Disability insurance with information delays

Oliver Lunding Sandqvist
Industrial PhD student
University of Copenhagen and PFA Pension

CONVENTION A
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Overview

1. Introduction
2. Available data
3. Reserving
4. Estimation
5. Data application
6. Closing remarks

Background

Talk is based on three papers (available on ArXiv):

- **Transaction time models in multi-state life insurance**
  - Jointly with Kristian Buchardt and Christian Furrer.
  - Published in SAJ 01/03/2023.
- **Estimation for multistate models subject to reporting delays and incomplete event adjudication**
  - Jointly with Kristian Buchardt and Christian Furrer.
- **A multistate approach to disability insurance reserving with information delays**

Background

Why disability insurance?

Competitive market

Rapid changes/trends

Long cash flows

Discrepancy between theory and practice

Classic semi-Markov disability model

- Semi-Markov model $\{Y(t)\}_{t\geq 0}$ for insurance events.
- State space:

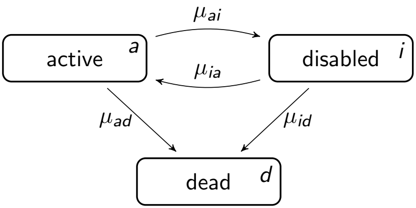


Figure 1: Illness-death model.

- Baseline covariates (age, gender, etc.) suppressed throughout, but could easily be included by conditioning on them.
- References: Janssen (1966), Hoem (1972), Haberman and Pitacco (1998), Helwich (2008), Christiansen (2012), and Buchardt et al. (2015).

Classic semi-Markov disability model

- Cash flow (specified in the insurance contract):

$$B(\mathrm{d}t) = \sum_{j} 1_{(Y(t-)=j)} b_j(t, U(t-))\,\mathrm{d}t + \sum_{j \neq k} b_{jk}(t, U(t-)) N_{jk}(\mathrm{d}t).$$

  - Counting processes $N_{jk}(t) = \#$jumps from j to k before time t.
  - Duration process $U(t) =$ duration since last jump at time t.

- Interest rate: $r(t)$.
- Present value: $PV(t) = \int_t^\infty e^{-\int_t^s r(v)\,\mathrm{d}v}\,\mathrm{d}B(s)$.
- Prospective reserve:

$$V(t) = V_{Y(t)}(t, U(t)) = \mathbb{E}\Big[PV(t) \mid Y(t), U(t)\Big].$$

Classic semi-Markov disability model

- Maximum likelihood estimation (MLE) based on $[0, t]$ observations discretized in $0 = t_0 < t_1 < \cdots < t_M = t$:
  - Occurrences and exposures:

$$
\begin{aligned}
O_{jk}(t_m) &= N_{jk}(t_{m+1}) - N_{jk}(t_m), \\
E_j(t_m) &= \int_{t_m}^{t_{m+1}} 1\{Y(s-) = j\} \, \mathrm{d}s.
\end{aligned}
$$

  - Input $(O_{jk}(t_m))_{j,k,m}$ into `glm` as independent observations where $O_{jk}(t_m)$ has mean $\mu_{jk}(t_m, U(t_m), \beta)E_j(t_m)$ for regressor β.

- References: Lindsey (1995).

Research problems

Problem: Y not fully observed due to reporting and processing delays.

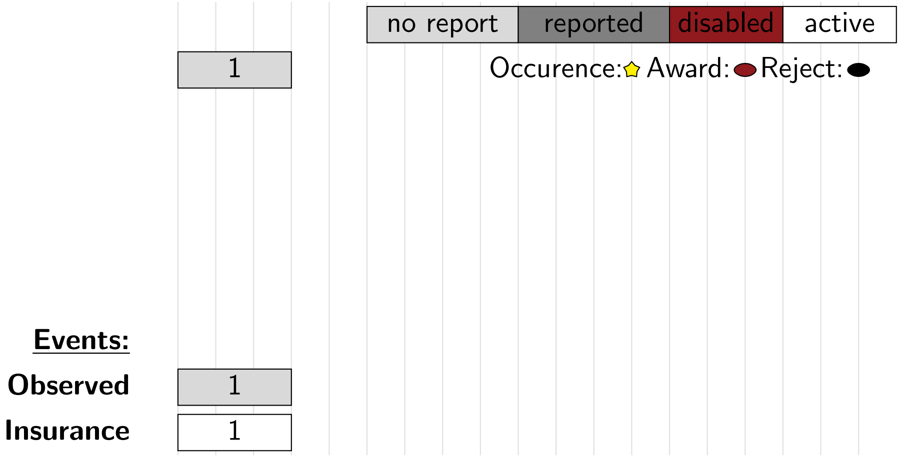
- Reserving: Can't compute $V(t)$ if $(Y(t), U(t))$ not fully known at time t and can't assume $B(t)$ is paid at time t.
- Estimation: Can't estimate using likelihood since occurrences and exposures not fully known at time t.

Content of presentation:

- Describe available data.
- Reserving proposal.
- Estimation proposal.
- Data application.

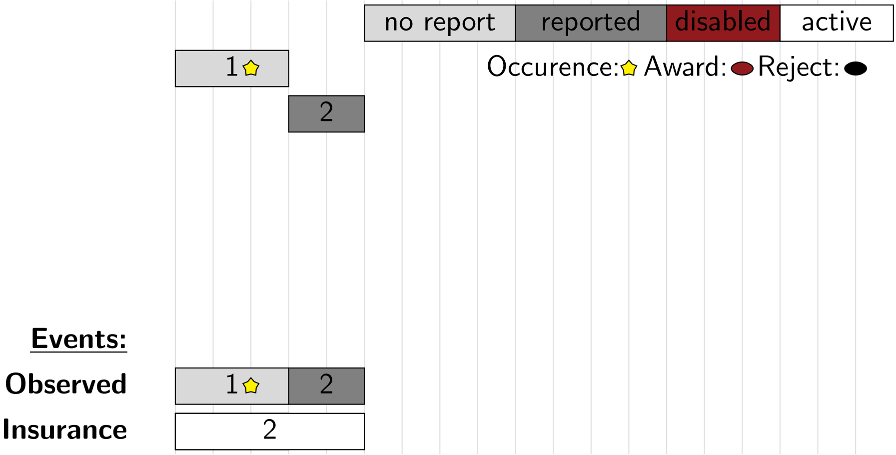
Available information

Example: Available information for one insured.



Available information

Example: Available information for one insured.



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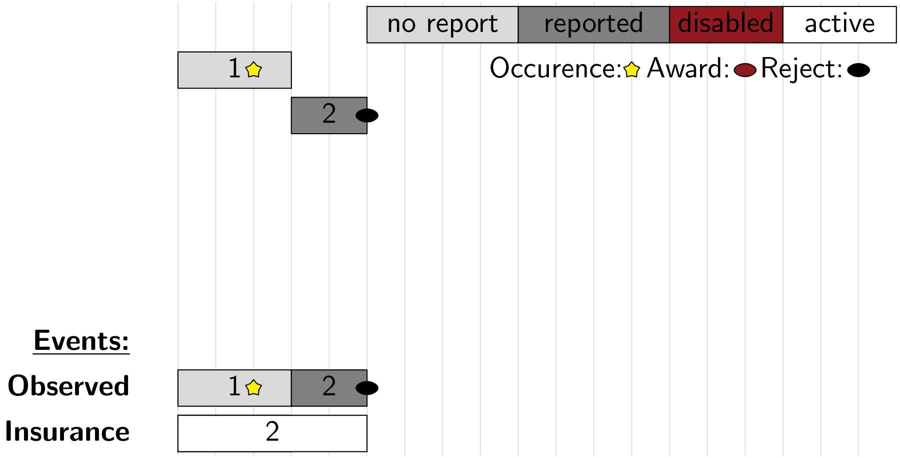

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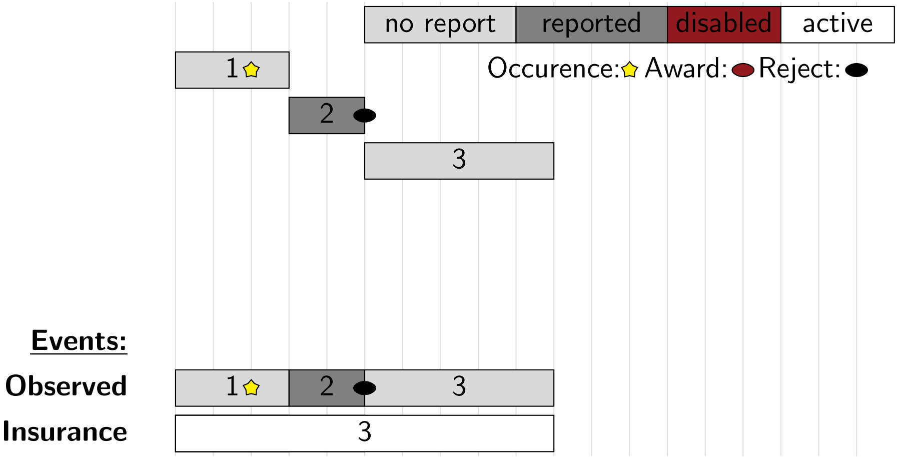

Available information

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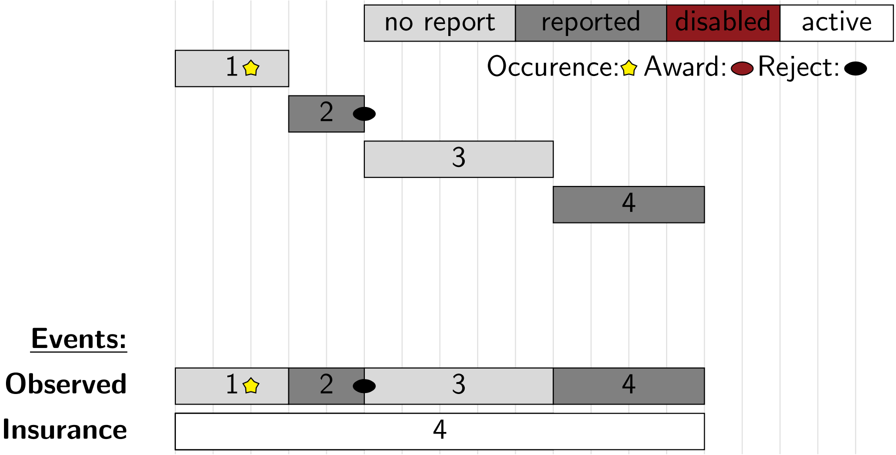

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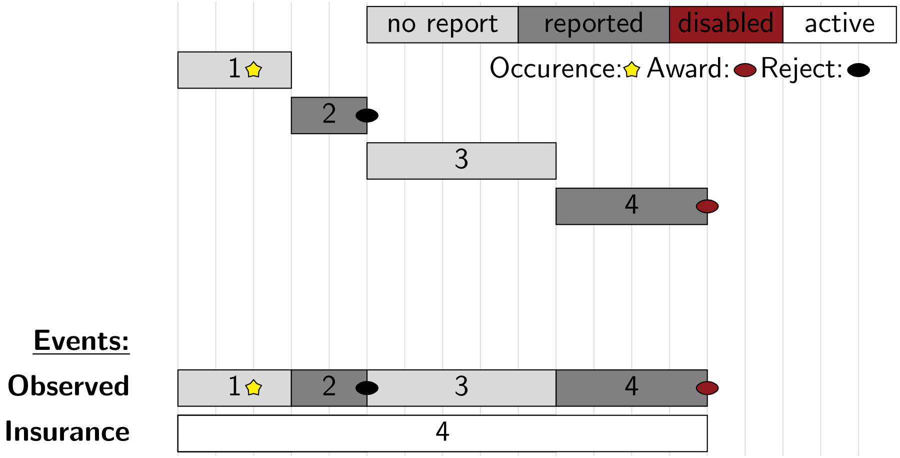

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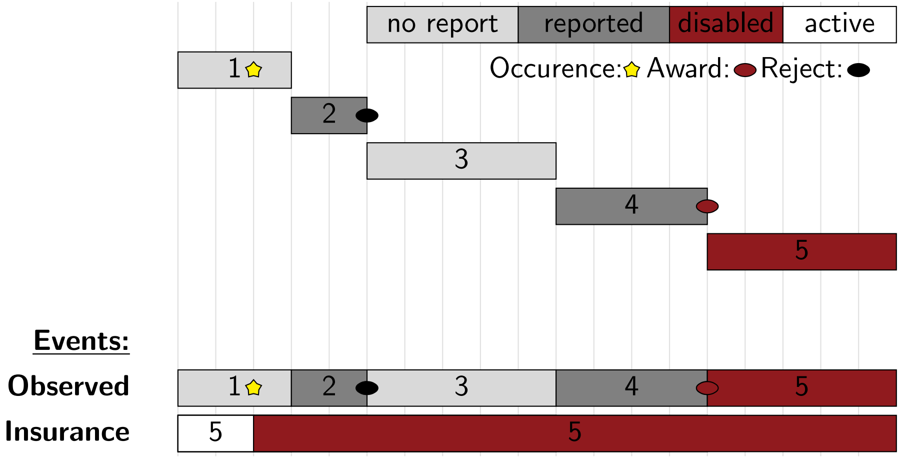

Disability insurance features

- Reasons for substantial reporting delays?
- Reasons for substantial adjudication processes?
- Disability insurance features:
  - Long cash flows $\sim$ life insurance.
  - Long reporting and adjudication delays $\sim$ non-life insurance.

- Modeling goal: Retain attractive properties of multistate models while accommodating reporting delays and adjudication processes.

Proposed disability model

- Jump process model $\{Z(t)\}_{t \geq 0}$ for claim settlement process.
- State space:

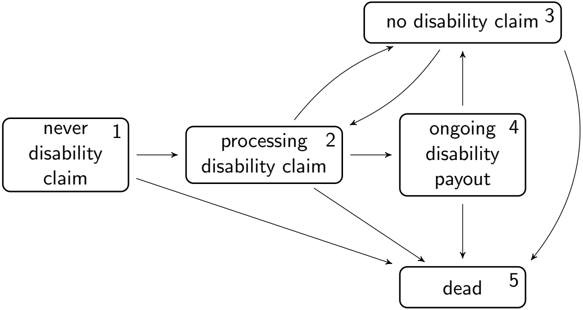

Figure 2: Claim settlement state space.

- Additional observed information:
  - The reported disability event time upon a transition into state 2.
  - What is awarded upon a transition out of state 2.

Proposed disability model

- Claim settlement reserve: $\mathcal{V}(t)$.
- Reserve $\mathcal{V}(t)$ differs from $V(t)$ in two ways:
  1. Reserve additionally to backpay (=*lump sum due to delay in payout of benefits that the insured was eligible for*).
  2. Reserve is a function of Z instead of (Y, U).

Proposed disability model

Proposed state-wise claim settlement reserves (simplified):

- State 1 (never disabiliy claim):

$$\mathcal{V}(t) = V_a(t) + \int_0^t V_i(s,0) I(t-s) \mu_{ai}(s) \, \mathrm{d}s.$$

  - $I(t-s) = \mathbb{P}(\text{Disability reporting delay } > t-s)$.

- State 2 & 3 (processing disability claim & no disability claim):

$$\mathcal{V}(t) = p(t) V_i(G(t), W(t)).$$

  - $p(t) = \mathbb{P}(\text{Award after time } t)$.
  - $G(t) =$ Start of next disability period if awarded after time t.
  - $W(t) =$ Length of awarded disability period by time t.

Proposed disability model

Proposed state-wise claim settlement reserves (simplified):

- State 4 (ongoing disability payout): $\mathcal{V}(t) = V_i(t, W(t))$.
- State 5 (dead): $\mathcal{V}(t) = V_d(t)$.

Estimation

- What to estimate:
  - Classic transition hazards: $(s, u) \mapsto \mu_{jk}(s, u)$.
  - Reporting delay distribution: $s \mapsto I(s)$.
  - Award probabilities: $s \mapsto p(s)$.
- $p(s)$ can be modeled as probability of hitting state 3 in Figure 3:

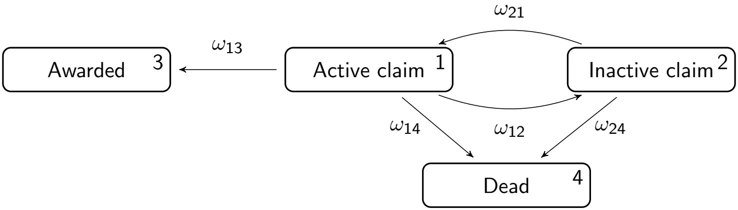

Figure 3: State space for adjudication.

Estimation procedure

Estimation procedure with time of analysis t:

1. $p(s)$: MLE for ω_{jk} and plug in.
2. $I(s)$: MLE for the reporting delays conditional on the delay being less than $t - T_{\text{disability}}$ and weighted with $p(t)$.
3. $\mu_{jk}(s,u)$: MLE based on the observed occurrences $O^{\text{obs}}_{jk}(t_m)$ and exposures $E^{\text{obs}}_{j}(t_m)$ after modifying as follows:

$$O^{\text{obs}}_{ai}(t_m) \leftarrow O^{\text{obs}}_{ai}(t_m)p(t),$$
$$O^{\text{obs}}_{ia}(t_m) \leftarrow O^{\text{obs}}_{ia}(t_m)(1 - p(t)),$$
$$E^{\text{obs}}_{a}(t_m) \leftarrow E^{\text{obs}}_{a}(t_m)(1 - I(t - t_m)).$$

Estimator properties

- Consistency and joint asymptotic normality follows by two-step M-estimation results.

- References: Newey and McFadden (1994).

Application to real data

- Anonymized and slightly altered real disability insurance data.
  - Disability exposure and occurrences.
  - Reactivation exposure and occurrences.
  - Disability reporting delays.
  - Adjudication exposure and occurrences.
- Time window $[0, t]$ is [31/01/2015, 01/09/2019].
- Available on `https://github.com/oliversandqvist/Web-appendix-estimation-contamination`.

Application: Estimation

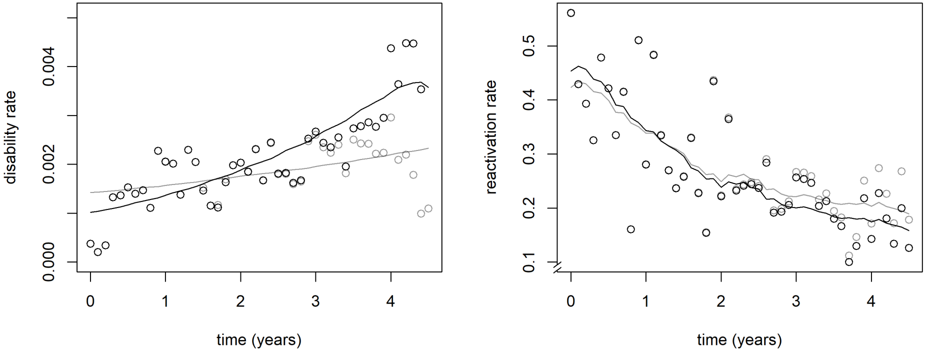

Figure 4: Fitted rates (lines) and occurrence-exposure rates (points) for the proposed method (black) and the naive method (gray). Disability rates are shown on the left and reactivation rates on the right.

Application: Reserving

Assume disability rate with 3 year coverage period

$$B(\mathrm{d}t) = 1\{Y(t-) = i\}1\{t - U(t-) \leq 3\}1\{\mathrm{Age}_0 + t \leq 67\}\,\mathrm{d}t.$$

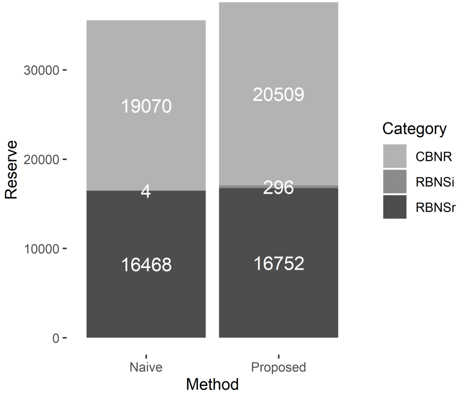

Figure 5: Portfolio level reserve decomposed by category.

Application: Reserving

- Proposed method leads to 5.7% increase in reserves compared to naive approach.
- Two large Danish insurers in 2019: Portfolio disability reserves of 1.4 billion USD and 2.6 billion USD.
- A 5.7% increase in these reserves is 80 million USD and 150 million USD respectively.

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Thank you for your attention!