Introducing AI in Pension Planning: A Comparative Study of Deep Learning and Mamdani Fuzzy Inference Systems for Estimating Replacement Rates

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The International Association of Consulting Actuaries (IACA) was formed following an informal meeting of several senior consultants attending the International Congress of Actuaries in 1968 who, because the business environment for consulting actuaries was changing so rapidly, felt that an international meeting once every four years was too infrequent. IACA was established as a separate organization and in 1970 started holding meetings every other year. IACA became a Section of the IAA in 2020.

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Introducing AI in Pension Planning: A Comparative Study of Deep Learning and Mamdani Fuzzy Inference Systems for Estimating Replacement Rates

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Contents

Introductory comments (1)

- Funded pensions are crucial for securing supplementary income in retirement.
- Many countries have introduced mandatory funded pension schemes.
- Post-2008 crisis reversals reduced the role of funding.
- Funded schemes promote economic growth.
- OECD stresses the importance of private pensions to growth.
- We compare two methods for estimating replacement rates:
    - Deep Learning Model
    - Fuzzy Mamdani Inference System (FIS)

Introductory comments (2)

- Deep learning model:
  - Trained on synthetic datasets
  - Demonstrated high accuracy
- FIS:
  - Relies on expert knowledge
  - Promising results, but needs refinement
- Importance of AI in pension planning:
  - AI techniques like neural networks and fuzzy logic are vital
  - Useful for developing decision support systems
  - Essential for managing large data volumes and making accurate plans

Some basic formulae

$$Acc = \sum_{k=1}^{n} \boldsymbol{X}(1+\boldsymbol{i})^{\boldsymbol{n-k}}$$

where:
Acc: is the accrued amount at retirement,
X: is the amount of contributions per year (assumed fixed),
i: is the yearly rate of return on investment,
n: is the number of years of contribution

https://www.mdpi.com/2227-9091/9/1/8

$$Acc_n = X_n * (1-E)(1+i)^{\frac{1}{2}} + Acc_{n-1(1+i)}(1+i)$$

where:
Acc_n: is the accrued amount at time n,
X_n: is the amount of contributions at time n as percentage of income, before expenses,
E: is the percentage of expenses on contributions,
i: is the yearly rate of return on investment,
n: is the number of years of contributions


risks
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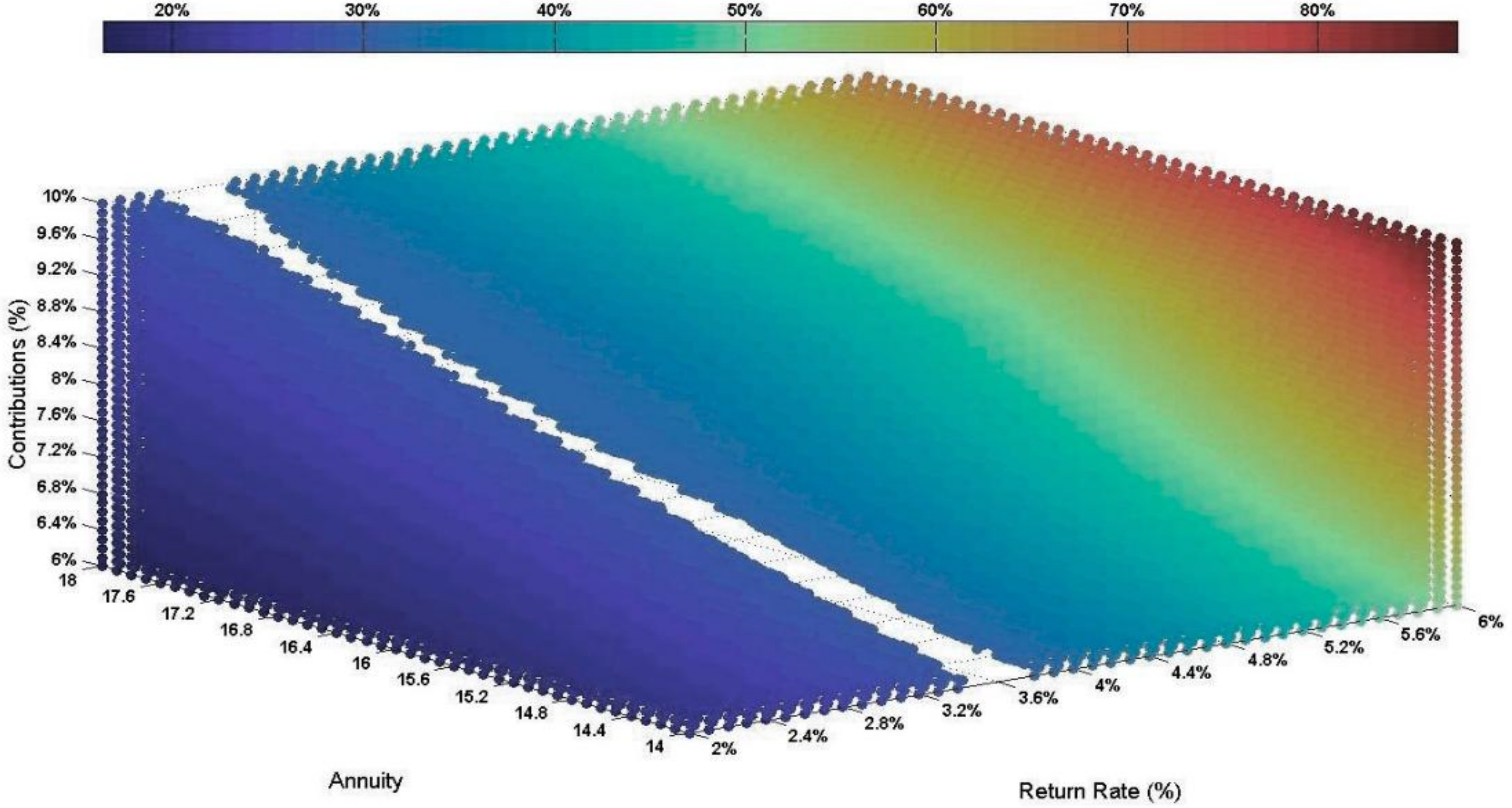
Enhancing Pension Adequacy While Reducing the Fiscal Budget and Creating Essential Capital for Domestic Investments and Growth: Analysing the Risks and Outcomes in the Case of Greece
Georgios Symeonidis; Platon Tinios; Panos Xenos
Risks 2021, Volume 9, Issue 1, 8

Baseline assumptions and sensitivity scenarios

- A paradigm: 40 years career, 6% contribution rate, 15.64% annuity, 3.5% return rate, 0.50% expenses on contribution and 0.50% income maturity
  - Calculated replacement rate: 26.02%
- Annuity Interval: Annuity values were sampled between 14.00 and 18.00.
- Return Rate Interval: The expected return on investment was varied from 2.0% to 6.0%.
- Contribution Rate Interval: The contribution rate was sampled between 6.00% and 10.00%.
- Career: 40 years

Actuarial Calculations - results



Modeling pension planning with Artificial Neural Networks (ANNs) (1)

- Parameters are estimated through optimization over training data
- Overview of Synthetic Dataset
  - **Dataset:** 500,000 samples
  - **Purpose:** Simulate realistic
  - pension scenarios
  - **Key Parameters:**

Parameter	Interval
Annuity	14.00 to 18.00
Return Rate	2.0% to 6.0%
Contribution Rate	6.00% to 10.00%

Years Fixed at 40

- Although the dataset is synthetic, the proposed computational framework is fully transferable to empirical pension data.
- The mathematical formulations of both the neural network and FIS pipelines are agnostic to the data source, requiring only consistent feature alignment and normalization.

Modeling pension planning with Artificial Neural Networks (ANNs) (2)

Details of the Deep Learning Model

- The primary goal of the neural network is to provide an alternative approach for estimating replacement rates
- The development of our deep learning (DL) model involved two primary phases:
  - an initial baseline neural network and
  - a more exhaustive search for an optimal model architecture

Modeling pension planning with Artificial Neural Networks (ANNs) (3)

- **Baseline Neural Network Approach**
  - **Libraries:** TensorFlow, Keras
  - **Data Split:** 80-20 (Training-Test)
  - **Feature Standardization:**
    - Mean = 0, Std = 1
  - **Architecture:**
    - 3 Hidden Layers (64, 32, 16 neurons)
    - Dropout Layers (Prevents Overfitting)
    - ReLU Activation (Hidden), Softplus (Output)
  - **Training:**
    - Adam Optimizer, MSE Loss
    - Early Stopping, 100 Epochs, Batch Size 32

Algorithm 1 Baseline Neural Network Approach

Load and Preprocess Data
Load the dataset into a DataFrame
Split the dataset into features (X) and target (y)
Split the data into training (80%) and testing (20%) sets using `train_test_split`
Standardize the features using `StandardScaler`
Build the Neural Network Model
Initialize a sequential model
Add a dense layer with 64 neurons and ReLU activation
Add a dropout layer with 20% dropout rate
Add a dense layer with 32 neurons and ReLU activation
Add a dropout layer with 20% dropout rate
Add a dense layer with 16 neurons and ReLU activation
Add a dropout layer with 20% dropout rate
Add a dense output layer with 1 neuron and softplus activation
Compile the model with Adam optimizer, MSE loss function, and MAE metric
Train the Model
Define early stopping with patience of 10 epochs
Train the model using training data with validation split of 20%, for up to 100 epochs, and batch size of 32, including early stopping
Evaluate the Model
Evaluate the model on the test set and print MAE
Plot training and validation loss over epochs
Plot training and validation MAE over epochs
Make Predictions and Visualize
Make predictions on the test set
Plot true vs. predicted values
Plot the residuals distribution
Save the Model and Scaler
Save the model to an H5 file
Save the scaler to a PKL file

Modeling pension planning with Artificial Neural Networks (ANNs) (4)

Exhaustive Search Approach

- Data Preparation: Similar to Baseline
- Architecture Search:
- Hidden Layers: 1 to 5
- Neurons per Layer: 64, 32,16, 8, 4
- Training:
  - Adam Optimizer, MSE Loss
  - Early Stopping
  - Metrics: MAE, MSE, RMSE, R^2, Adjusted R^2, MAPE

Algorithm 2 Exhaustive Search Neural Network Approach

```
Load and Preprocess Data
Load the dataset into a DataFrame
Split the dataset into features (X) and target (y)
Split the data into training (80%) and testing (20%) sets using
  train_test_split
Standardize the features using StandardScaler
Define Model Building Function
Initialize a sequential model
for each hidden layer do
    Add a dense layer with specified number of neurons and ReLU activation
    Add a dropout layer with 20% dropout rate
end for
Add a dense output layer with 1 neuron and softplus activation
Compile the model with Adam optimizer, MSE loss function, and MAE
  metric
Experiment with Different Topologies
Define the range of hidden layers and neurons per layer
Initialize variables to track the best model and lowest MAE
Train and Evaluate Models
for each configuration of hidden layers and neurons do
    Build the model with the current configuration
    Define early stopping with patience of 10 epochs
    Train the model using training data with validation split of 20%, for up
      to 100 epochs, and batch size of 32, including early stopping
    Evaluate the model on the test set
    if the model's MAE is lower than the best MAE then
        Update the best MAE
        Save the current model and training history as the best model
    end if
end for
Plot Training History of Best Model
Plot training and validation loss over epochs
Plot training and validation MAE over epochs
Make Predictions and Visualize
Make predictions with the best model on the test set
Evaluate the model using various metrics (MAE, MSE, RMSE, R², adjusted
  R², MAPE)
Plot true vs. predicted values
Plot the residuals distribution
Save the Best Model and Scaler
Save the best model to an H5 file
Save the scaler to a PKL file
```

Modeling pension planning with Fuzzy Logic (1)

- Mamdani Fuzzy Inference System (FIS)
- human-in-the-loop
- rule-based, enabling interpretable predictions without the need for iterative training
- Fuzzy Logic:
  - Handles imprecise information with linguistic terms (’low’, ’medium’, ’high’, ’low to medium’, ’medium to high’)

Modeling pension planning with Fuzzy Logic (2)

- Purpose:
  - Model complex relationships using FL
- Input Variables:
  - Annuity, Return Rate, Contribution Rate
- Output Variable:
  - Replacement Rate
- Membership Functions:
  - Triangular functions for all variables

Algorithm 3 Mamdani Fuzzy Inference System (FIS)

```
Define Universe Variables and Membership Functions
Define antecedent variables (e.g., X1, X2, X3) and consequent variable (Y)
with linguistic terms (e.g., low, medium, high)
Define membership functions for each antecedent variable using triangular
or Gaussian functions
Define membership functions for the consequent variable with linguistic
terms and corresponding fuzzy sets
Define Fuzzy Rules
Define a set of fuzzy rules linking antecedent variables to consequent vari-
ables:
- Rule 1: If X1 is low and X2 is high, then Y is medium
- Rule 2: If X1 is medium and X3 is low, then Y is high
- ...
Create Fuzzy Control System
Create a fuzzy control system using the defined rules
Initialize the fuzzy control system simulation
Apply Inputs and Compute Outputs
for each input set in the dataset do
   Pass input values to the fuzzy control system simulation
   Compute the inferred output using the fuzzy inference mechanism
   Store computed fuzzy output in the dataset
end for
```

Baseline Neural Network Approach Results

- Computer Specifications
  - Processor: AMD Ryzen 5 5500U (2.10 GHz)
  - Graphics: Radeon Graphics
  - RAM: 8.00 GB (5.83 GB usable)
  - Operating System: Windows 11 Pro (64-bit)
  - Programming Language: Python
- Dataset Size: 500,000 rows
- Annuity: Mean: 16.00 Std. Dev.: 1.15 Range: 14.00 - 18.00
- Return Rate: Mean: 4.00 Std. Dev.: 1.15 Range: 2.00 - 6.00
- Contribution Rate: Mean: 8.00 Std. Dev.: 1.15 Range: 6.00 - 10.00
- Purpose: Basis for model training and evaluation

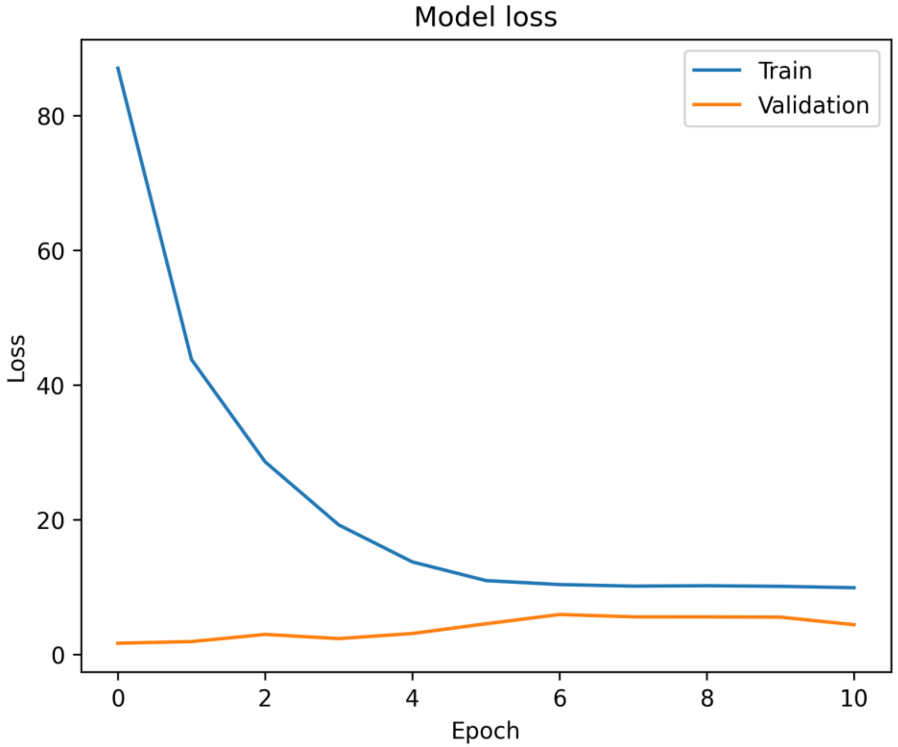
Baseline Neural Network Approach Results

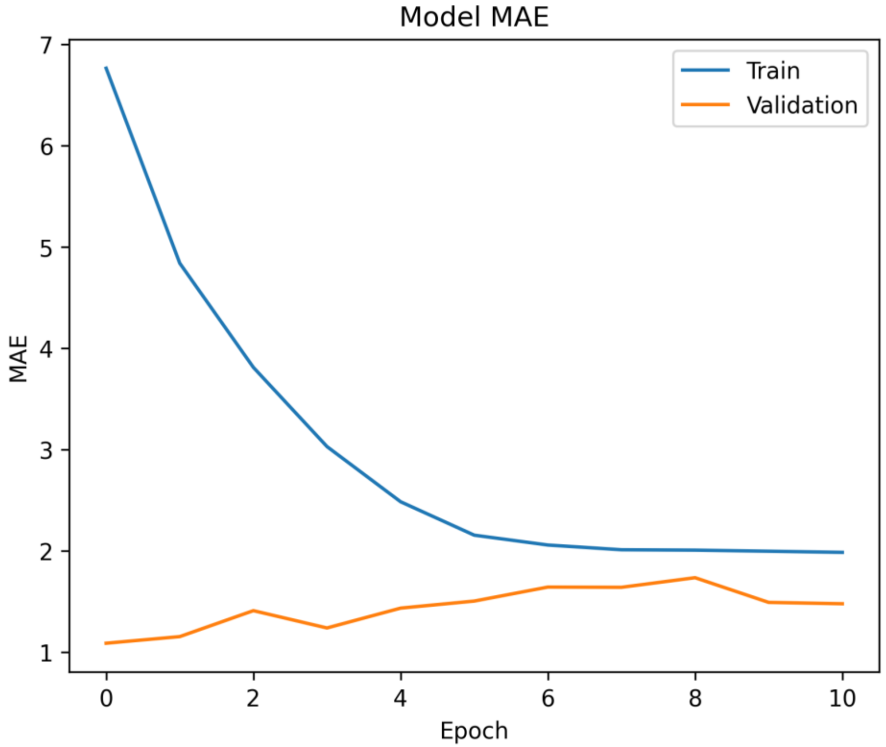
- Baseline Neural Network Approach
- Architecture:
  - Input Layer: input variables
  - Hidden Layers: 64, 32, 16 neurons
  - Output Layer: Single neuron with softplus activation
- Activation Functions:
  - Hidden Layers: ReLU (Rectified Linear Unit f (x) = max(0, x))
  - Output Layer: Softplus (f (x) = log(1 + ex))
- Dropout Rate: 0.2
- Epochs: 100 (each epoch represents one complete pass through the entire dataset)
- Batch Size: 32
- Loss Function: Mean Squared Error (MSE)
- Optimizer: Adam (Learning Rate: 0.001)

Baseline Neural Network Approach Results

Model Performance: Loss and Mean Absolute Error (MAE)

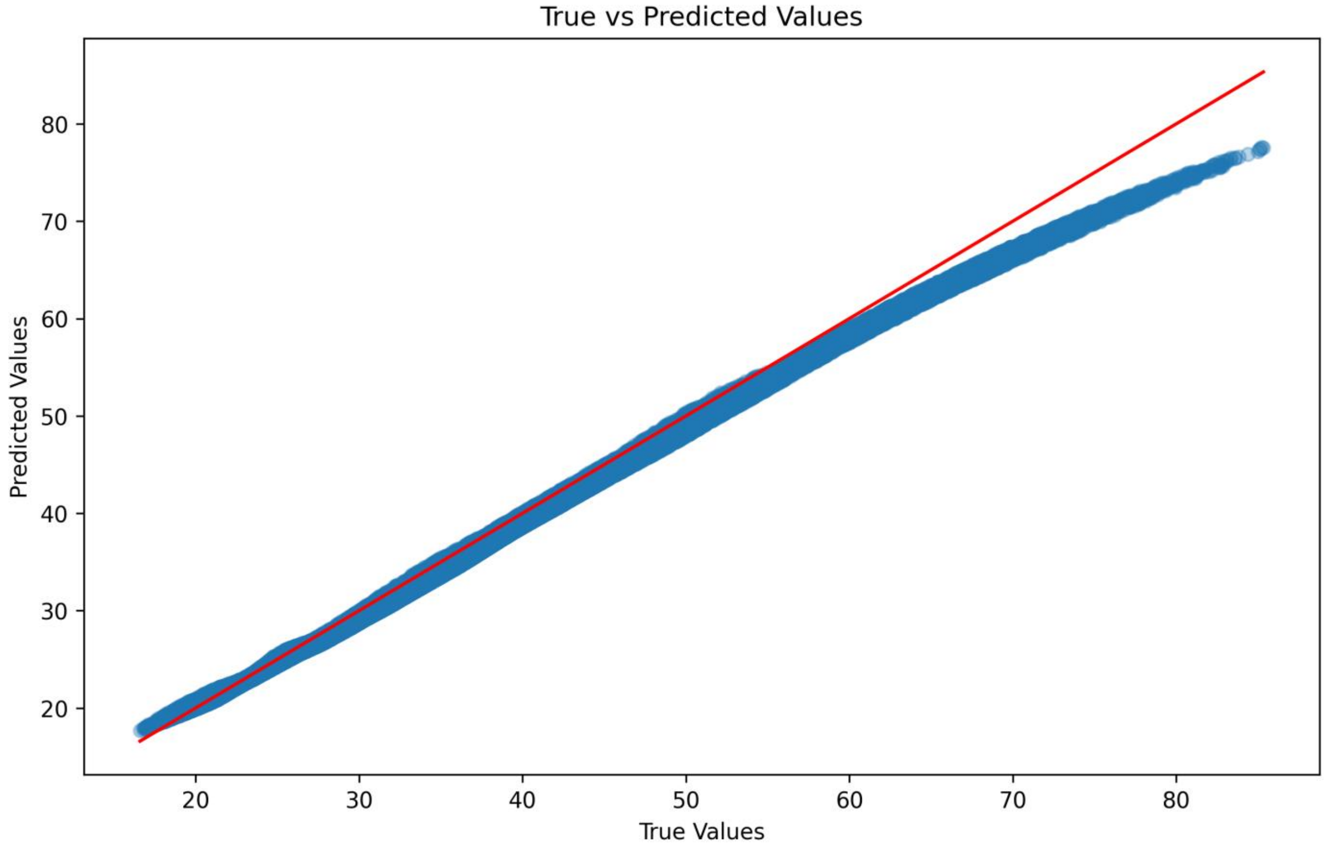
- Loss and MAE across epochs
- Goal: Decrease in both training and validation loss/MAE





Convergence of model loss and MAE (baseline neural network).

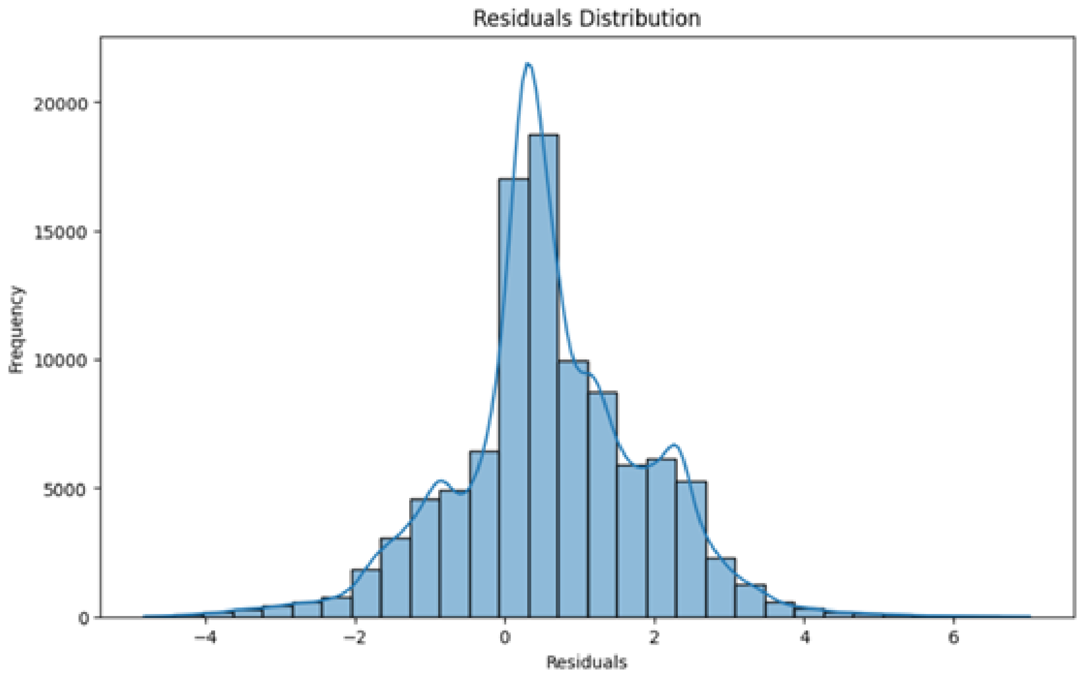
Baseline Neural Network Approach Results



- Residuals Line Plot:
  - Displays fluctuations in prediction errors
  - Observation:
    - Deviations from actual values
  - Implication:
    - Need for model refinement

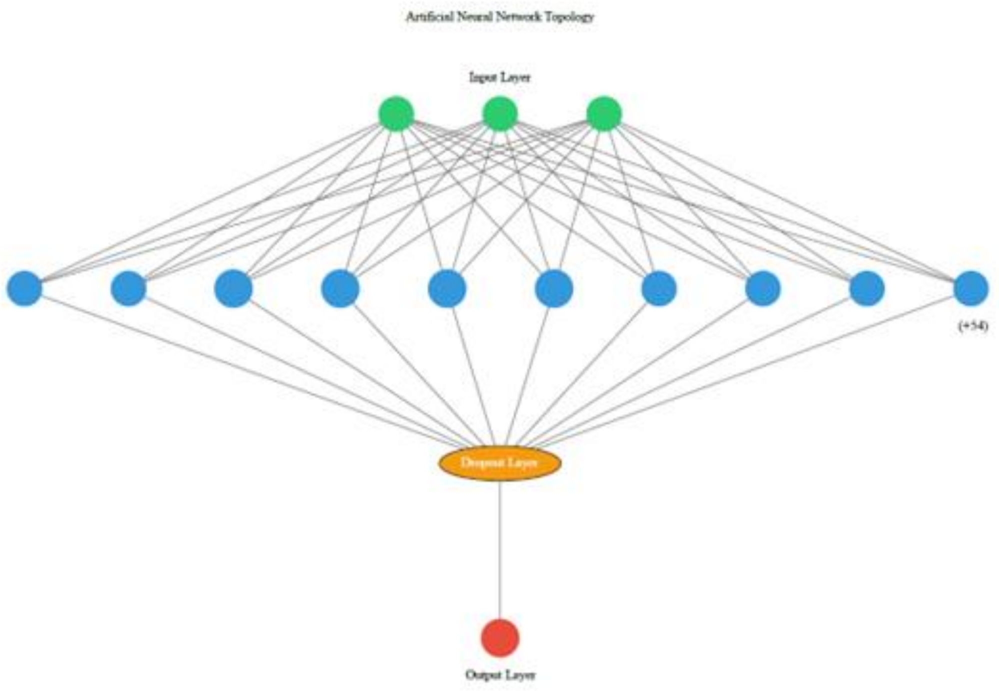
Baseline Neural Network Approach Results

- Residuals Distribution - Histogram of Residuals
  - Majority clustered between -2 and 2
  - Outliers: Significant deviations in tails (-4 to -2 and 4 to 6)
  - Insight: Potential for model or feature enhancement



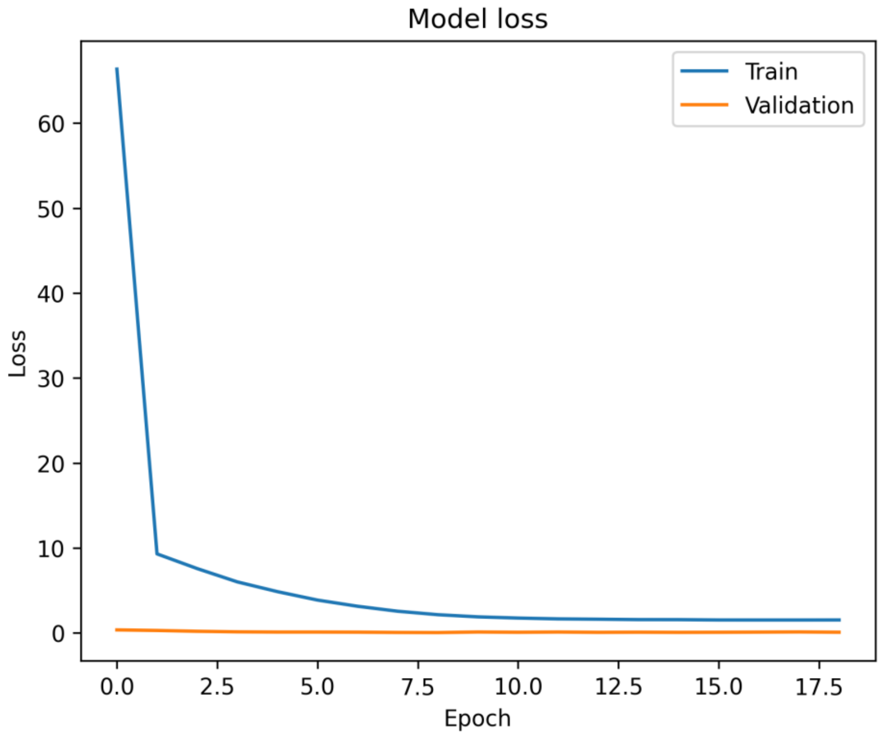
Optimized Neural Network Approach Results (1)

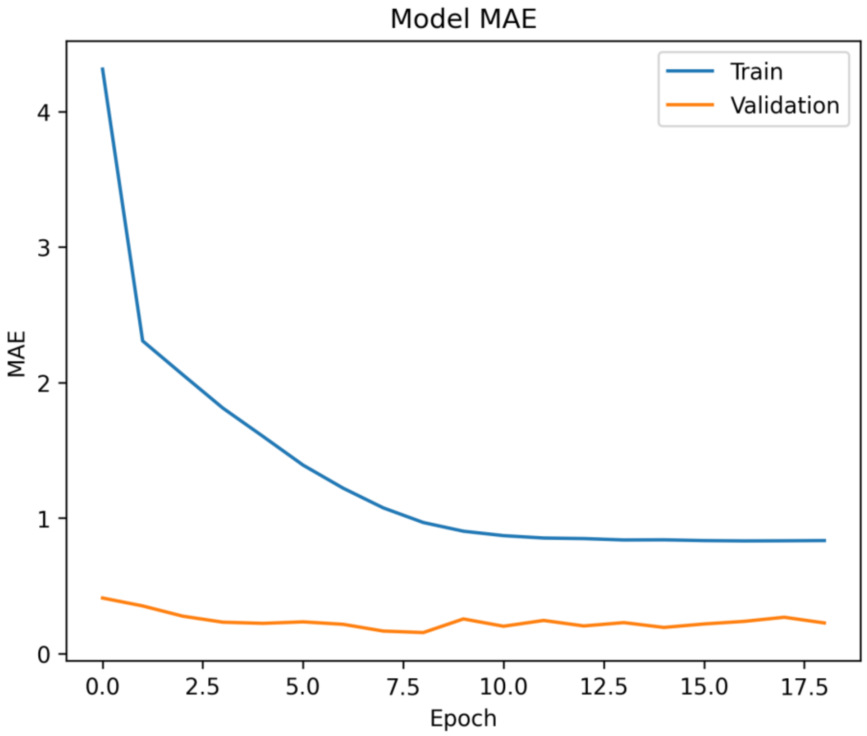
- Best Model: 1 hidden layer with
  - 64 neurons
- Mean Absolute Error (MAE): 0.2006
- Additional Metrics:
  - Mean Squared Error (MSE): 0.0718
  - Root Mean Squared Error (RMSE): 0.2680
  - R-squared (R^2): 0.9995
  - Adjusted R-squared (R^2adj): 0.9995
  - Mean Abs Percentage Error (MAPE): 0.0055
- Observation:
  - Exceptional performance with near-perfect R-squared and minimal MAPE



Optimized Neural Network Approach Results (2)

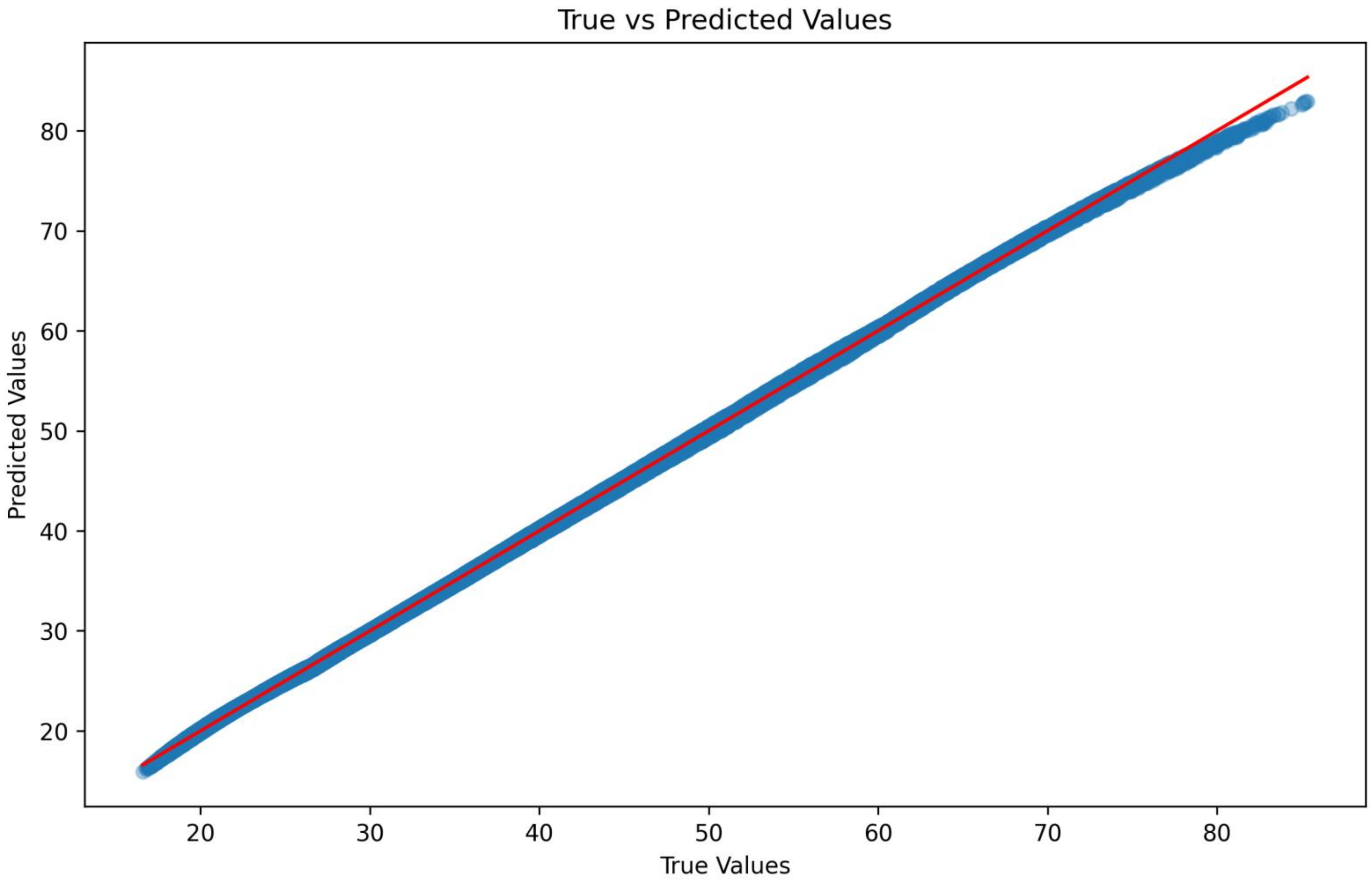
- Enhanced Stability:
  - Improved loss and MAE convergence
  - Faster convergence during training and validation





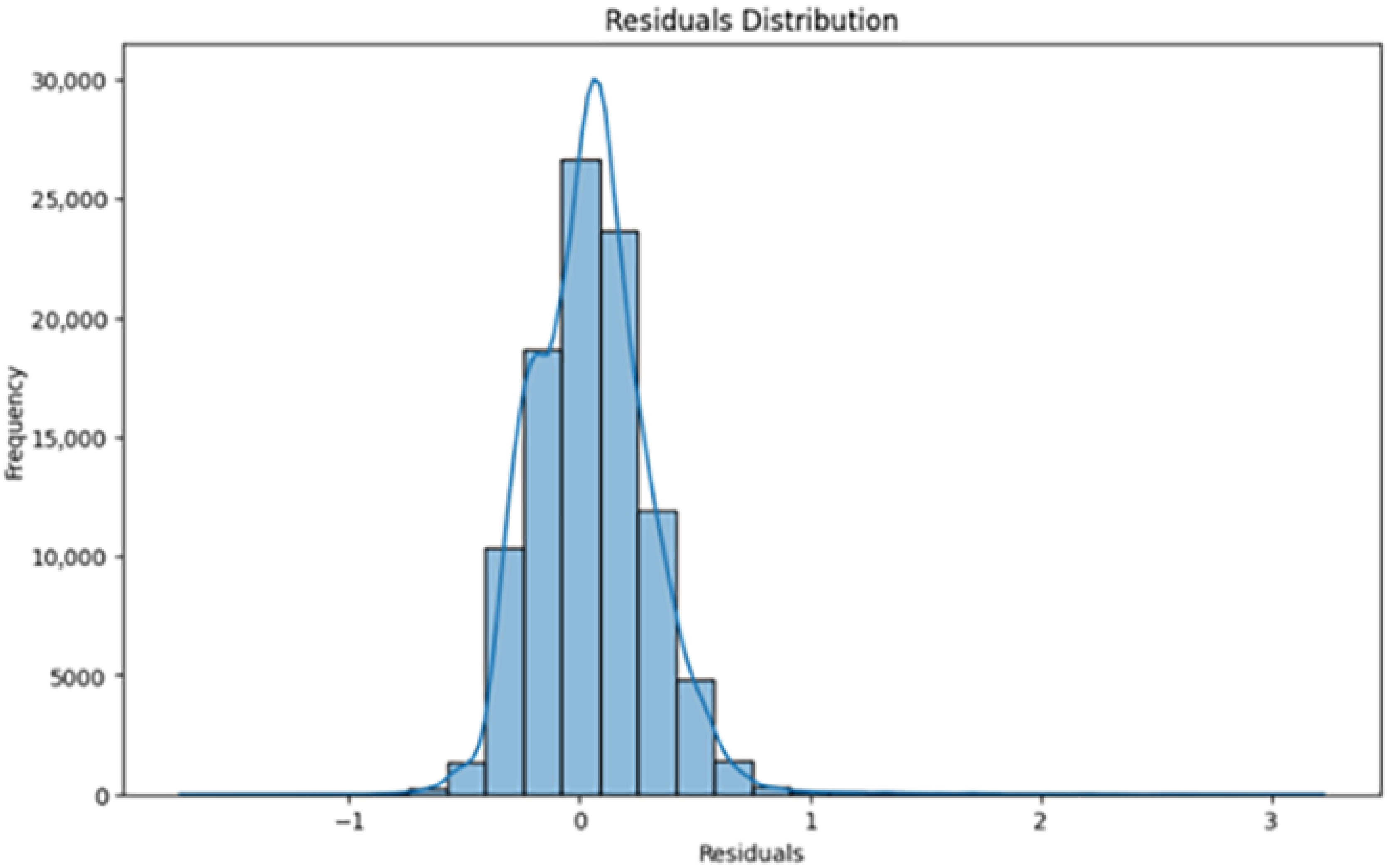
Optimized Neural Network Approach Results (3)

- Improved Line Plot:
  - Comparison of true vs predicted values
  - Demonstrates model's predictive accuracy



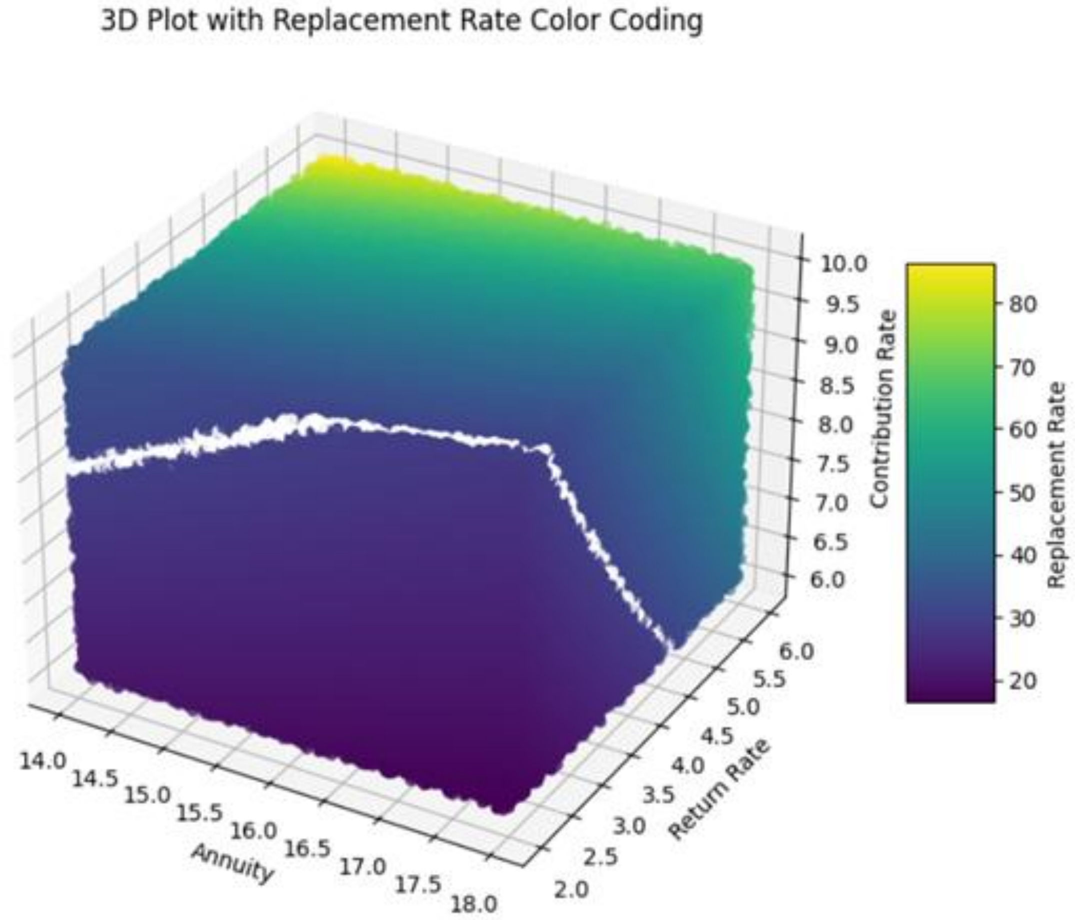
Optimized Neural Network Approach Results (4)

- Residuals Distribution:
- Majority within -1 to 1
- Outliers up to 3, but reduced frequency compared to baseline
- Indicates improved model accuracy and consistency



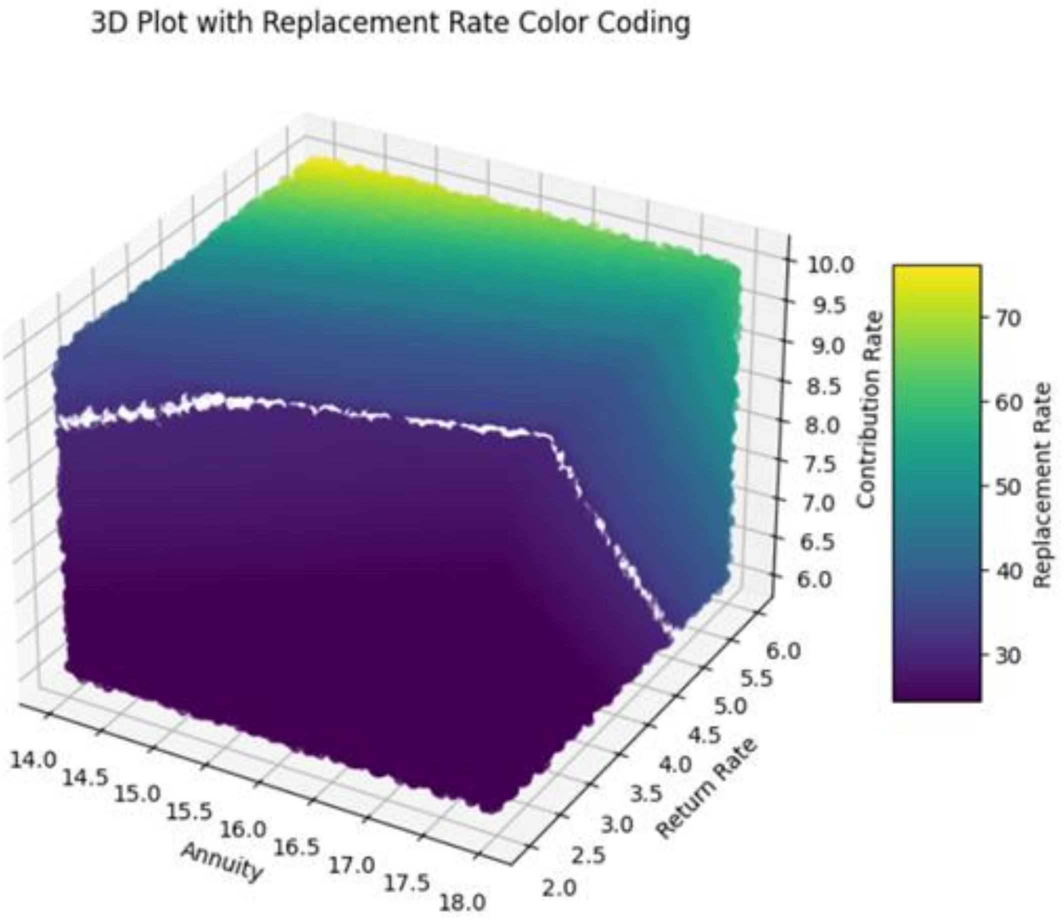
Optimized Neural Network Approach Results (5)

- Actual Values:
  - Visualizes relationship between annuity, return rate, and contribution rate



Optimized Neural Network Approach Results (6)

- Predicted Values:
- Compares predicted replacement rates against actual values



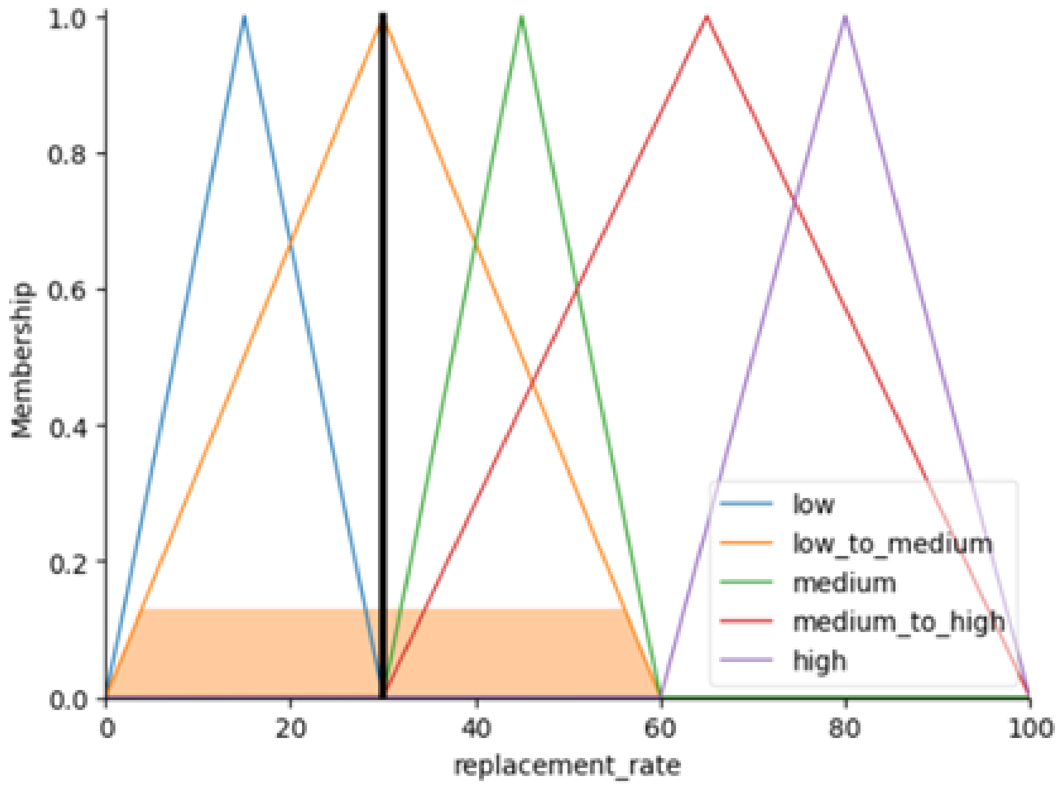
Mamdani Fuzzy Inference System Overview (1)

- Inputs:
  - Annuity: Low [13.99 - 15.0], Medium [14.0 - 17.0], High [17.0 - 18.01]
  - Return Rate: Low [1.99 - 3.0], Medium [2.0 - 5.0], High [5.0 - 6.01]
  - Contribution Rate: Low [5.99 - 7.0], Medium [6.0 - 9.0], High [9.0 - 10.01]
- Output:
  - Replacement Rate: Low [0.0 - 30.0], Low to Medium [0.0 - 60.0], Medium [30.0 - 100.0], Medium to High [30.0 - 100.0], High [60.0 - 100.0]
- Rules:
  - 27 fuzzy rules mapping inputs to output replacement rate
  - Example rule: ”If annuity is low and return rate is low and contribution rate is low, then replacement rate is low.”

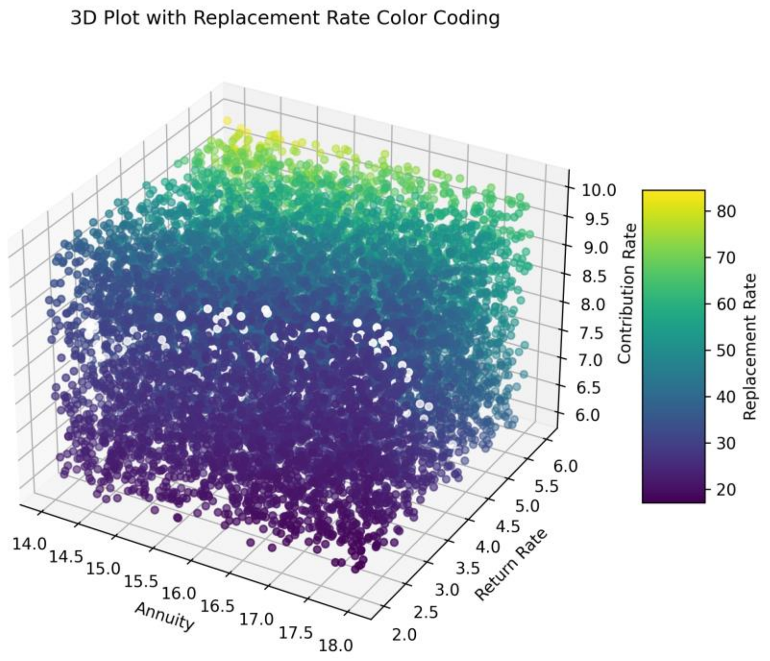
Mamdani Fuzzy Inference System Overview (2)

Example Prediction and Results

- Example Values:
  - Annuity: 16.75
  - Return Rate: 3.67
  - Contribution Rate: 7.13
- FIS Prediction:
  - Replacement Rate: Approximately 30.

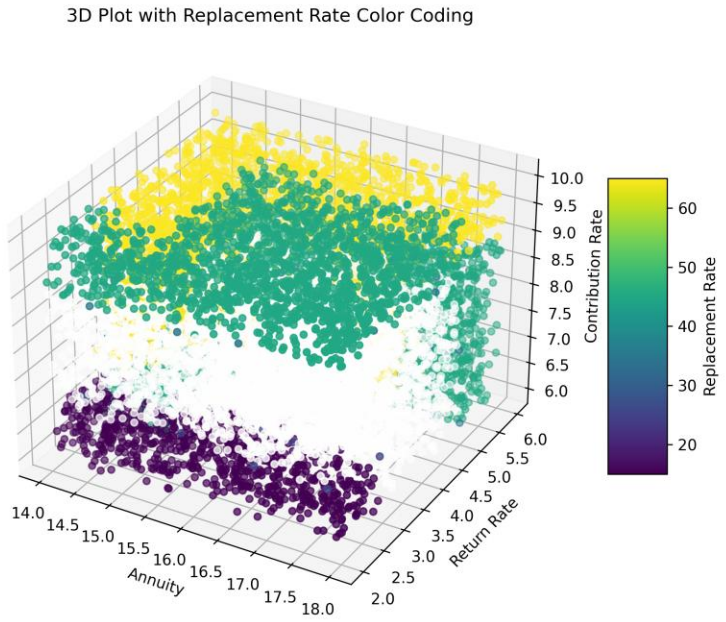


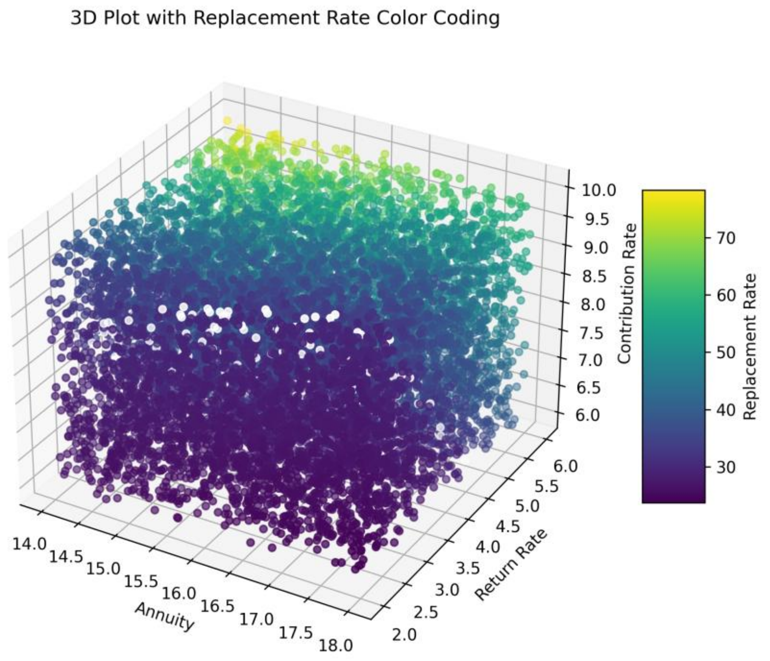
Mamdani Fuzzy Inference System Overview (3)



Actual Values of Replacement Rates (Sample of 11,000 rows)

Predicted Values by Optimized Neural Network





Predicted Values by Fuzzy Inference System

Conclusions and future research direction (FIS)

Limitations of Fuzzy Logic Approach and Next Steps

- Fuzzy Inference System Findings:
  - Effective in handling imprecise data with predefined rules and membership functions
  - Observed significant fluctuations in FIS predictions compared to optimized neural network
- Next Steps:
  - Refine fuzzy rules and membership functions
  - Invest in more detailed rule crafting and parameter tuning
  - Continue research to enhance predictive accuracy and stability
- Note: Expert knowledge is crucial for optimizing FIS models to ensure robust predictions.

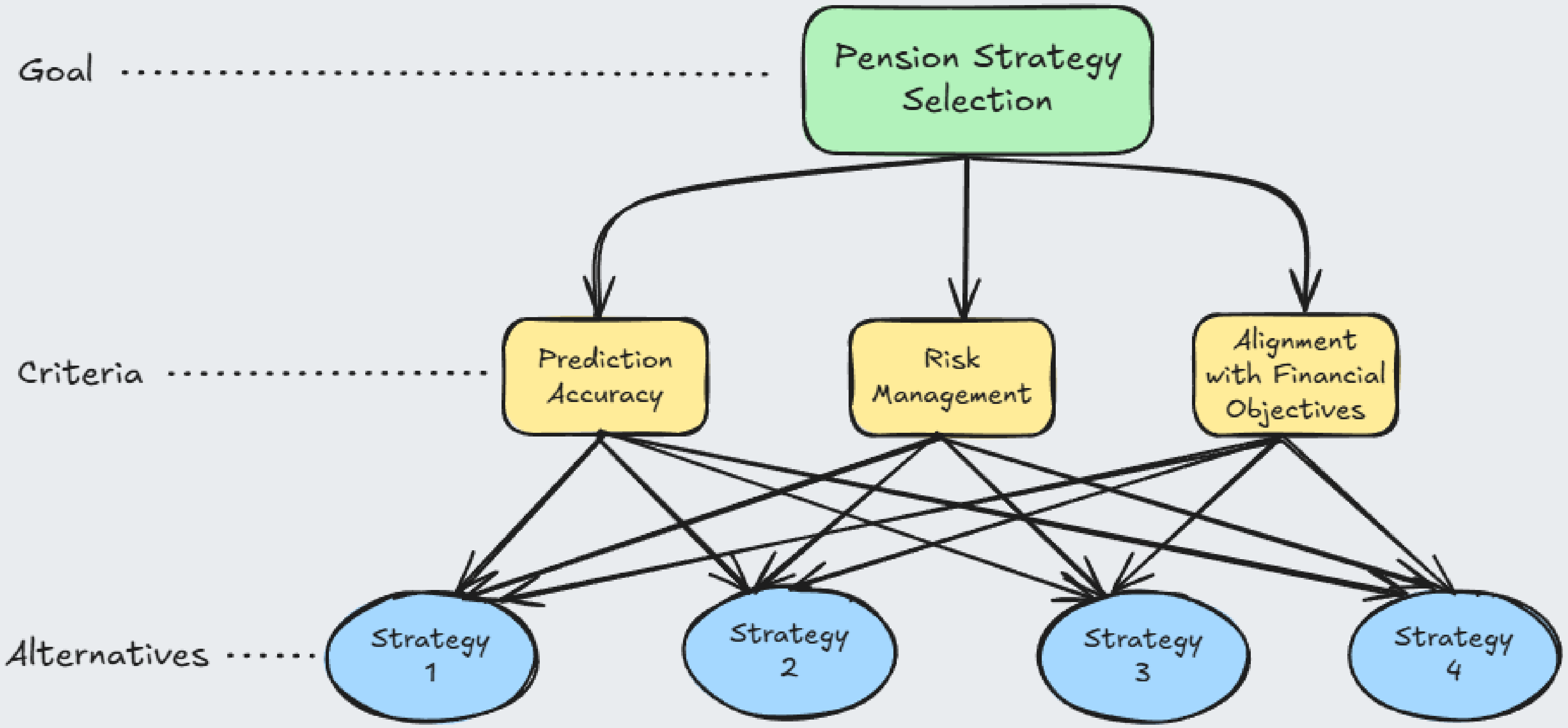
Conclusion Overview

- Objective: Accurately estimate replacement rates in retirement planning.
- Approaches Explored:
  - Neural Networks:
    - Demonstrated high efficacy in estimating replacement rates
    - Achieved low Mean Absolute Errors (MAE)
    - Robust convergence during training
    - Effective in capturing complex patterns
- Mamdani Fuzzy Inference Systems (FIS):
  - FIS Shows promise with potential improvements
  - Exhibited fluctuations in prediction accuracy
  - Potential for handling uncertainties
  - Need for thorough rule generation and parameter tuning

Future Research Directions

- Refine FIS Models:
  - Integrate expert insights
  - Enhance predictive accuracy and stability
  - Define rules and membership functions more precisely
- Explore Multi-Criteria Decision Analysis Approaches:
  - Integration of expert knowledge and decision analysis frameworks is crucial
  - Use methods like Analytic Hierarchy Process (AHP) and ensure robust and informed decision-making for retirement planning
  - Evaluate pension strategies based on multiple criteria like estimated replacement rates, risk tolerance, and financial sustainability

Future Research Directions - Analytic Hierarchy Process (AHP)



Q1: With respect to the goal, which criterion is the most significant?

Q2: Given a specific criterion, which alternative is the most significant?

Q3: How are the alternatives ranked to select the best one?

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