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# Sum-Insured Weighted and Post-code Based Mortality Models: Application and Implications on Liability Estimates


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# Motivation

Think, for instance, of an insurer with just 2 policyholders for a product whose pay-out depends on death/survival:

- First person has a benefit of  €1 000 euro
- Second person has a benefit of €1 000 000 euro

Should mortality assumptions be estimated giving these two individuals the same level of importance, or should models account for the difference in benefits when setting assumptions?

# Motivation

- ✓ Some practitioners think that mortality assumptions should try to predict as well as possible the survival of the second individual even if this means increasing the chances of getting wrong the survival of the first one.
- ✓ They advocate for the use of insured amount-weighted mortality models
- ✓ It is usually said that these models will result in lower mortality- see, for example, Richards (2008).
- ✓ Practitioners expect sum insured to be an indicator of affluence (Haberman et al., 2014).

# Maximum likelihood estimation refresher

# Maximum likelihood estimation

Under maximum likelihood estimation:

- We are interested in estimating the unknown parameters P of a function g. We do this in a way that the parameters can be considered the “most likely to be true” given data observed.
- Consider a random variable T, and a subscript i denoting an individual, out of a group of N members.
- Assume the existence of a related probability (density) function that can be written as g(t;P)

MLE will find the value of P that maximizes the probability of observing the data collected, which can be written as

$$L(P) = \prod_{i=1}^{N} g(t_i; P)$$

# Maximum likelihood estimation

Note how we have assumed that the experience of every observation is equally important. This is what some insurers/practitioners do not like. Because of this, they resort to a methodology where they weight the observations by the sum insured/benefit. Consider for this a variation of the previous model, where $w_i$ refers to a weight associated to each individual. In this particular case, $w_i$ is a function of the size of the sum insured.

$$L_w(P) = \prod_{i=1}^{N} g(t_i; P)^{w_i}$$

# Maximum likelihood estimation

Consider a data set on survival of N individuals, where individual i is observed for $t_i$ units of time. Moreover, assume that individual survival for individual i is weighted by $w_i$, which is some function of the sum insured. Then the likelihood equation can be written as

$$L = \prod_{i=1}^{N} \{Pr(T_{x_i} = t_i)^{\delta_i} Pr(T_{x_i} > t_i)^{1-\delta_i}\}^{w_i} = \prod_{i=1}^{N} \{f_{x_i}(t_i)^{\delta_i} S_{x_i}(t_i)^{1-\delta_i}\}^{w_i} =$$

$$\prod_{i=1}^{N} \{\{S_{x_i}(t_i)\mu_{x_i+t_i}\}^{\delta_i} S_{x_i}(t_i)^{1-\delta_i}\}^{w_i} = \prod_{i=1}^{N} \{\mu_{x_i+t_i}^{\delta_i} S_{x_i}(t_i)\}^{w_i} = \prod_{i=1}^{N} \{\mu_{x_i+t_i}^{\delta_i} e^{-H_{x_i}(t_i)}\}^{w_i}$$

Where $T_{xi}$ is the future lifetime of individual i, $F_{xi}(t)=Pr(T_{xi} \leq t)$ is the probability of the individual surviving at most t years, $S_{xi}(t)= Pr(T_{xi} > t)$ is the probability of surviving at least t years, and $\delta_i$ is an indicator of whether the individual survived ($\delta_i=0$) or died ($\delta_i=1$) during the period of observation. We have assumed that individuals aged x are subject to a force of mortality $\mu_x$ and a cumulative hazard function $H_x(t)$.

# Maximum likelihood estimation

We will be assuming classical mortality laws from literature as shown in the table below.

| Law | $\mu_x$ | $H_x(t)$ |
|---|---|---|
| Gompertz | $e^{\alpha+\beta x}$ | $\frac{(e^{\beta t}-1)}{\beta} e^{\alpha+\beta x}$ |
| Makeham | $e^{\epsilon} + e^{\alpha+\beta x}$ | $te^{\epsilon} + \frac{(e^{\beta t}-1)}{\beta} e^{\alpha+\beta x}$ |
| Makeham-Perks | $\frac{e^{\epsilon}+e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$ | $te^{\epsilon} + \frac{1-e^{\epsilon}}{\beta} log\{\frac{1+e^{\alpha+\beta(x+t)}}{1+e^{\alpha+\beta x}}\}$ |
| Makeham-Beard | $\frac{e^{\epsilon}+e^{\alpha+\beta x}}{1+e^{\alpha+\rho+\beta x}}$ | $te^{\epsilon} + \frac{e^{-\rho}-e^{\epsilon}}{\beta} log\{\frac{1+e^{\alpha+\rho+\beta(x+t)}}{1+e^{\alpha+\rho+\beta x}}\}$ |

Table: Key functions per mortality law

# Generalized Additive Models

# Generalized additive models (GAM)

The objective of making use of a GAM is to study a model of the form

$$\eta_i = g(\mu_i) = g(E[D_i]) = log(t_i)+\theta_0+\theta_1 C_{1,i}+\ldots+\theta_n C_{n,i}+s_{n+1}(C_{n+1,i})+\ldots+s_m(C_{m,i}),\ n \leq m$$

Where $D_i$ is the number of deaths, $C_{1,i}$, …, $C_{n,i}$ are variables, and $C_{n+1}$,…, $C_{m,i}$ are continuous variables for the i-th person. In addition, g(x)=log(x) is a link function and $s_{n+j}(x)$ is a smooth, non-parametric function associated to the j-th continuous covariate.

# Implementation of experiments

# Implementation of experiments

In our experiments, we will…

- estimate mortality rates using both a classical and a weighted approach in maximum likelihood estimation.
- We estimate mortality rates using pension information as a covariate instead of a weight.
  - We begin with models omitting socio-economic information as it is typically done in practice. We finish by incorporating models with socioeconomic information on participants.
- Since weighting by sum-insured is used claiming that rates are more liability tuned, we compute the liabilities associated to an annuity portfolio.
- We will analyze the fit of estimates from a financial perspective.

# Data

# Data

For our purposes, we will use a dataset provided by one of the largest insurance companies in the Netherlands. This data set offers valuable information on the survival of individuals buying annuity products from the company.

- Survival information given in the form of yearly snapshots.
- The time period covers from 2015 to 2022
- Estimations consider information on 630 112 individuals (428 563 men and 201 549 women).
- Total exposure used amounts to 4 454 820 years lived.
- Ages go from 30 to 100

The data includes policyholder characteristics:

- Age
- Gender
- Estimated salary
- Sum insured
- Postcode based characteristics

# Data

| Variable | Label | Description (categories) | Provenance |
|---|---|---|---|
| $AG$ | Age | Age | Personal records |
| $GE$ | sex | Male or female | Personal records |
| $PE$ | Pension amount | Amount (to be) received per annuity payment | Personal records |
| $PQ$ | Pension Quantile | Computed by age and sex. Class 1 (20% lowest pension),Class 2, Class 3, Class 4, Class 5 (20% highest pension) | Personal records |
| $SA$ | Salary | Annual income of less than 25k, 25k to less than 50k,50k to 250k, Over 250k, Unknown | Personal records |
| $OR$ | Origin of individuals | Dutch (born in the Netherlands with and without Dutch background), other (born outside of the Netherlands) | Area records |
| $SB$ | Percentage of individuals receiving social benefits | Less than 33%, 33% to 66%, 66% or more, unknown | Area records |
| $BY$ | Building year of houses | Before 1945, 1945 to 1995, after 1995 | Area records |
| $VH$ | Value of house | Less than 250k, 250k to 400k, over 400k | Area records |
| $UI$ | Urban Indicator | Indicator of concentration of human activities. Class 1 (more densely populated areas), Class 2, Class 3, Class 4, Class 5 (least densely populated areas) | Area records |

- The variables related to postcode correspond to the dominant category in the zipcode of the person. For instance, if most houses were built before 1945 in the postcode of a policyholder, the policyholder counts in the category "Before 1945" for variable BY.

# Computation of liabilities

# Computation of liabilities

Based on mortality estimations, we compute the value of a portfolio:
The benefit of the product is an annuity payable yearly from age 65 until death

**Characteristics of the portfolio**

- 63 012 policyholders
- From all ages

We compute the liability associated to person i under a deterministic approach as

$$P_i * AF_{xi}$$

Where $AF_{xi}$ is defined as an annuity factor per unit of sum insured and $P_i$ the corresponding benefit amount. Taking $d_i = \max(0, 65 - x_i)$ as the deferred period, we get

$$AF_{x_i} = \begin{cases} a_{x_i} & \text{if } x_i \geq 65 \\ {}_{d_i}E_{x_i} a_{65} & \text{if } x_i < 65 \end{cases}$$

# Weight determination

- Weights can take multiple forms
- Hu (1997) stresses on the importance of weights used to construct estimators. For their theorical work, they define their weights as positive and estimated so that they add up to 1.
- Harrel (2015) mentions that weighted maximum likelihood estimators can be obtained using weights that do not necessarily have to be a frequency or an integer and give the example of weights in the case of sample surveys.
- The industry tends to use directly the sum insured of the individual. This is to try to model mortality in amounts.
- As suggested by Richards (2008), however, it makes sense to use weights that add up to the total number of individuals. We adopt this.

# Weight determination –Weight 1

- For individual i, we define the weight

$$w_i = \frac{B_i}{\frac{\sum_{j=1}^{N} B_j}{N}} = \frac{B_i}{\bar{B}}$$

This is equivalent to the weights typically used by the industry in the sense that weights are entirely dependent on sum insured only.

# Weight determination –Weight 2

- For individual i, we define a weight that considers age and gender structure as

$$w_i^{(g,x)} = \frac{B_i^{g,x}}{\frac{\sum_{j=1}^{N_{g,x}} B_j^{g,x}}{N_{g,x}}} = \frac{B_i^{g,x}}{\bar{B}^{g,x}}$$

where the superscripts g and x are used to differentiate benefits associated to a particular gender g and age x.

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# Financial Test of Fit

- For people active in the population at the beginning of 2022 ($AP_{2022}$), we compute the difference between the expected and actual liability at the end of 2022 as

$$ID^M = (\delta_i^{2022} \cdot P_i \cdot \bar{AF}_{x_i} - p^M_{x_i,2022} \cdot P_i \cdot \bar{AF}_{x_i})^2 = (\delta_i^{2022} - p_{x_i,2022})^2 \cdot (P_i \cdot \bar{AF}_{x_i})^2$$

Where $\delta_i^{2022}$ denotes the survival indicator to year 2022, $p_{xi,2022}^M$ denotes the probability of surviving year 2022 for the individual under mortality model M. In addition, the average annuity factor considers all the annuity estimations for person i associated to model M (average of all 4 weighted, unweighted versions).
We therefore adopt the indicator of overall difference relative to the total value of liabilities as

$$RD^M = \sqrt{\frac{\sum_{i=1}^{AP_{2022}} (\delta_i^{2022} - p^M_{x_i,2022})^2 \cdot (P_i \cdot \bar{AF}_{x_i})^2}{\sum_{i=1}^{AP_{2022}} (P_i \cdot \bar{AF}_{x_i})^2}}$$

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# Results

# Estimated liabilities with models considering only age, gender and pension amount

| Discount rate: 1%<br>Maximum attainable age ($\omega$): 120 years<br>Model Covariates: Only Age and Sex | Unweighted (in MM EUR) | $W_1$ (in MM EUR) | $W_2$ (in MM EUR) | Pension as covariate (in MM EUR) |
|---|---|---|---|---|
| *Gompertz* | | | | |
| Best estimate liability | 36 230 | 37 127 | 37 297 | 37 165 |
| $RD^M$ | 0.08436 | 0.08421 | 0.08418 | 0.08418 |
| *Makeham* | | | | |
| Best estimate liability | 36 267 | 37 211 | 37 435 | 37 267 |
| $RD^M$ | 0.08407 | 0.08391 | 0.08388 | 0.08387 |
| *Makeham-Perks* | | | | |
| Best estimate liability | 36 298 | 37 243 | 37 465 | 37 292 |
| $RD^M$ | 0.08420 | 0.08403 | 0.08400 | 0.08400 |
| *Makeham-Beard* | | | | |
| Best estimate liability | 36 276 | 37 210 | 37 397 | 37 277 |
| $RD^M$ | 0.08408 | 0.08391 | 0.08389 | 0.08388 |
| *GAM model* | | | | |
| Best estimate | 36 846 | 39 920 | 40 053 | 37 674 |
| $RD^M$ | 0.08344 | 0.08320 | 0.08318 | 0.08332 |

Table: Summary of estimated liability values under a simplified model.

Taking $YR$ as a year regressor defined as $YR = y - 2019$, with $y \in \{2015, ..., 2022\}$, the equation describing our GAM models corresponds to

$$log(E[D_{i,y}]) = \log(t_{i,y}) + YR \cdot \theta_{YR} + s_{AG}(AG_{i,y}, by = GE) + \theta_{GE} GE_i$$

# Estimated liabilities with models considering socioeconomic information

| Discount rate: 1%<br>Maximum attainable age ($\omega$): 120 years<br>Model Covariates: All included in Table 2 | Unweighted (in MM EUR) | $W_1$ (in MM EUR) | $W_2$ (in MM EUR) | Pension as covariate (in MM EUR) |
|---|---|---|---|---|
| *Gompertz* | | | | |
| Best estimate liability | 37 702 | 37 701 | 38 171 | 38 009 |
| $RD^M$ | 0.08312 | 0.08298 | 0.08295 | 0.08304 |
| *Makeham* | | | | |
| Best estimate liability | 37 568 | 37 322 | 37 731 | 37 579 |
| $\mathrm{RD}^M$ | 0.08306 | 0.08294 | 0.08291 | 0.08305 |
| *Makeham-Perks* | | | | |
| Best estimate liability | 37 976 | 37 616 | 37 998 | 38 368 |
| $RD^M$ | 0.08313 | 0.08301 | 0.08297 | 0.08305 |
| *Makeham-Beard* | | | | |
| Best estimate liability | 37 550 | 37 352 | 37 734 | 37 582 |
| $RD^M$ | 0.08314 | 0.08302 | 0.08300 | 0.08307 |
| *GAM model* | | | | |
| Best estimate | 39 510 | 40 197 | 40 397 | 39 624 |
| $RD^M$ | 0.08302 | 0.08279 | 0.08277 | 0.08300 |

Table: Summary of estimated liability values.

# Weighting with $W_1$ results in lower liabilities: why?

- Let's take a look at some basic descriptive statistics to understand what $W_1$ is doing.

| Pension band | Pension value | Headcount | Total weight | Headcount females | Weight females | Proportion females (%) |
|---|---|---|---|---|---|---|
| B1 | Less than 1 000 | 278 172 | 27 021.26 | 111 537 | 10 027.22 | 40.09 |
| B2 | [1 000, 2 000[ | 97 541 | 35 751.66 | 33 096 | 12 058.02 | 33.93 |
| B3 | [2 000, 3 000[ | 55 798 | 34 871.02 | 17 387 | 10 823.28 | 31.16 |
| B4 | [3 000, 4 000[ | 36 720 | 32 353.89 | 10 207 | 8 972.51 | 27.79 |
| B5 | [4 000, 5 000[ | 26 844 | 30 513.77 | 6 809 | 7 736.14 | 25.37 |
| B6 | [5 000, 6 000[ | 20 217 | 28 112.51 | 4 621 | 6 416.50 | 22.86 |
| B7 | [6 000, 7 000[ | 16 208 | 26 677.64 | 3 416 | 5 614.45 | 21.08 |
| B8 | 7 000 or more | 98 612 | 414 810.25 | 14 476 | 50 219.94 | 14.68 |

This suggests that the model is paying disproportionate attention to the survival experience of a (relatively) small group of males, which is resulting in higher mortality under a more granular model. Using directly sum insured as weights yields the same result.

# Key take aways

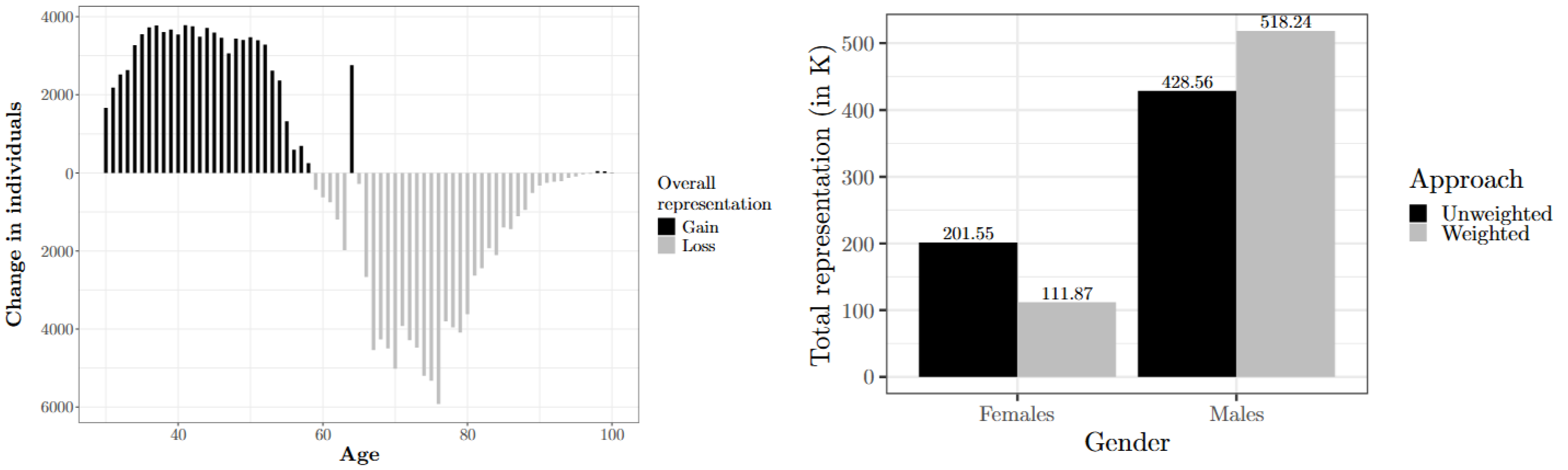
# Key takeaways

- Weighting has a clear impact on the value of liabilities, and the choice of weights used will affect financial results. Model choices will also make a difference.

- Let's talk about the fundamental difference between using pension information as a covariate and as a weight.

# Key takeaways

- Weights that only depend on sum-insured may introduce bias in representation when estimating parameters.




(a) Change in representation by age when switching from $W_1$ to $W_2$

(b) Change in representation by sex when weighting using $W_1$

# Key takeaways

- Pension amount by itself is not the best token of liability value in annuity business: size of payment vs. Number of remaining payments left?

- Multiplying by sum-insured does not necessarily transform mortality from a person basis to sum-insured basis: sum insured is only used as an indicator of relevance when estimating parameters, but focus continues to be human survival.

- Weights do not address the value of insight and information.

# Sources

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Thank you! Obrigado!

# Questions?

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