DAV/DGVFM
Herbsttagung
2024

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FINANZ- UND VERSICHERUNGSMATHEMATIK
FINANCIAL AND ACTUARIAL MATHEMATICS

Einige Optimierungsprobleme unter Vorgabe einer Zielverteilung

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Introduction

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Target: Opportunistic Mode With a Compound Poisson Surplus

The Surplus

We look at an insurance company, whose surplus is assumed to follow

a) **A Brownian motion with drift:**

$$X_t = \mu t + \sigma W_t \,, \quad t \in [0, T] \,,$$

where $\mu, \sigma > 0$ and W is a standard Brownian motion.

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b) **A classical risk model:**

$$X_t = ct - \sum_{i=1}^{N_t} Z_i\,, \quad t \in [0, T]\,.$$

where $c > 0$, $N = \{N_t\}$ is a Poisson process independent of the iid claim sizes $(Z_i)_{i \geq 1}$.

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In both cases, the initial capital is set to 0.

Target: A Specific Terminal Surplus Distribution

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Target: The Terminal Surplus Is Normally Distributed

Let the surplus be given by a Brownian motion with drift.

Some optimisation problems under the constraint of a normal terminal surplus:

- Maximise the expected discounted dividends.
- Minimise the ruin probability over dividend payments **!!!**
- Minimise the ruin probability over proportional reinsurance.

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The solution to the optimisation problems are:

- For a positive preference rate $\delta > 0$: "pay on the maximal rate until some critical time t^* and do not pay after" in order to maximise the expected dividends.
- "Do not pay until some critical time t^* and pay on the maximal rate after" to minimise the ruin probability.
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DAV Deutsche Aktuarvereinigung e.V.
DGVFM Deutsche Gesellschaft für Versicherungs- und Finanzmathematik e.V.

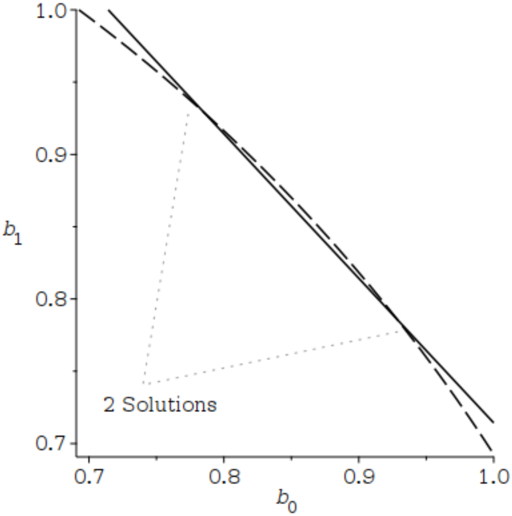
A 2-Period Model With Reinsurance

The strategy in $[0, T]$ can be changed only once, at $T/2$.

Because the mean, the variance and the distribution are fixed, we look at the strategies of the form: (b_0, b_1), where b_0 is a constant reinsurance strategy in $[0, T/2)$ and b_1 in $[T/2, T]$ with some $\Lambda > 0$:

$$\mathbb{E}[X_1]b_0 + \mathbb{E}[X_1]b_1 = 2(\mu + \Lambda), \quad \text{and} \quad b_0^2 \mathrm{Var}(X_1) + b_1^2 \mathrm{Var}(X_1) = \sigma^2 .$$

The solutions have the form



Target: The Terminal Surplus Close to a Desired Distribution

☞ We are looking at a compound Poisson process under proportional reinsurance $\{X_t^b\}$ and fix a desired terminal (at $t = 1$) distribution Q.

☞ The squared Wasserstein distance to the desired target should be minimised over constant deductibles:

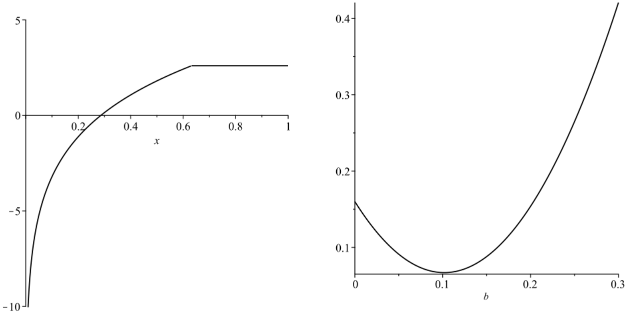
$$\inf_b \inf_{Y\sim\Psi} \mathbb{E}\left[(X_1^b - Y)^2\right] = ?$$

☞ Note that it holds

$$\inf_{Y\sim\Psi} \mathbb{E}\left[(X_1^b - Y)^2\right] = \mathbb{E}\left[(G^{-1}(U;b) - \Psi^{-1}(U))^2\right],$$

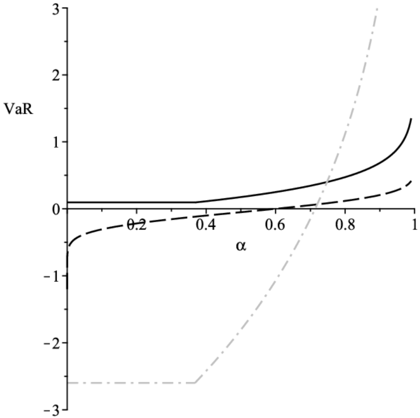
where G^{-1} is the inverse of the distribution of X_1^b, and U is an on $[0,1]$ uniformly distributed random variable.

The Modified Problem



The inverse distribution function of X_1 (left) and the Wasserstein distance in dependence on b.

VaR for the Normal Distribution as the Target and Exponential Claims



VaR for X_1 (gray dashdotted), VaR for the optimal strategy (solid black) and VaR for the normal target (dashed black).

Further Problem Set I:

Minimise the variance for a given expectation (and correlation coefficient).
A non time-consistent problem.

Dynamic strategies.

Achievability of distribution choices.

Further Problem Set II:

For a dividend optimisation problem:

Fix a strategy that seems reasonable. Now, solve the HJB backwards and find the corresponding drift and variance. The diffusion with these parameters features the chosen strategy as the optimal dividend choice behaviour.

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Vielen Dank für Ihre Aufmerksamkeit.

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