

LIFE INSURANCE DEMAND UNDER AMBIGUOUS MORTALITY RISK

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Do you have life insurance contracts?

WHAT IS THE LIFE INSURANCE CONTRACT?

What is the life insurance contract? (one sentence)



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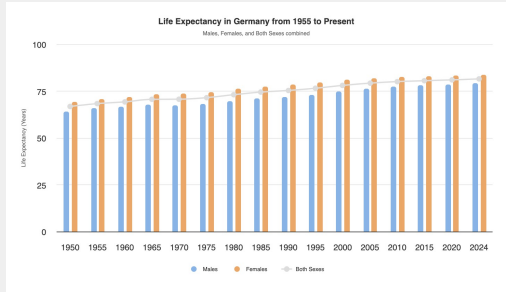


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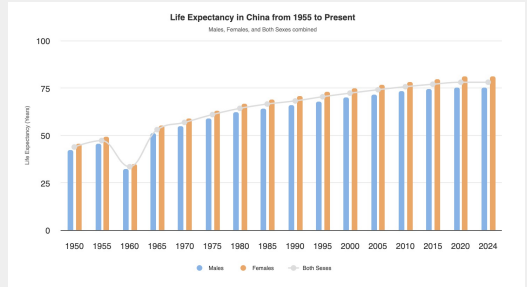


- The probability of death is the key!

HIGHLY UNPREDICTABLE MORTALITY RISK: POSITIVE

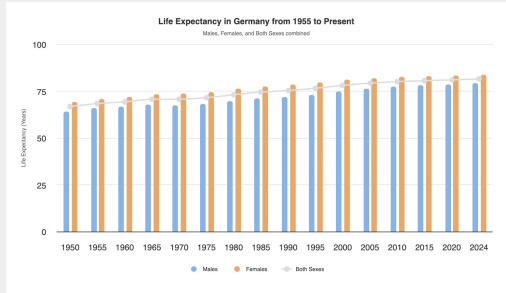


(a) Life expectancy in Germany

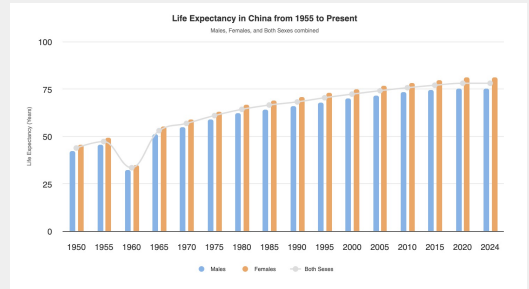


(b) Life expectancy in China

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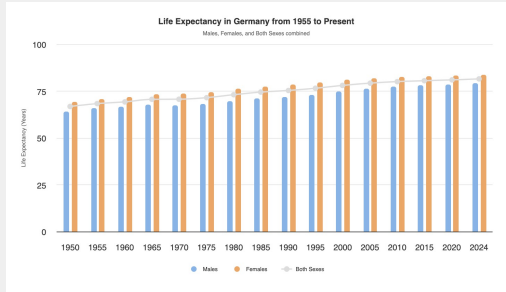
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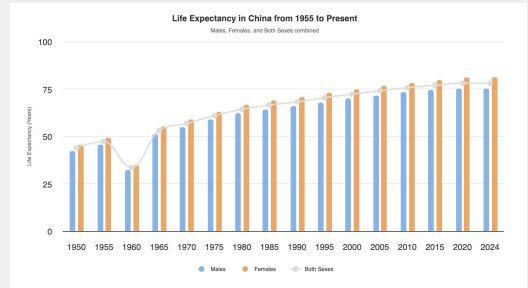
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- Medical technology innovations;

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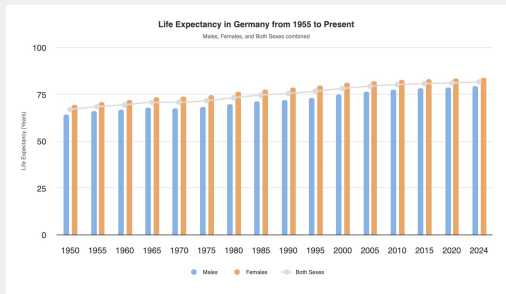
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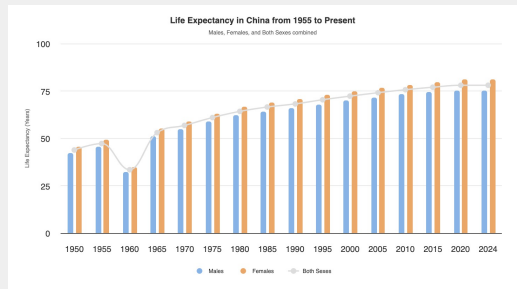
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- Medical technology innovations;
- Preventive healthcare investment;

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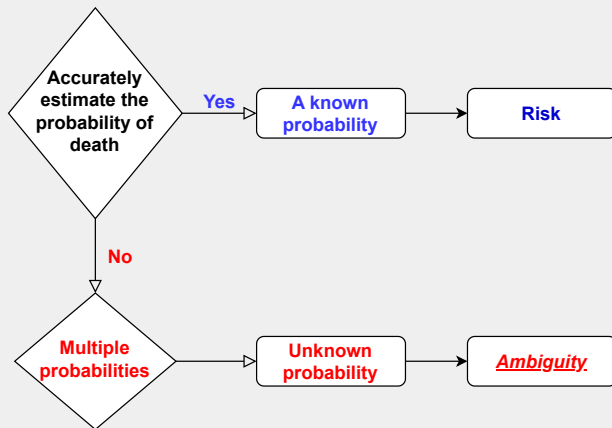
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- We assume that the utility is state-dependent, which relies on the **survival** or **death** states;
- We characterize the **optimal life insurance demand** under the **smooth ambiguity framework**.

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- We find that **ambiguity aversion could explain the under-insurance puzzle**.

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Our work:

- Introduction of **ambiguous mortality risk** into the **life insurance contract** and explain the under-insurance puzzle by **ambiguity aversion!**

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 - ▶ k is interpreted as the strength of bequest motive.

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Proposition 1 (No ambiguity)

Let $\tau \geq 0$ and $(1 + \tau)\pi < 1$. When $k \leq \frac{(1+\tau)(1-\pi)V'(w_0+y)}{(1-(1+\tau)\pi)V'(w_0)}$, the optimal contract $I^* = 0$. If $\frac{(1+\tau)(1-\pi)V'(w_0+y)}{(1-(1+\tau)\pi)V'(w_0)} < k < \frac{(1+\tau)(1-\pi)}{1-(1+\tau)\pi}$, the optimal contract $0 < I^* < y$. If $k > \frac{(1+\tau)(1-\pi)}{1-(1+\tau)\pi}$, the optimal contract $I^* > y$. In particular, $I^* = y$ if and only if $k = \frac{(1+\tau)(1-\pi)}{1-(1+\tau)\pi}$.

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- The smooth ambiguity model ([Klibanoff et al., 2005]):

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- The function ϕ characterizes the individual's attitude towards ambiguity:
 - ▶ An increasing and **linear ϕ** : **ambiguity neutrality**;
 - ▶ An increasing and **concave ϕ** : **ambiguity aversion**.

The probability of death Π is a **random variable!**

- Π with n possible outcomes $\pi_i \in (0, 1)$, $i = 1, \dots, n$, with known probabilities $(q_1, \dots, q_n) > 0$, $\sum_{i=1}^n q_i = 1$.

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- The insurer has information about the insured's **distribution** of uncertain mortality:
 - ▶ Expected value premium principle, i.e.,

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- The smooth ambiguity objective for the insured with the life insurance contract:

$$\begin{aligned} W(I) &:= \sum_{i=1}^n q_i \phi \left((1 - \pi_i)V(w_0 + y - P(I)) + \pi_i B(w_0 + I - P(I)) \right) \\ &:= \mathbb{E}_F[\phi(U(\Pi, I))], \end{aligned}$$

where \mathbb{E}_F is the expectation under the distribution F , i.e., $\mathbb{P}(\Pi = \pi_i) = q_i$ and

$$U(\Pi, I) := (1 - \Pi)V(w_0 + y - P(I)) + \Pi B(w_0 + I - P(I)).$$

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Proposition 2 (Ambiguity neutrality)

Let $\tau \geq 0$, $\phi(x) = x$ and $(1 + \tau)\hat{\pi} < 1$. When $k \leq \frac{(1+\tau)(1-\hat{\pi})V'(w_0+y)}{(1-(1+\tau)\hat{\pi})V'(w_0)}$, the optimal contract $I_{neutral}^* = 0$. When $\frac{(1+\tau)(1-\hat{\pi})V'(w_0+y)}{(1-(1+\tau)\hat{\pi})V'(w_0)} < k < \frac{(1+\tau)(1-\hat{\pi})}{1-(1+\tau)\hat{\pi}}$, the optimal contract $0 < I_{neutral}^* < y$. When $k > \frac{(1+\tau)(1-\hat{\pi})}{1-(1+\tau)\hat{\pi}}$, the optimal contract $I_{neutral}^* > y$. In particular, we have $I_{neutral}^* = y$ if and only if $k = \frac{(1+\tau)(1-\hat{\pi})}{1-(1+\tau)\hat{\pi}}$.

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Proposition 3

Let $(1 + \tau)\hat{\pi} < 1$. The optimal life insurance demand for a risk-and-ambiguity-averse insured is higher in the presence of ambiguity than in its absence, i.e., $I_{\phi}^* > I_{neutral}^*$ if and only if $\frac{\partial U(\Pi, I_{neutral}^*)}{\partial \Pi} < 0$. Moreover, $I_{\phi}^* < I_{neutral}^*$ if and only if $\frac{\partial U(\Pi, I_{neutral}^*)}{\partial \Pi} > 0$. In particular, $I_{\phi}^* = I_{neutral}^*$ if and only if $\frac{\partial U(\Pi, I_{neutral}^*)}{\partial \Pi} = 0$, where

$$\frac{\partial U(\Pi, I_{neutral}^*)}{\partial \Pi} = -V(w_0 + y - P(I_{neutral}^*)) + B(w_0 + I_{neutral}^* - P(I_{neutral}^*)).$$

Example 1

Let $(1 + \tau)\hat{\pi} < 1$ with $\tau \geq 0$. When $V = B$, we have $I_{\phi}^* \geq I_{neutral}^*$. If further requires $\tau = 0$, we have $I_{\phi}^* = I_{neutral}^* = y$.

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Example 2

Let $(1 + \tau)\hat{\pi} < 1$ with $\tau > 0$. When $k > \frac{(1+\tau)(1-\hat{\pi})}{1-(1+\tau)\hat{\pi}}$, $c \geq 0$ and $V(x) > 0$ for all $x \geq 0$, we have $I_{\phi}^* < I_{neutral}^*$.

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Proposition 4 (Greater ambiguity aversion)

Let $(1 + \tau)\hat{\pi} < 1$. In the presence of ambiguity, optimal life insurance demand for a risk-and-ambiguity averse insured increases with greater ambiguity aversion, i.e., $I_{R\phi}^* > I_\phi^*$ if $\frac{\partial U(\Pi, I_\phi^*)}{\partial \Pi} < 0$. Moreover, $I_{R\phi}^* < I_\phi^*$ if $\frac{\partial U(\Pi, I_\phi^*)}{\partial \Pi} > 0$. In particular, we have $I_{R\phi}^* = I_\phi^*$ if $\frac{\partial U(\Pi, I_\phi^*)}{\partial \Pi} = 0$, where $\frac{\partial U(\Pi, I_\phi^*)}{\partial \Pi} = -V(w_0 + y - P(I_\phi^*)) + B(w_0 + I_\phi^* - P(I_\phi^*))$.

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- **Outlook:**
 - ▶ Incorporating other **risks**: non-performance risk...
 - ▶ Considering other **frameworks**: multi-period model; dynamic model...
 - ▶ Other **insurance contracts**: long-term care insurance; climate-related insurance...

THANK YOU FOR YOUR ATTENTION!

PREPRINT IS COMING SOON...

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