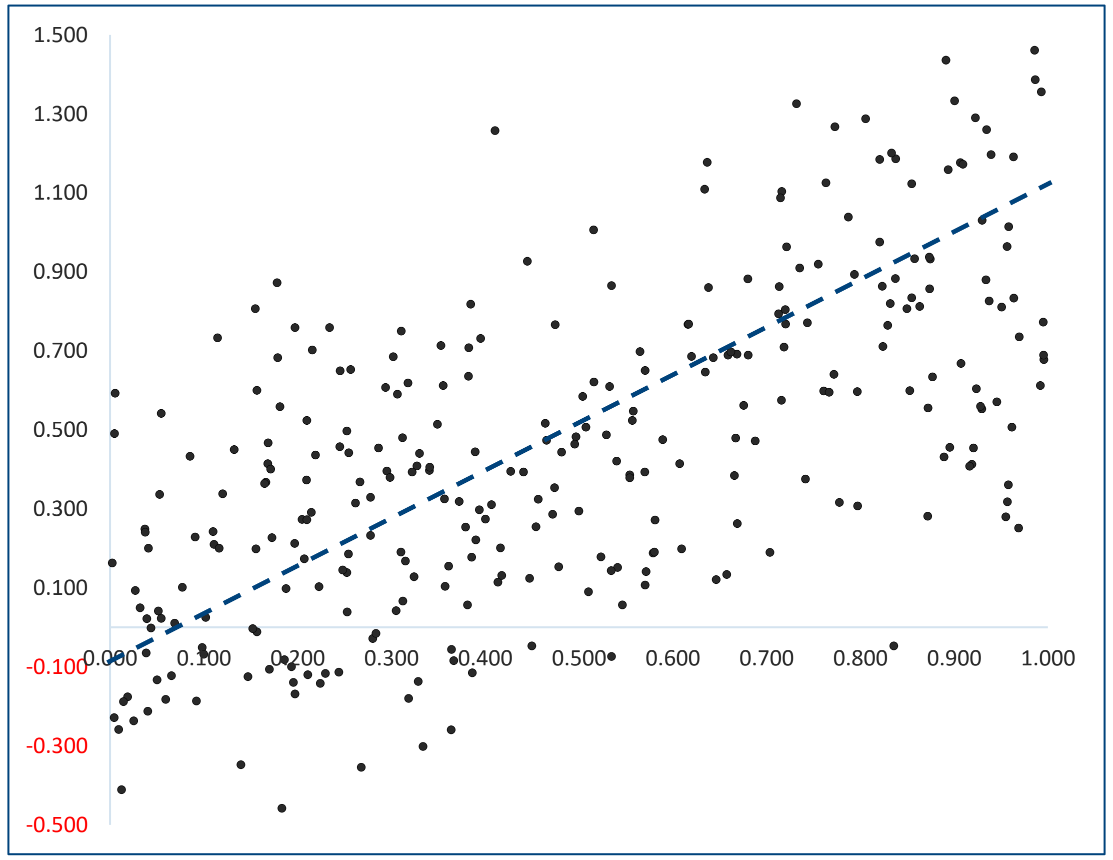
Parameter Estimation Method - Least Squares

Minimise $$S = \sum_{x=0}^{\omega} \sum_{t=1}^{T} \left[ln(m_{x,t}) - a_x - b_x \kappa_t \right]^2$$

However, just because it is obtained using the least squares method does not necessarily mean it is a good approximation.

For example, the following case…





Singular Value Decomposition (SVD)

For any matrix , the Singular Value Decomposition (SVD) is given by:

$$\mathrm{Z} = \mathrm{U\,S\,V^T}, \qquad (Z)_{xt} = \sum_{i=1}^{r} s_i\, u_{xi}\, v_{ti}\ , \qquad s_1 \geq s_2 \geq s_3 \geq \cdots \geq s_r$$

where:

- U is an orthogonal matrix whose columns are the left singular vectors, $\mathrm{U} = [\vec{\boldsymbol{u}}_1, \vec{\boldsymbol{u}}_2, \cdots]$
  The vector $\vec{\boldsymbol{u}}_i$ is an eigenvector of the matrix $\mathrm{Z\,Z^T}$ corresponding to the eigenvalue λ_i .
  $$\mathrm{Z\,Z^T}\,\vec{\boldsymbol{u}}_i = \lambda_i \vec{\boldsymbol{u}}_i\ , \quad \sqrt{\lambda_i} = s_i$$
- S is a diagonal matrix with singular values on the diagonal, $\mathrm{S} = diag(s_{1,} s_2, \cdots, s_r)$
- V is an orthogonal matrix whose columns are the right singular vectors. $\mathrm{V} = [\vec{\boldsymbol{v}}_1, \vec{\boldsymbol{v}}_2, \cdots]$
  The vector $\vec{\boldsymbol{v}}_i$ is an eigenvector of the matrix $\mathrm{Z^T Z}$ corresponding to the eigenvalue λ_i .
  $$\mathrm{Z^T Z}\,\vec{\boldsymbol{v}}_i = \lambda_i \vec{\boldsymbol{v}}_i\ , \quad \sqrt{\lambda_i} = s_i$$

Rank-1 approximation

The singular values compactly represent the information in the matrix and are closely related to its rank.

$$Z = U\,S\,V^T, \qquad (Z)_{x,t} = \sum_{i=1}^{r} s_i\, u_{xi}\, v_{ti} = s_1 u_{x1}\, v_{t1} + s_2 u_{x2}\, v_{t2} + s_3 u_{x3}\, v_{t3} + \cdots$$

If $s_1 \gg s_2 \geq s_3 \geq \cdots \geq s_r$, it is possible to perform an approximate calculation with rank 1

$$\sim s_1 u_{x1}\, v_{t1}$$ **Rank-1 Approximation**

$$b_x \kappa_t = s_1 u_{x1}\, v_{t1}$$

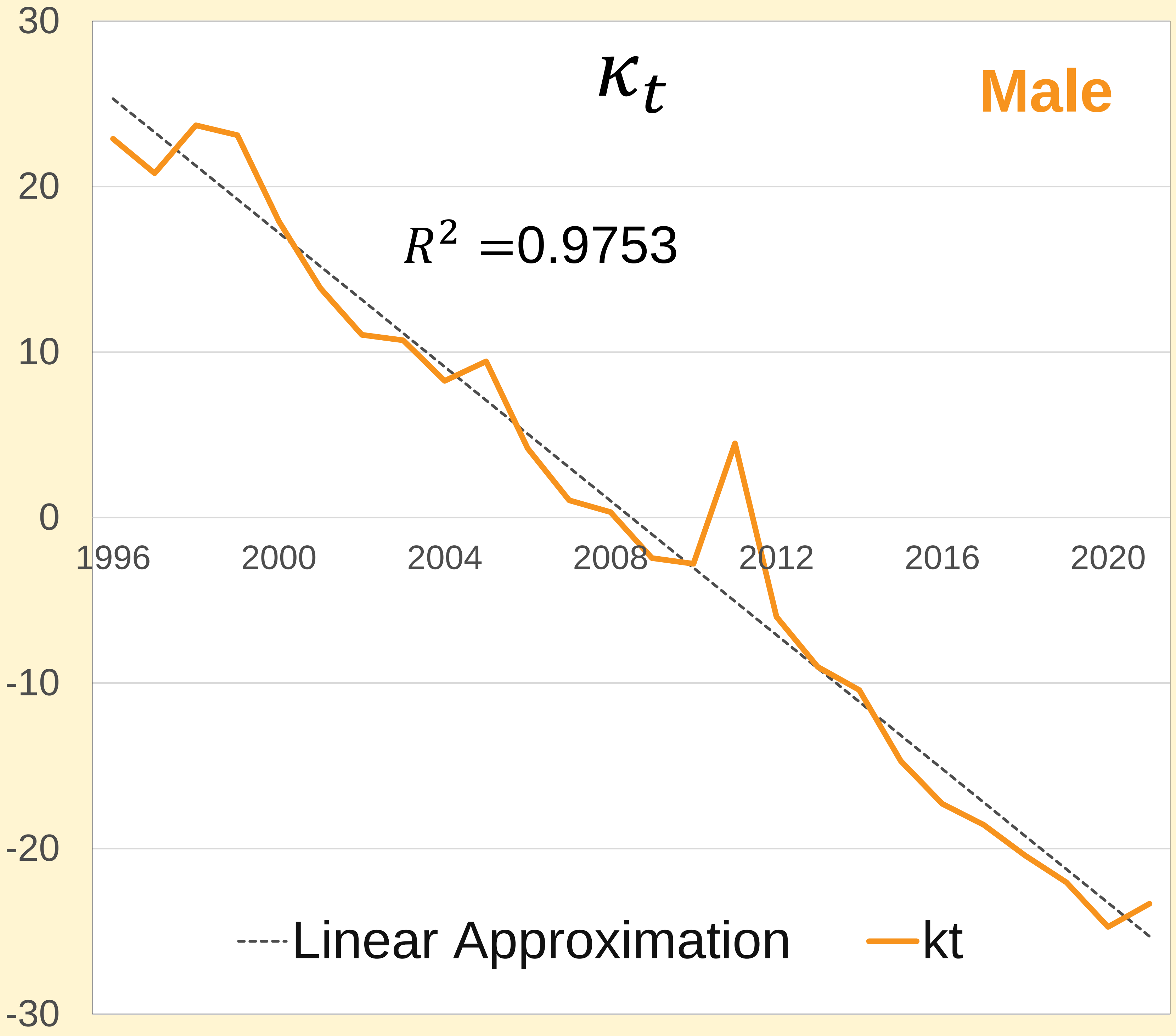
$$b_x \propto u_{x1}$$
$$\kappa_t \propto v_{t1}$$
$$\varepsilon_{x,t} = \sum_{i=2}^{r} s_i\, u_{xi}\, v_{ti}$$

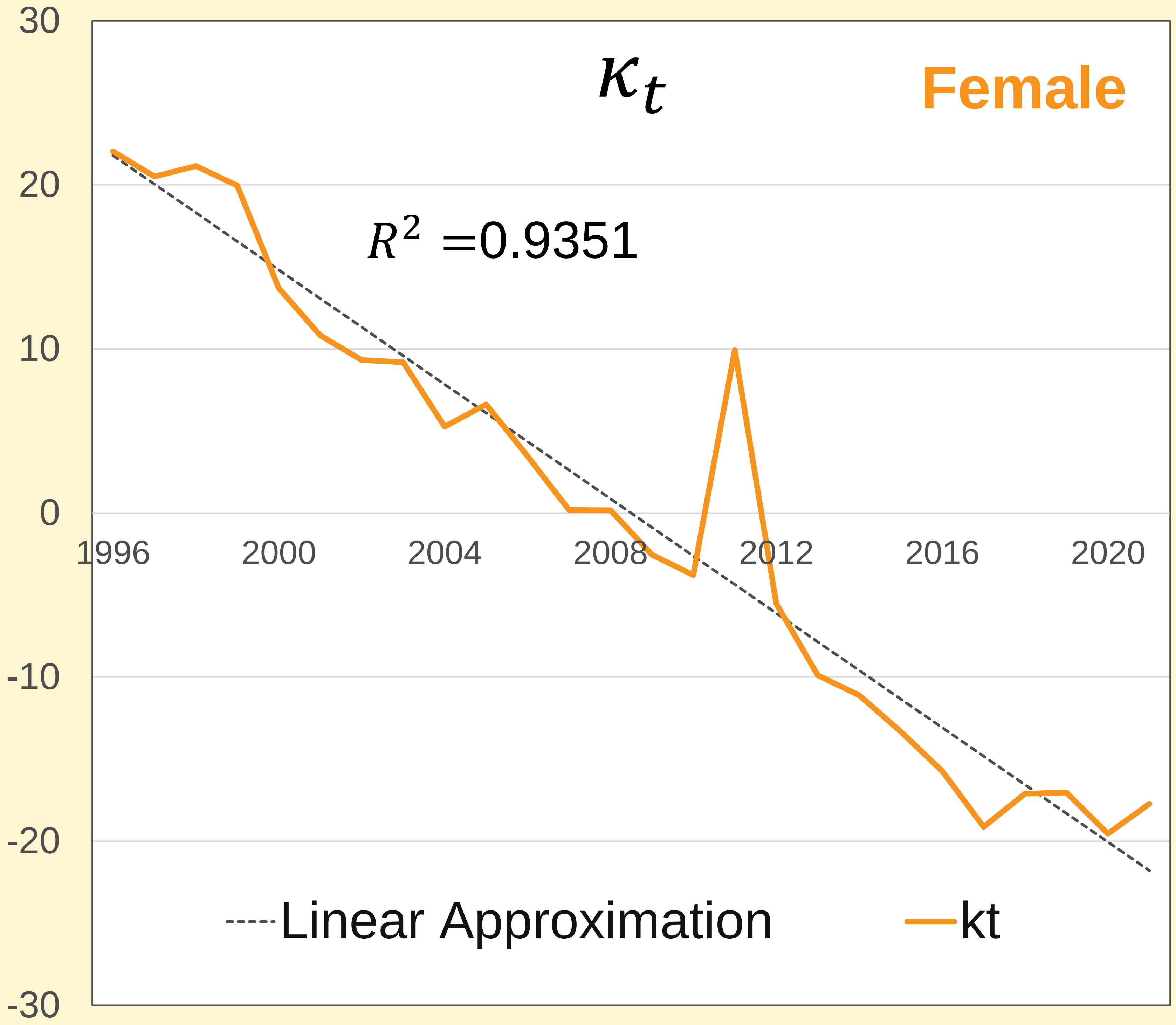
From $\sum_{x=0}^{\omega} b_x = 1$, b_x is determined.

From $\sum_{t=1}^{T} \kappa_t = 0$, κ_t is determined.

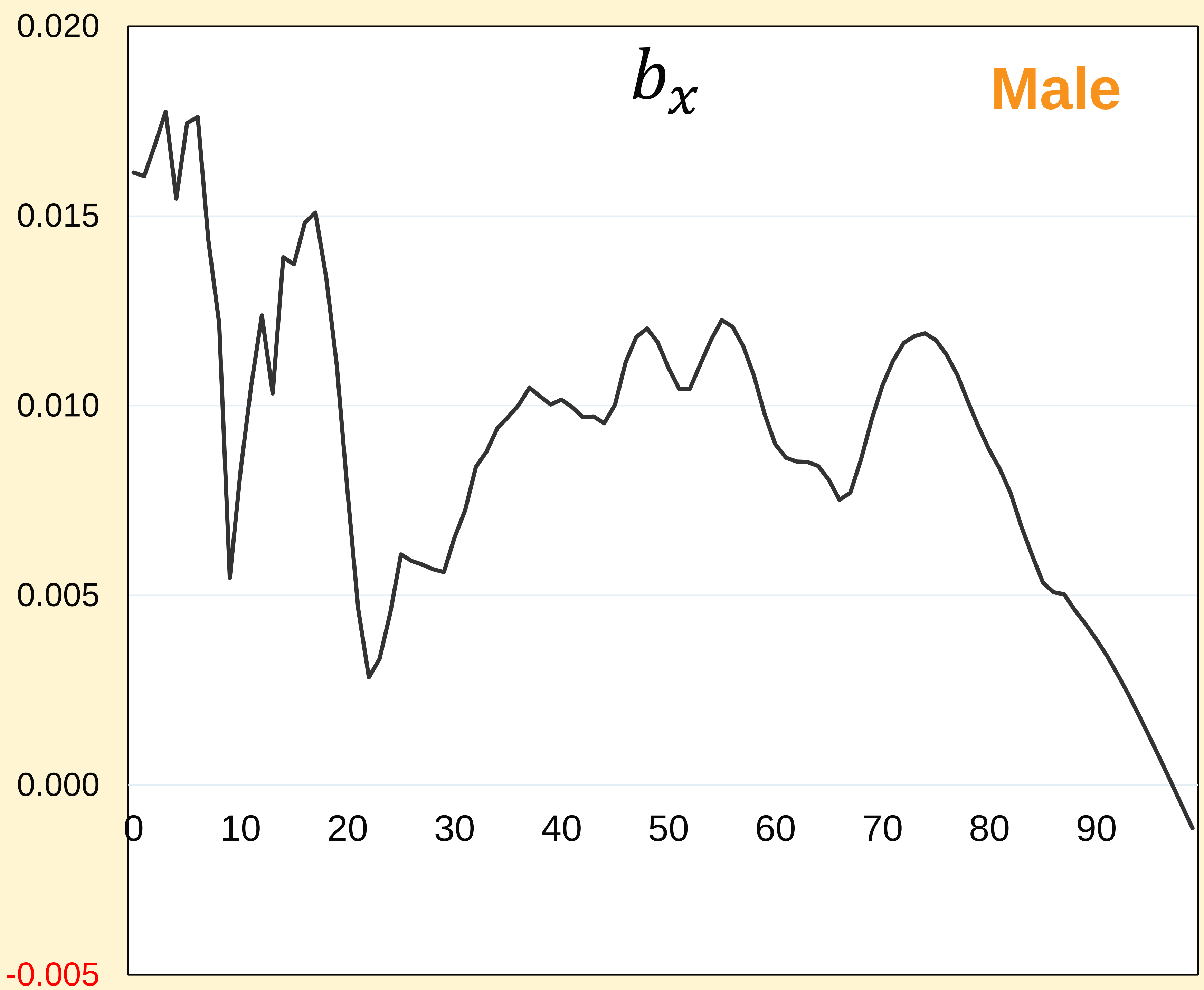
s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	…
8.526	1.873	1.533	0.943	0.778	0.595	0.536	0.466	…
46.6%	10.2%	8.4%	5.2%	4.3%	3.3%	2.9%	2.5%	…

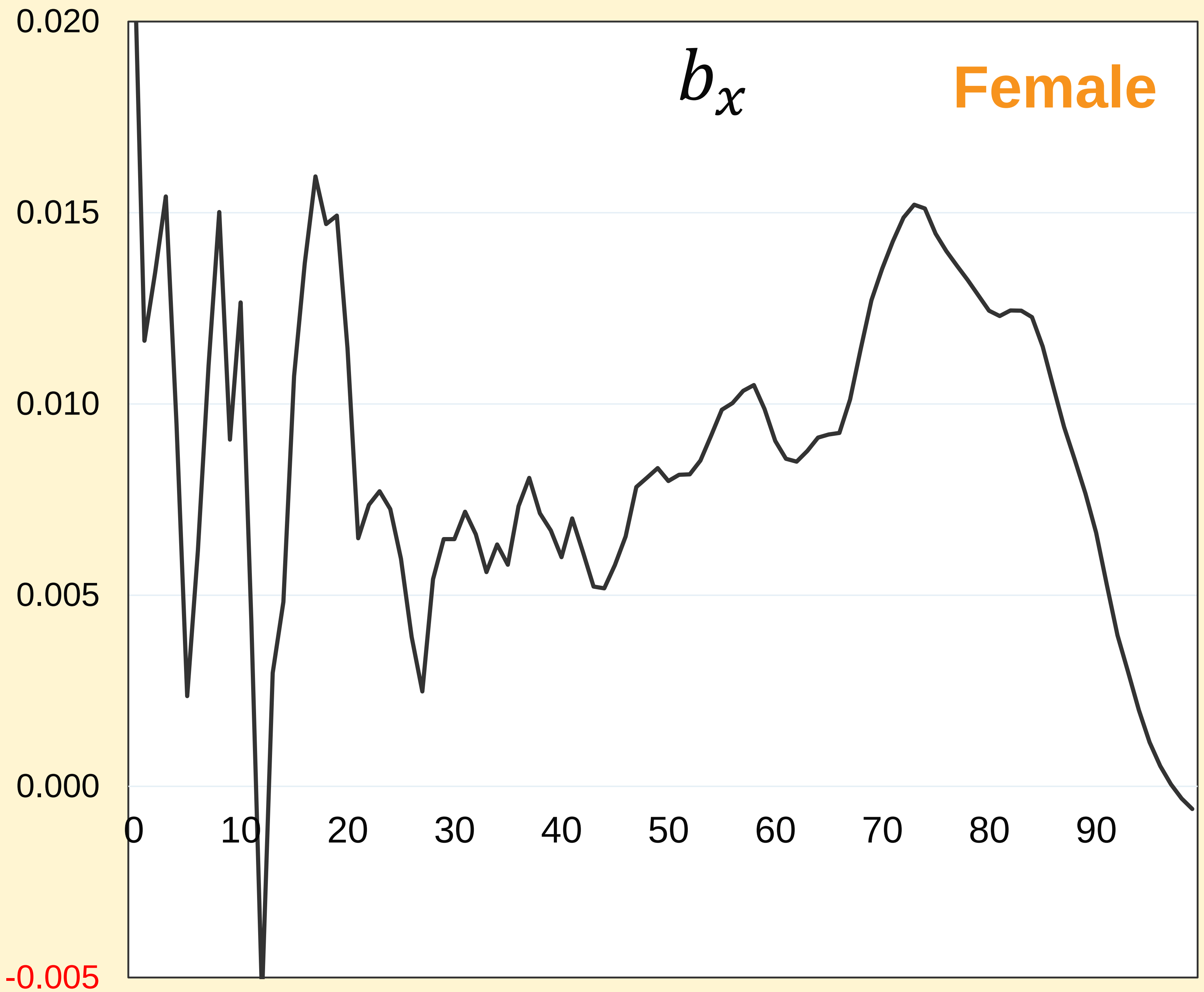
Parameter Estimate κ_t





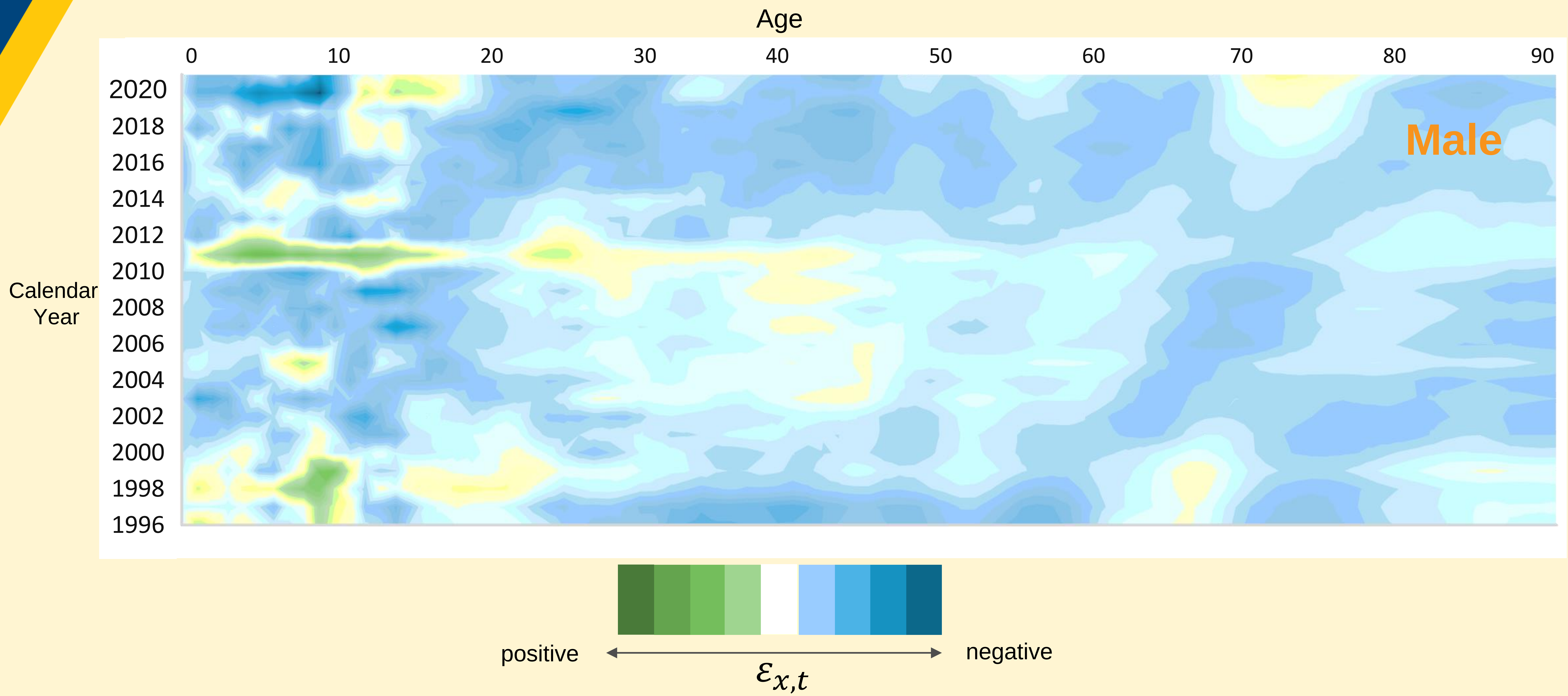
Parameter Estimate b_x





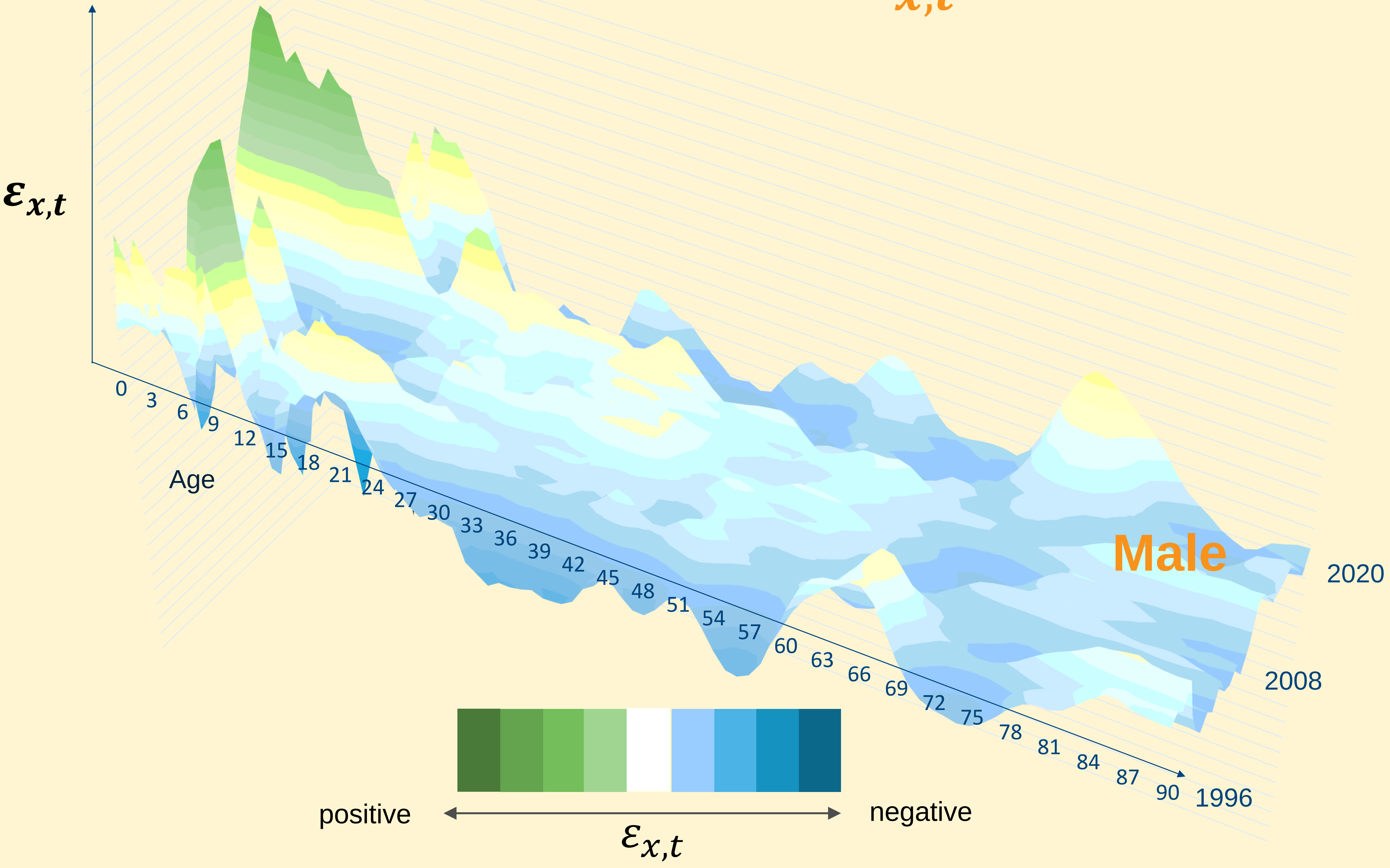
Ⅳ. Numerical Examples

Lee-Carter Residual Term $\varepsilon_{x,t}$



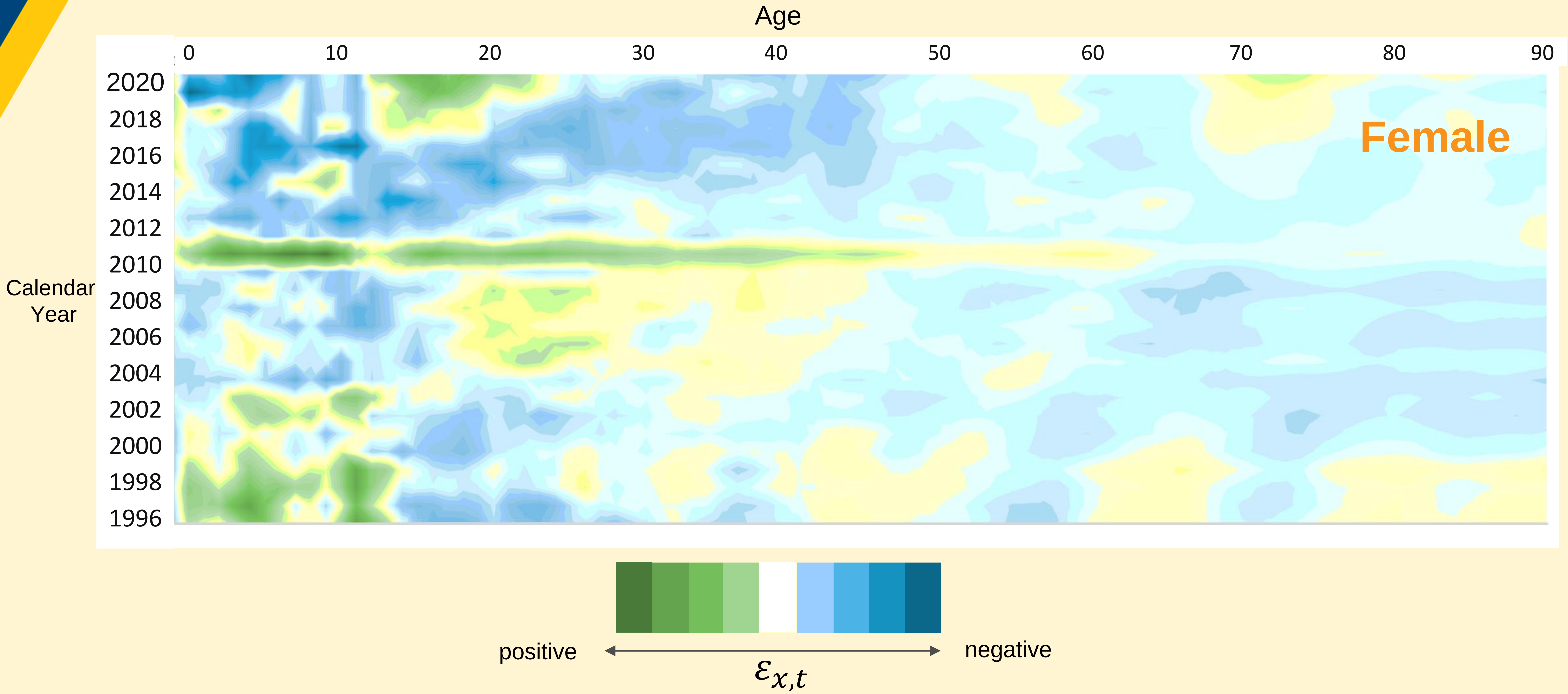
Source: Ministry of Health, Labor and Wealth

Lee-Carter Residual Term $\varepsilon_{x,t}$



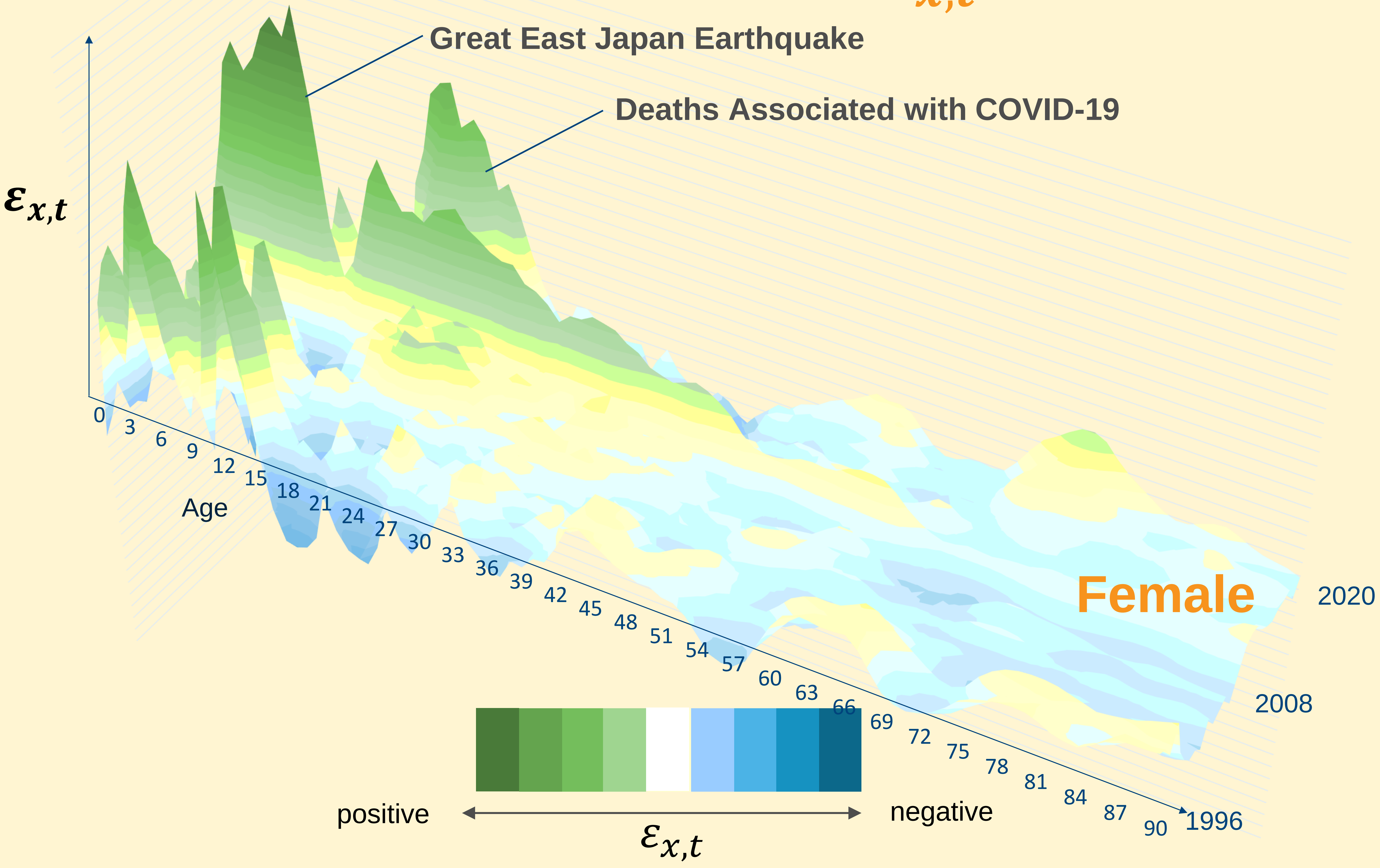
Source: Ministry of Health, Labor and Wealth

Lee-Carter Residual Term $\varepsilon_{x,t}$



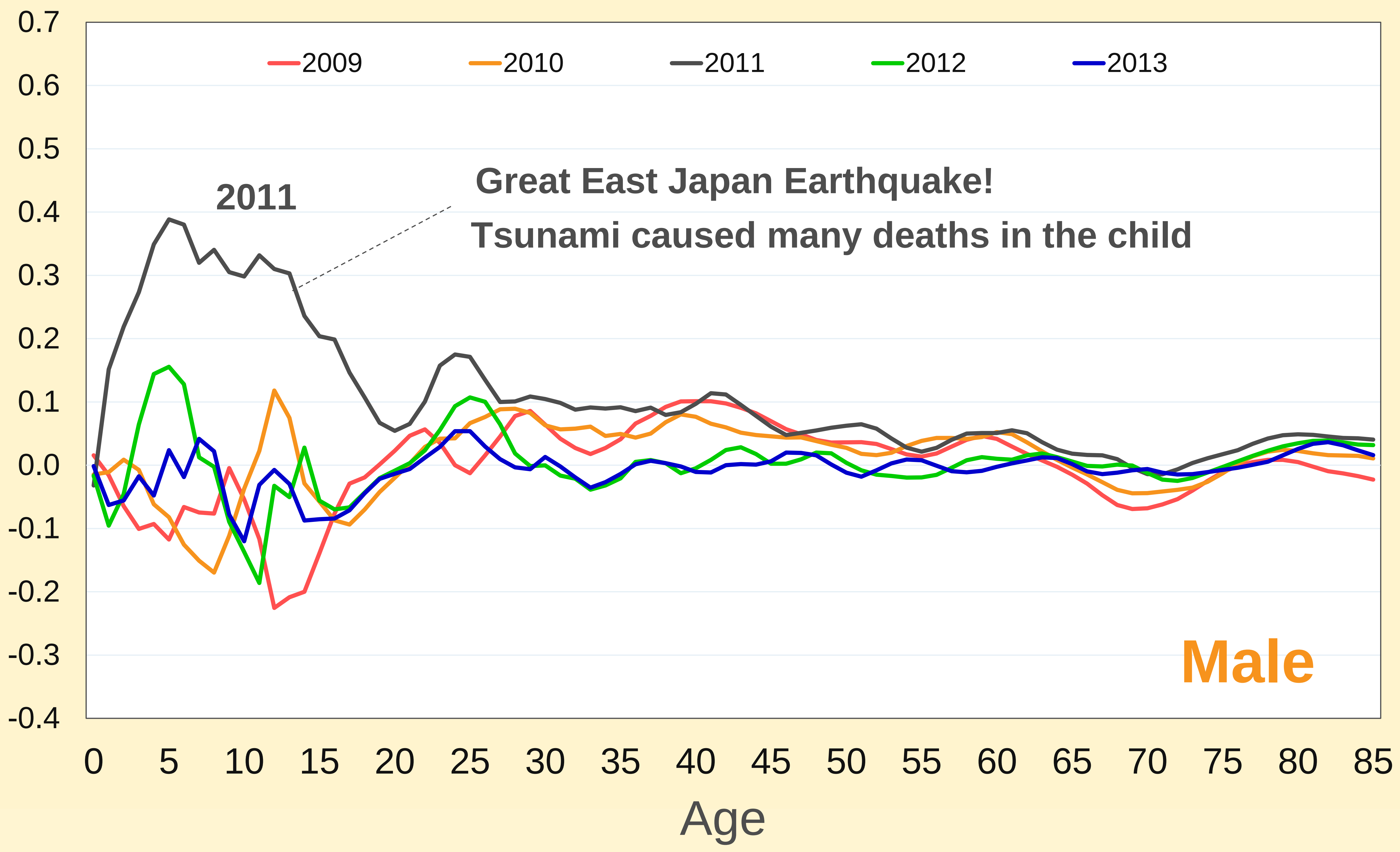
Source: Ministry of Health, Labor and Wealth

Lee-Carter Residual Term $\varepsilon_{x,t}$

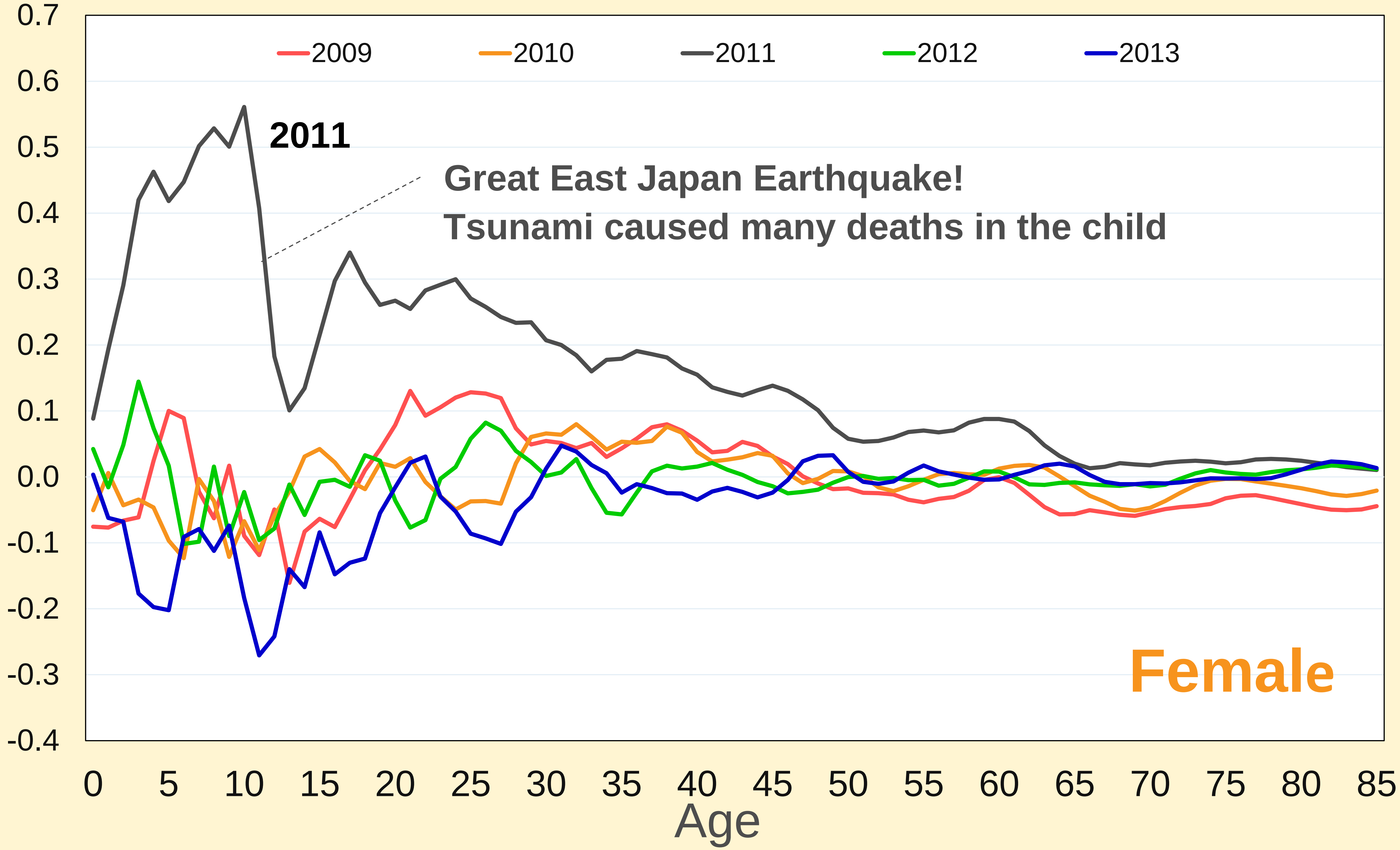


Source: Ministry of Health, Labor and Wealth

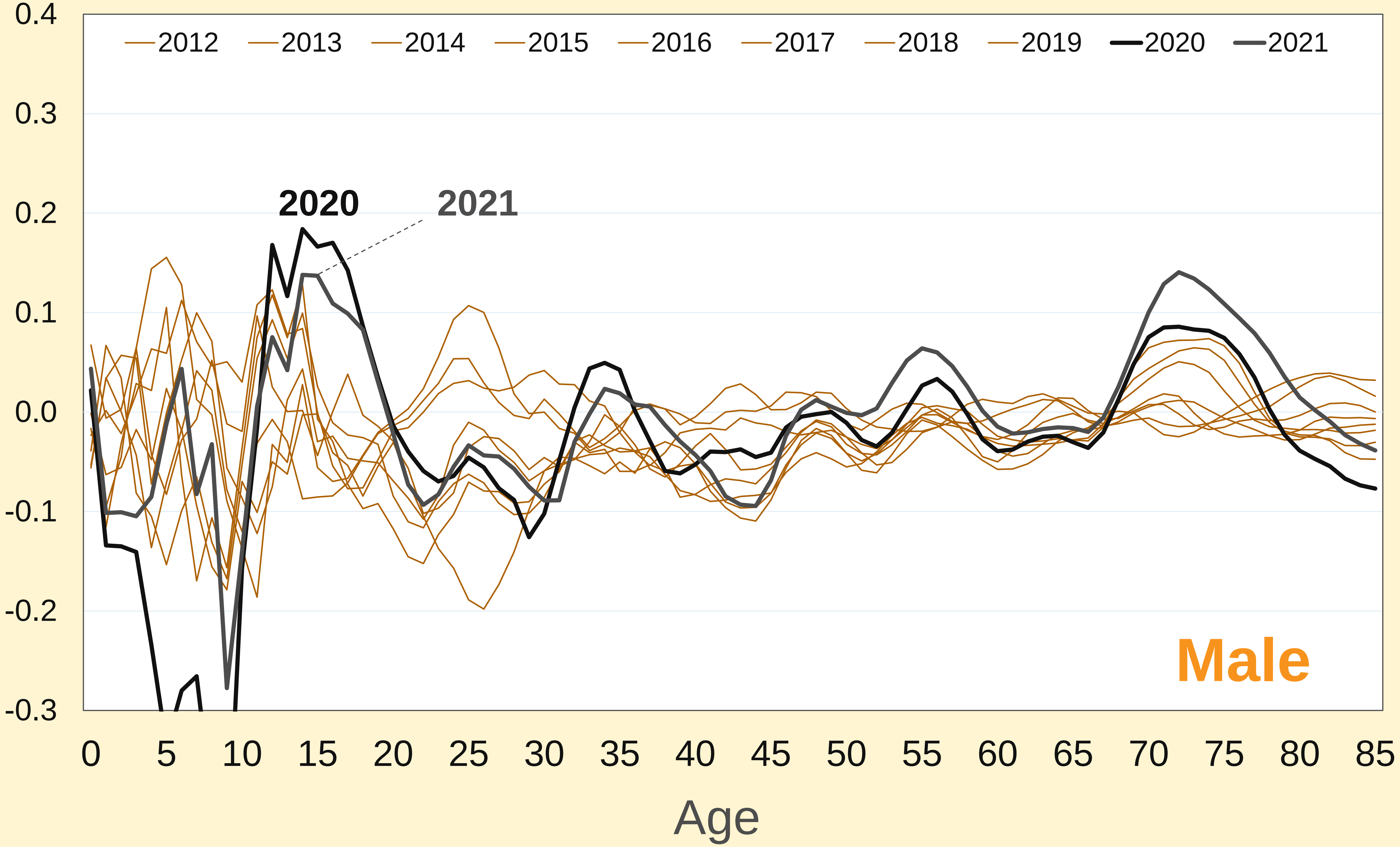
Lee-Carter Residual Term $\varepsilon_{x,t}$ with Fixed Calendar Year 2009 - 2013



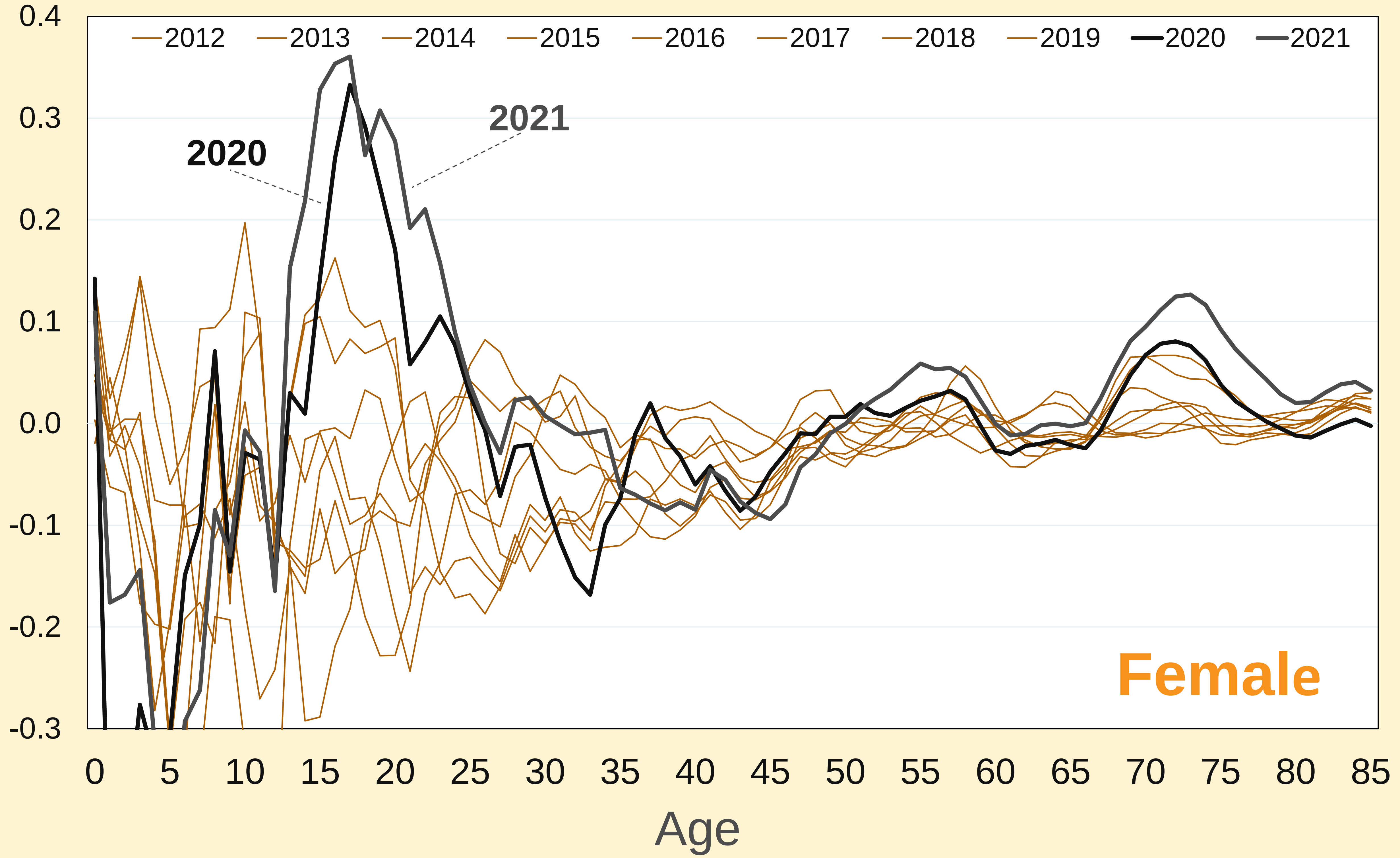
Lee-Carter Residual Term $\varepsilon_{x,t}$ with Fixed Calendar Year 2009 - 2013



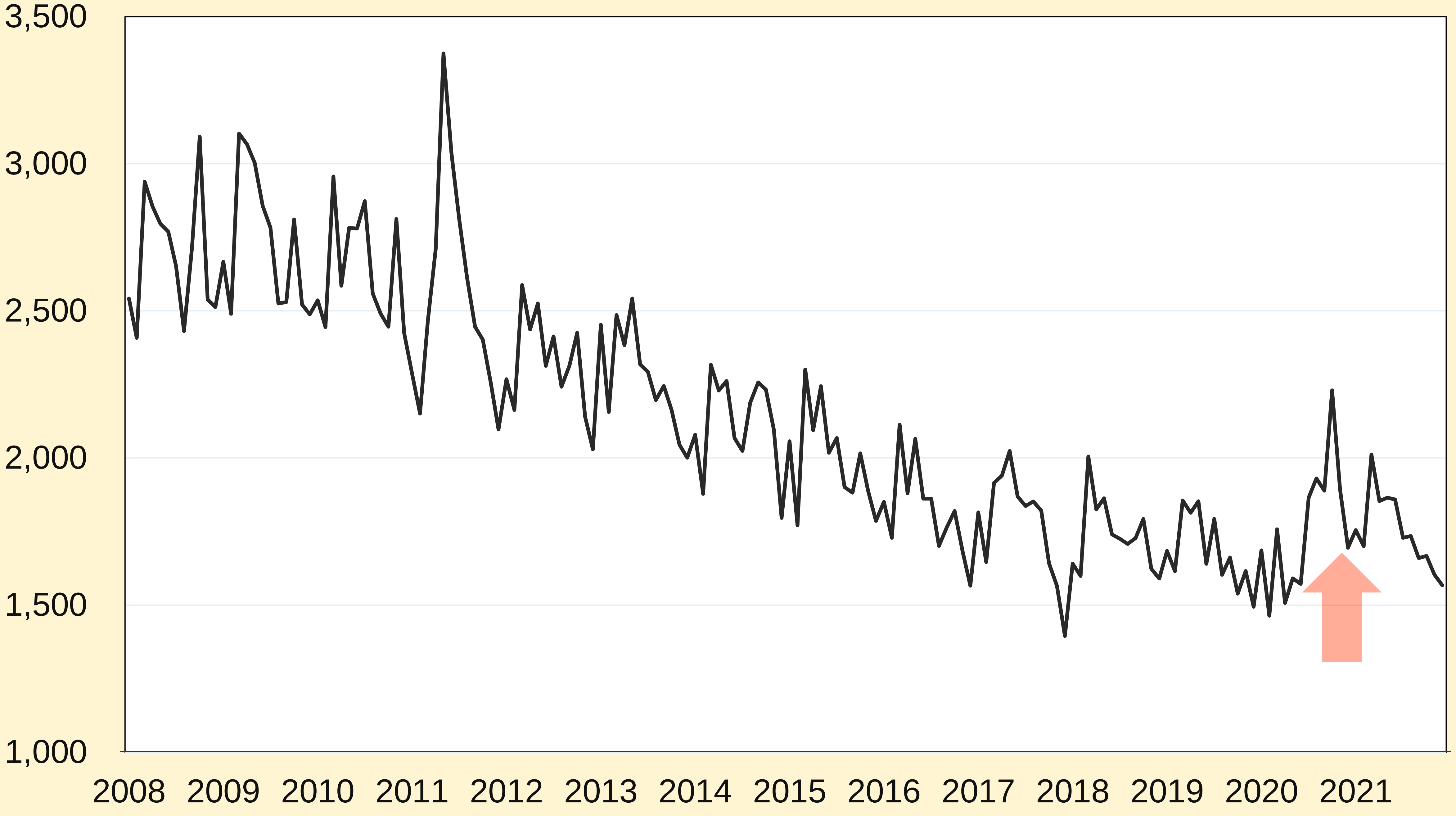
Lee-Carter Residual Term $\varepsilon_{x,t}$ with Fixed Calendar Year 2018 - 2021



Lee-Carter Residual Term $\varepsilon_{x,t}$ with Fixed Calendar Year 2018 - 2021

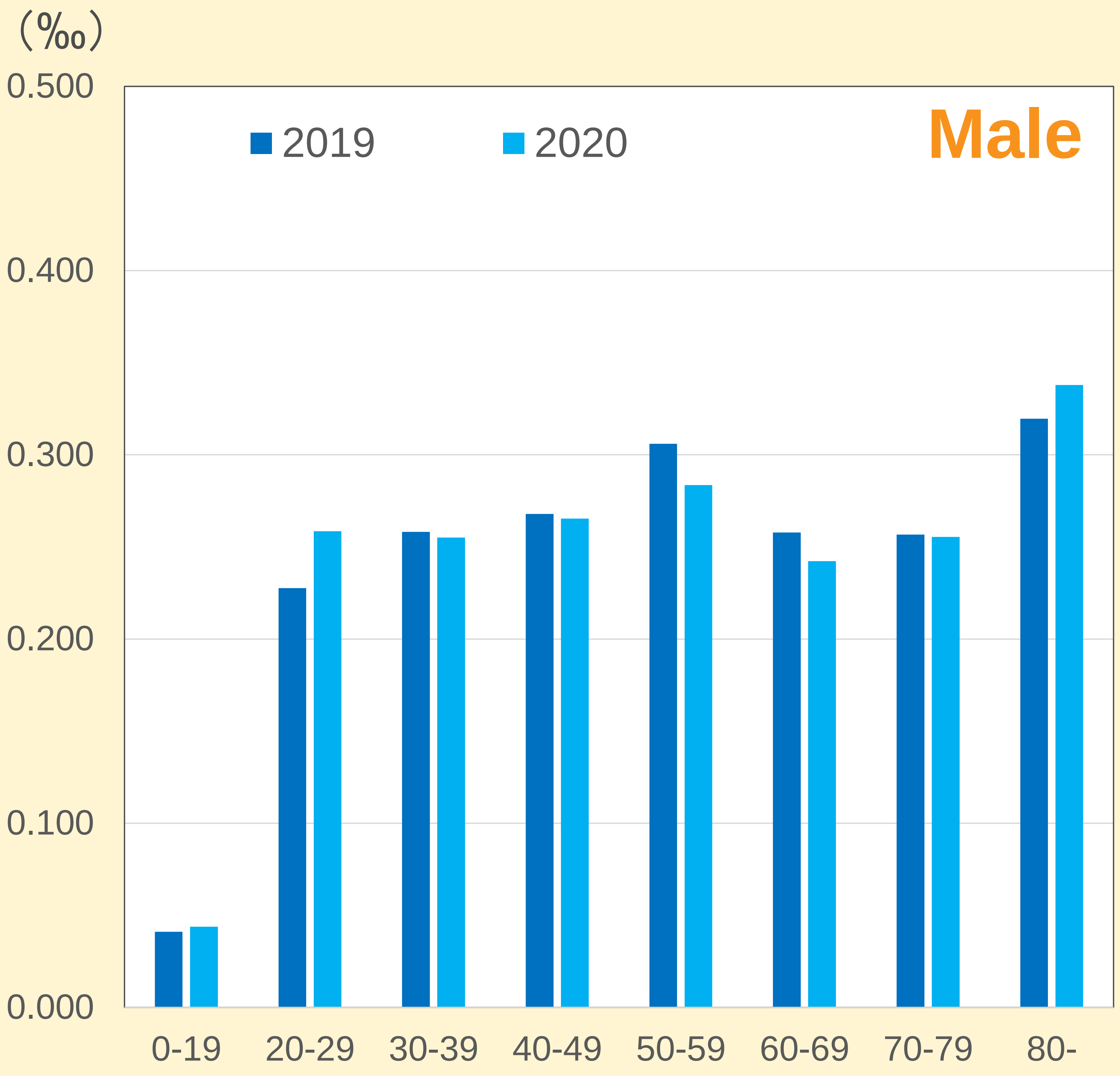


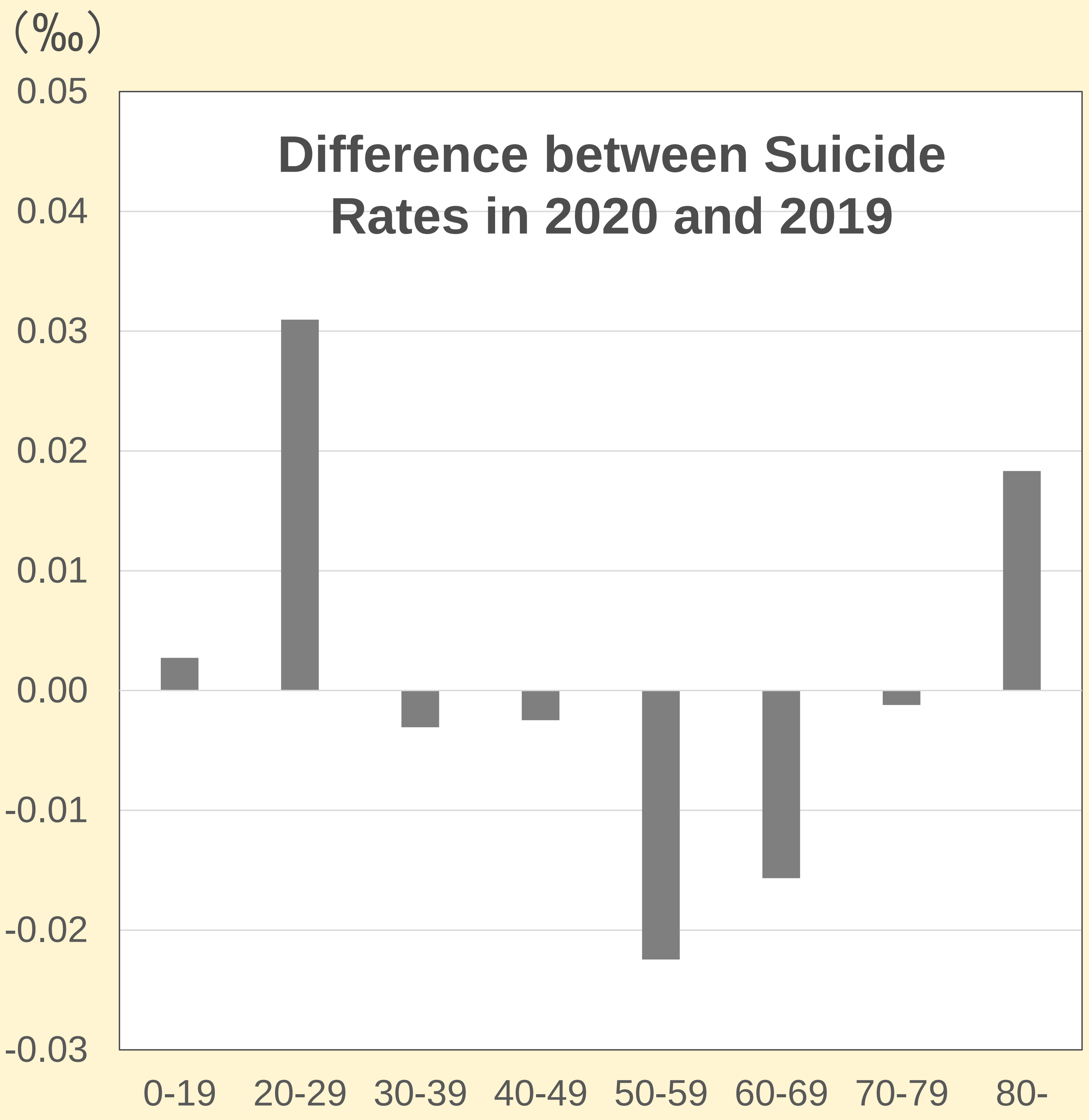
The number of Suicide in Japan



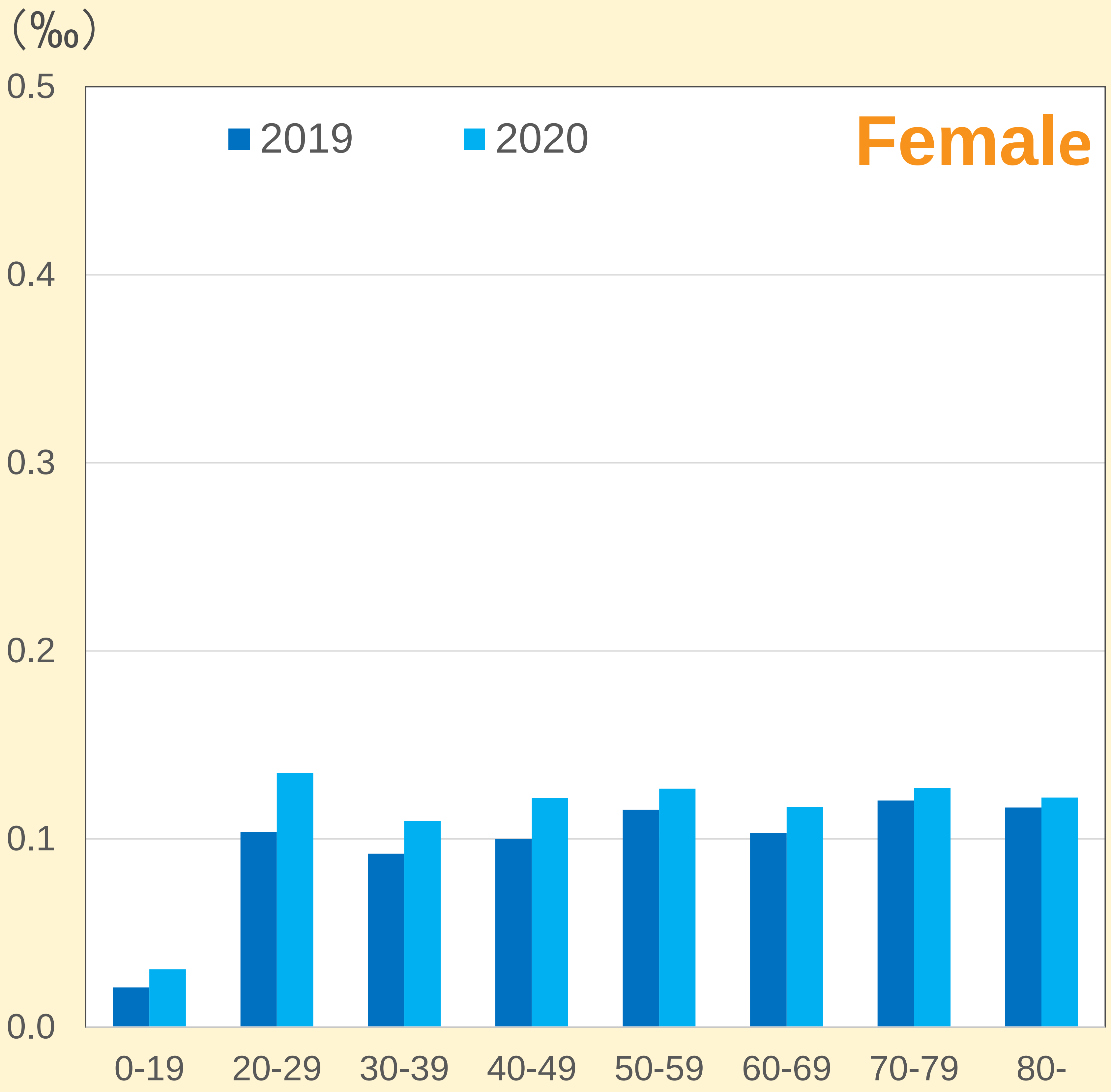
Source: National Police Agency

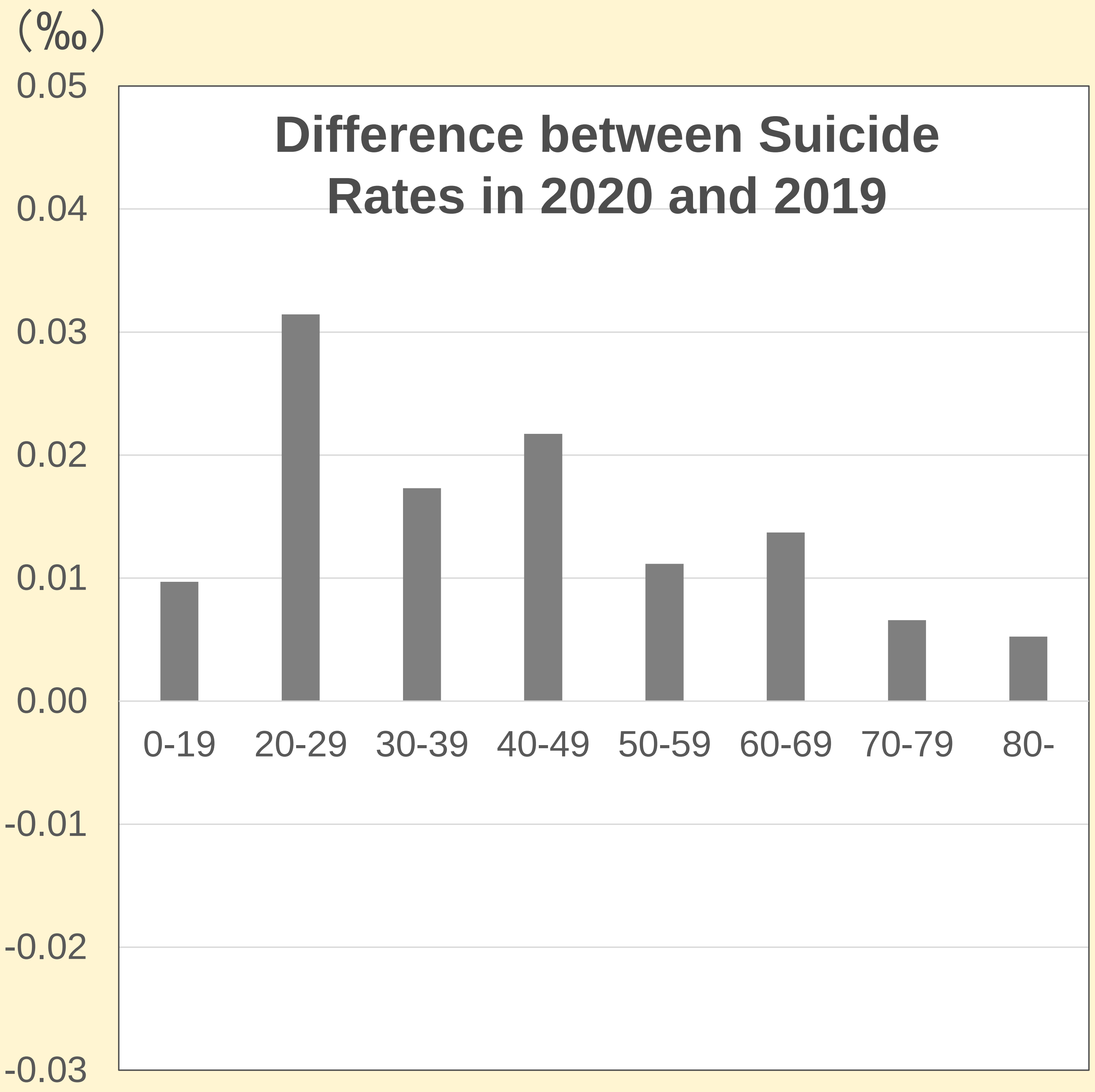
Suicide Rates in 2019 and 2020





Suicide Rates in 2019 and 2020





Contribution of Suicide Rate to Residual Term $\varepsilon_{x,2020}$ in 2020

<u>*t* =2020</u>

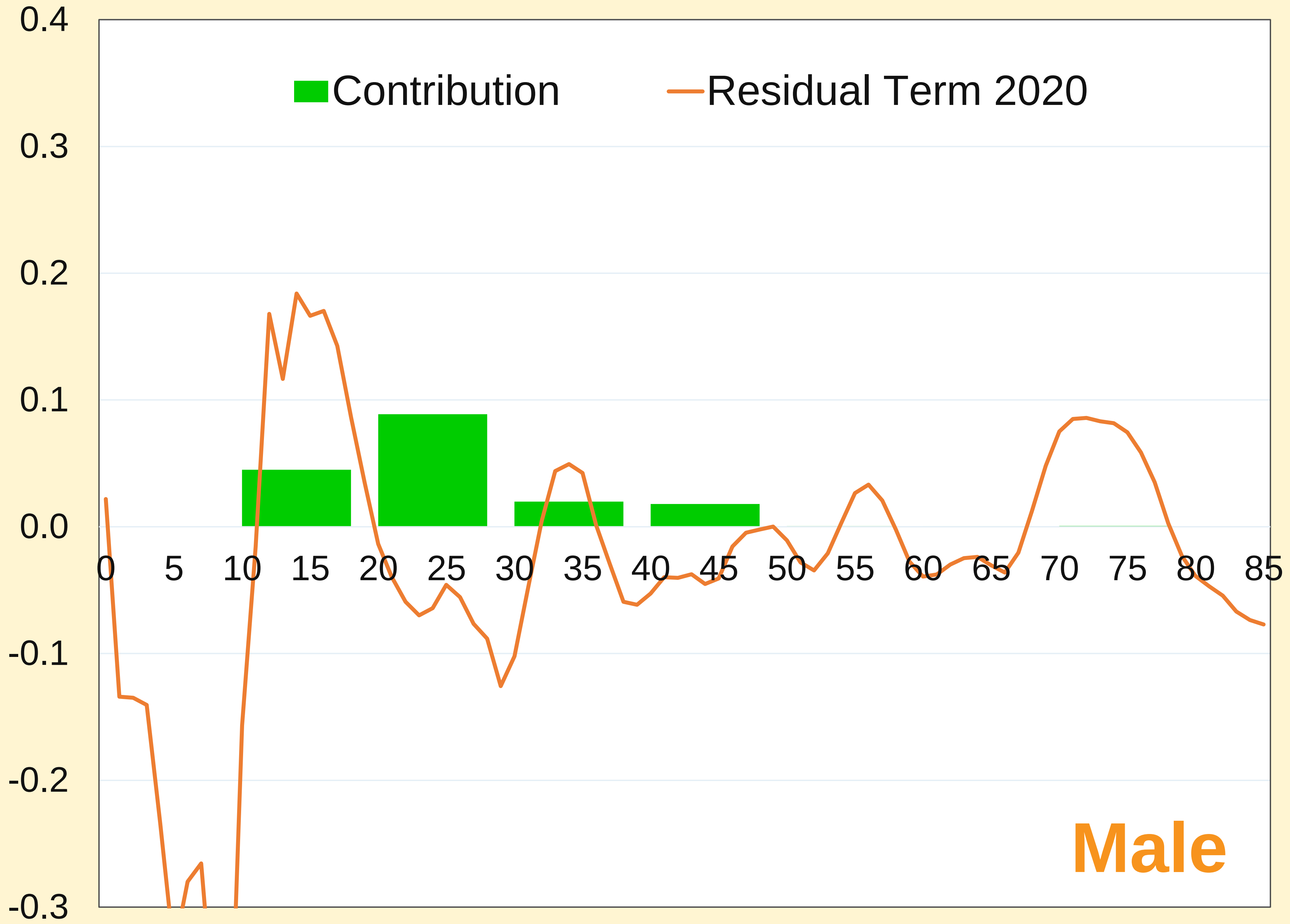
$$ln(m_{x,t}) = a_x + b_x\kappa_t + \varepsilon_{x,t}$$

$$ln(m_{x,t} - \Delta_{x,t}) = a_x + b_x\kappa_t + \delta_{x,t}$$

Actual: $q_{x,t}^{Suicide} \overset{def}{=} q_{x,t}^{trend} + \Delta_{x,t}$

Contribution of Suicide Rate to Residual Term

Contribution: $\varepsilon_{x,t} - \delta_{x,t}$



Contribution of Suicide Rate to Residual Term $\varepsilon_{x,2020}$ in 2020

$\underline{t = 2020}$

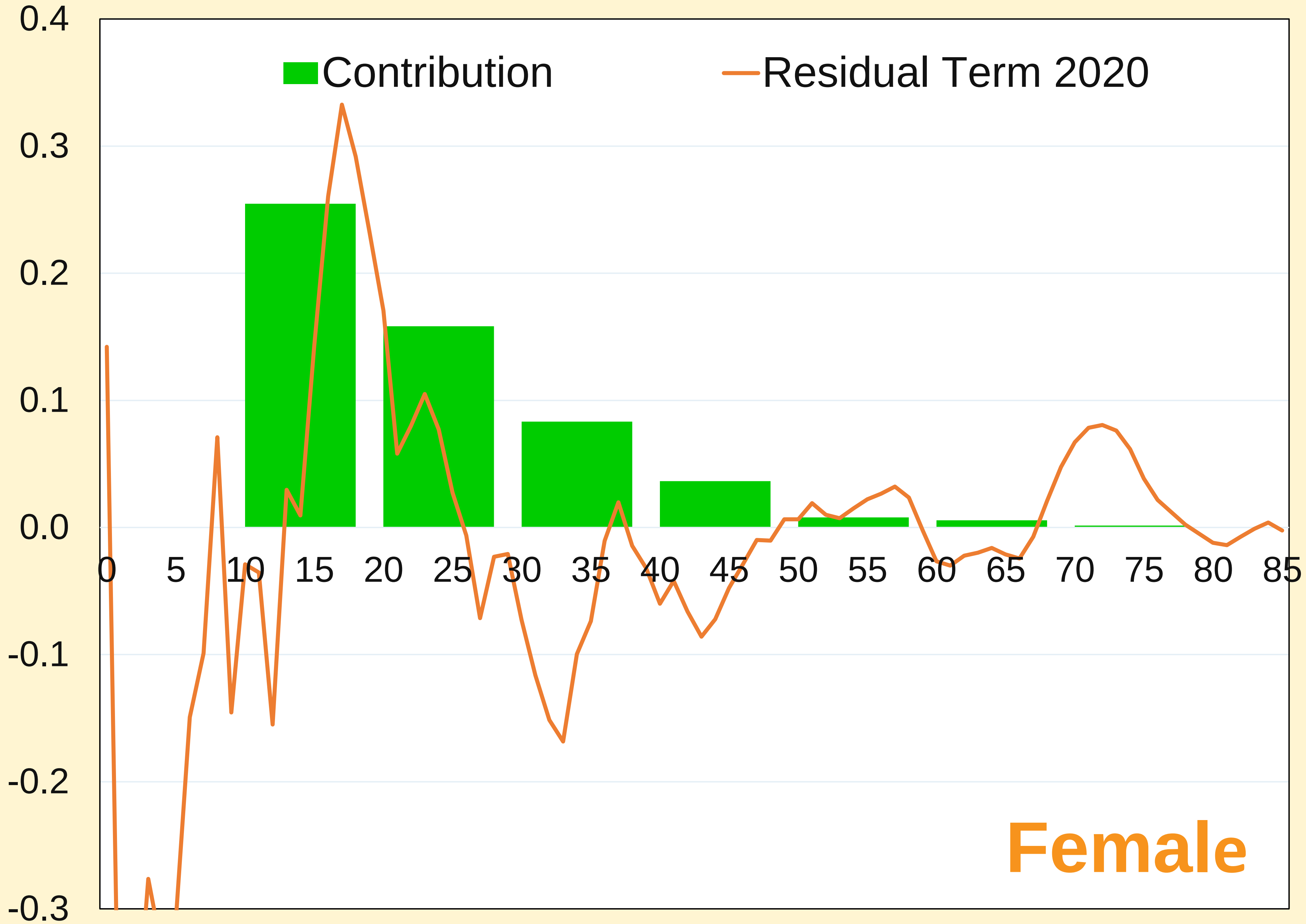
$$ln(m_{x,t}) = a_x + b_x \kappa_t + \varepsilon_{x,t}$$

$$ln(m_{x,t} - \Delta_{x,t}) = a_x + b_x \kappa_t + \delta_{x,t}$$

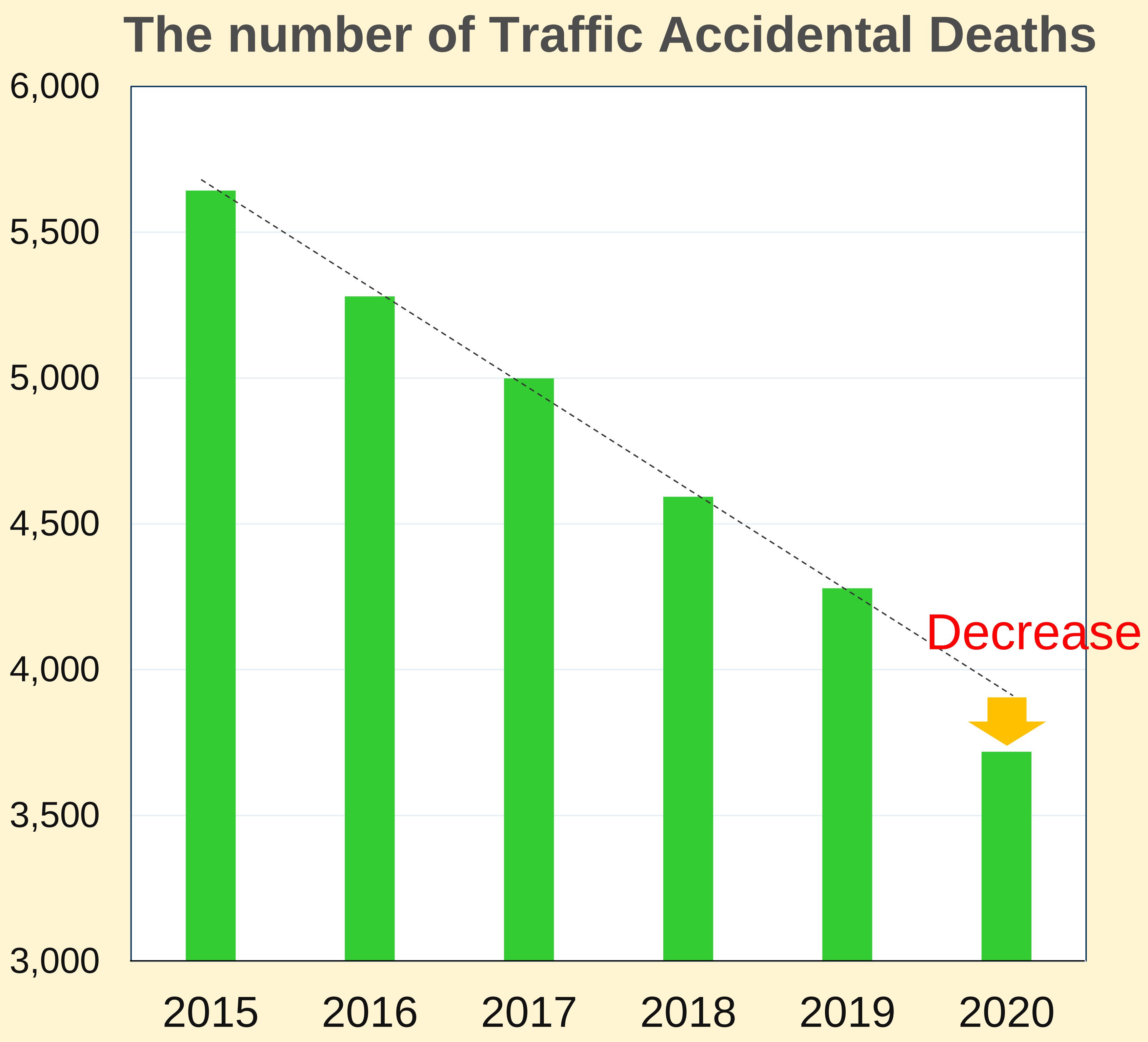
Actual: $q_{x,t}^{Suicide} \overset{def}{=} q_{x,t}^{trend} + \Delta_{x,t}$

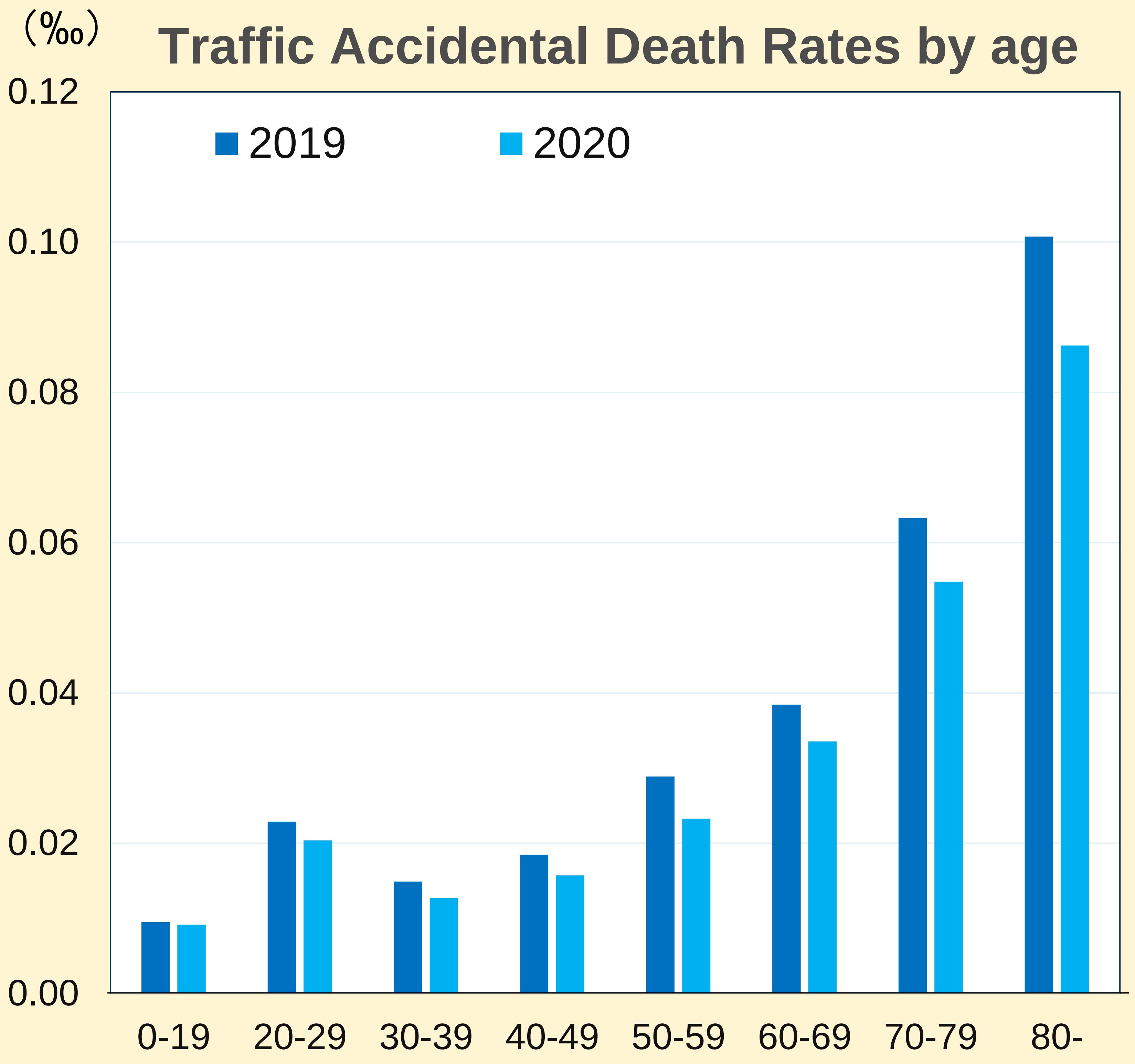
Contribution of Suicide Rate to Residual Term

Contribution: $\varepsilon_{x,t} - \delta_{x,t}$

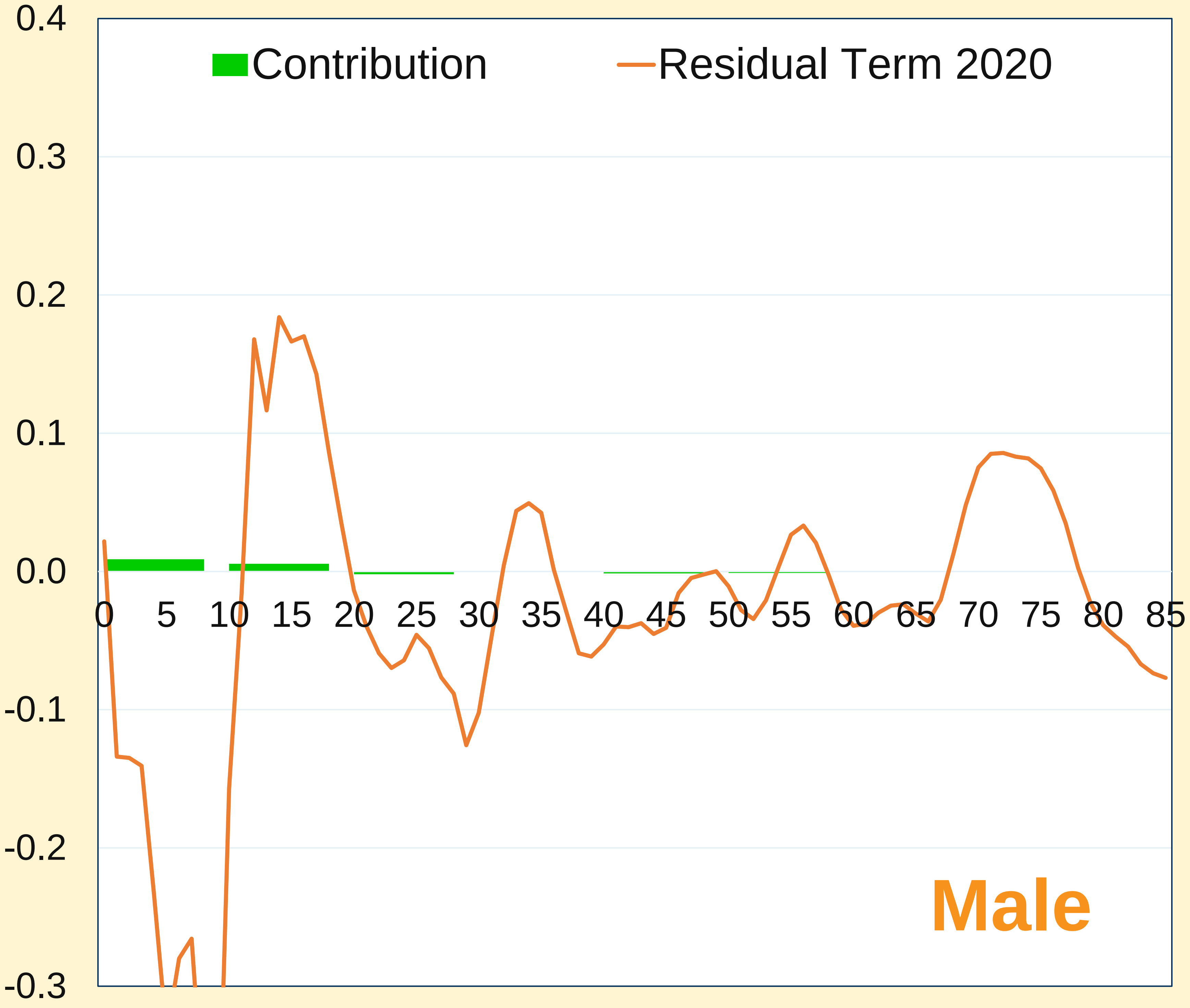


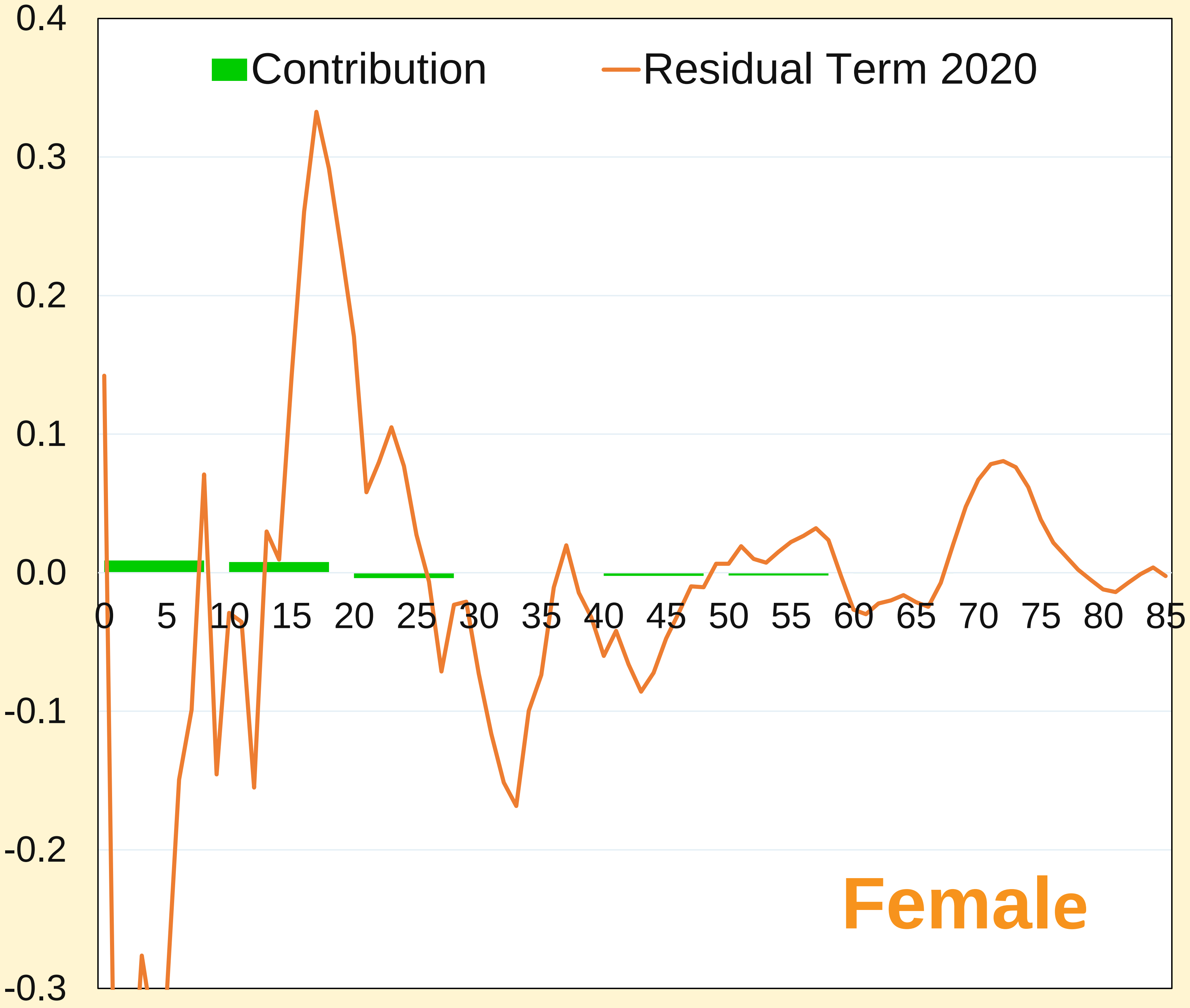
The number of Traffic Accidental Deaths





Contribution of Traffic Accidental Death Rate to Residual Term $\varepsilon_{x,2020}$ in 2020





Ⅴ. Conclusion

Results and Implications

- Insurance companies often have to remove outliers in premium rate calculations and soundness assessments.

- The results suggest that the analysis of outliers from baseline should consider not only deaths directly due to COVID infection, but also secondary factors such as increased suicide and decreased traffic accidents.

- As an example of analysing the secondary impact, we have shown that the residual term of the Lee-Carter Model is a useful tool.

Thank you! Obrigado!

Any Questions?

Contact Us
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