Long-Term Asset Allocation in Pension Funds

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RAILPEN

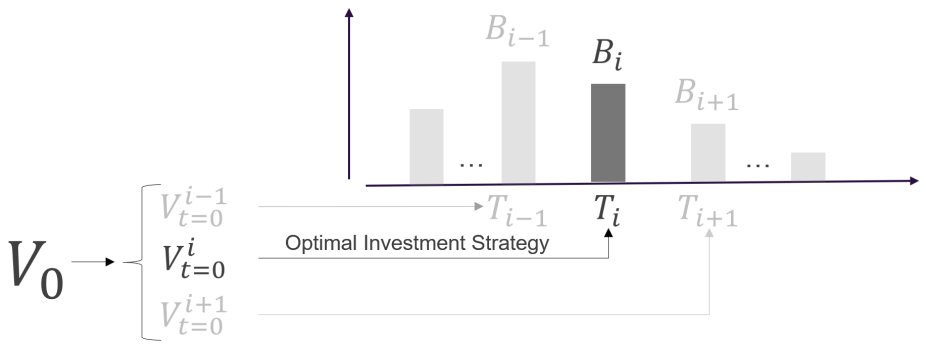
THE 'USUAL' PROBLEM

- Pensions is usually about
  - How much money do you need today to cover future obligations?
- This problem falls (at least) in two categories
  - *Solvency/Liquidation Approach (Life Insurance)*
    - Discount benefit payments by risk-free (bond) rate
    - Investment strategy: Buy bonds to match benefit cashflow
    - 'Safe' - but very (very) expensive
  - Going-Concern Approach (Defined Benefit)
    - Discount benefit payments by (prudent) expected return
    - Investment strategy: Diversified portfolio of 'stocks and bonds'
    - 'Tail risk' - dependent on sponsor/member additional financial capacity

- This is where research curiosity kicks in:
  - Discounting neglects the timing of events as they unfold in future
  - Investment strategy is 'prudent' - not optimal (no optimality criterion is stated)

RAILPEN RESEARCH QUESTION

- Instead of discounting 'backwards' in time, we ask the 'forward' question:
  - *What is the* optimal *investment strategy to meet each individual benefit payment?*



DUAL PROBLEM

- How much capital, $V^i_{t=0}$ to put into each investment strategy at $t = 0$?
  - How to formulate a coherent notion of risk across investment horizons
- The sum must match total capital available

  $V_0 = \sum V^i_{t=0}$

- What is the optimal strategy by investment horizon T_i?
  - Classical results *do not* depend on horizon
- Well known, that optimal strategy depends on investment horizon
  - if assets returns are mean-reverting
- Coincides with the investment belief that pension funds are *long-term* investors
  - Able to *hold* assets during ups and downs is rewarded

PAPER SUMMARY

Preisel (2024) Long-Term Mean-Variance Optimization Under Mean-Reverting Equity Returns, https://arxiv.org/abs/2309.07488

- Pension funds set long-term investment strategy as the *Strategic Asset Allocation*
  - The SAA specifies the allocation to all major asset classes
  - The SAA may - and often does - vary *deterministically* over time
- Examples
  - De-risking in DB pension plans
  - Life-cycle products in DC
  - Target date funds

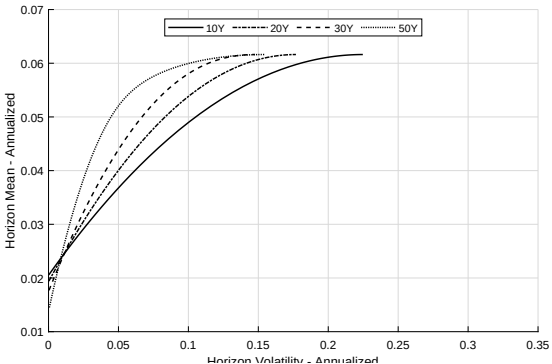
- The paper derives the mean-variance optimal Strategic Asset Allocation
  - if long-term investing is rewarded *aka*
  - if equity return displays mean reversion
- Methodological contribution to MVO
  - Equivalent Eigenvalue problem
  - Theory of matrix polynomials and solvents
- Key findings
  - Optimal strategies are unique by each investment horizon
  - Explicit result for optimal policy and efficient frontier

EFFICIENT FRONTIER BY INVESTMENT HORIZON

Preisel (2024) Long-Term Mean-Variance Optimization Under Mean-Reverting Equity Returns, https://arxiv.org/abs/2309.07488

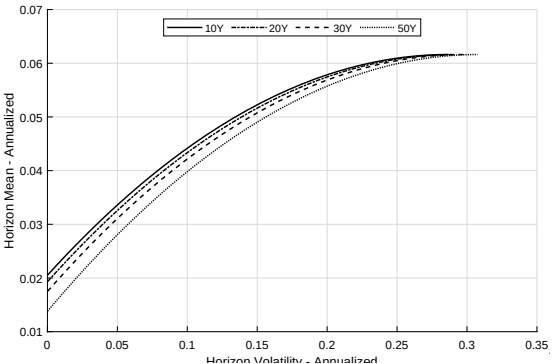
Slow Mean-Reversion

After a market shock, the risk-premium is kicked above it's average and reverts slowly back to normal.



Fast Mean-Reversion

After a market shock, the risk-premium is kicked but reverts quickly back to normal (expected return remains constant over time).



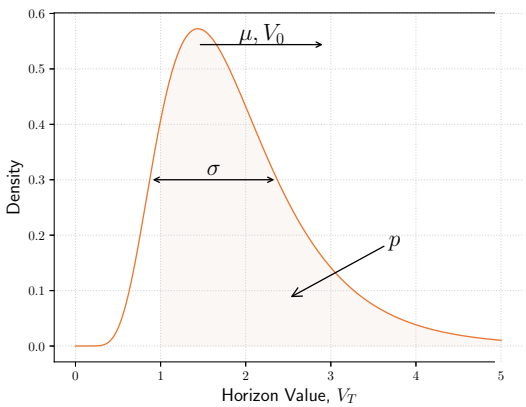
DIVIDING INITIAL CAPITAL

Objective: Divide initial capital 'equally' between investment strategies

- We break the division of initial capital into three questions
  - What is a reasonable measure of success across investment horizons?
  - What is the appropriate success rate?
  - Which (optimal) investment strategy to pick?
- Working hypothesis
  - Each investment strategy should expire with same probability, p, of success

$$p = P(V^i_{T_i} > B_i | V^i_0)$$

Horizon Distribution



THE LONG-TERM OPTIMAL ASSET ALLOCATION

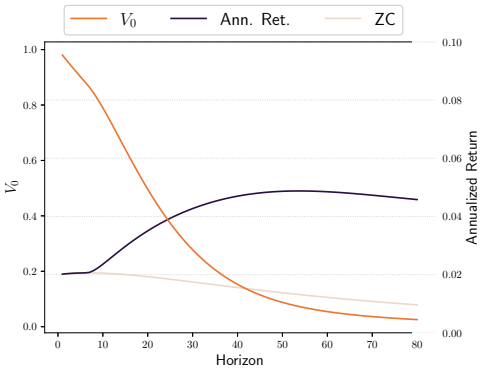
It seems reasonable $p \approx 0.80$

- Given a target probability, p, the final step is to use an optimizer to minimize V_0 for each investment horizon.
- The most striking finding is that it is optimal to hedge *early* payments
  - Here, implicit returns coincide with the zero-coupon term structure
- whereas longer-dated payments are better met with a diversified portfolio.

- In this way, this new approach truly interpolates the two dominant regulatory regimes in pension management.

Minimal Initial Capital to Deliver 1£

$p = 0.80$



The (implicit) annualized return is $-\log V^0/T$.