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Benefit volatility-targeting strategies in lifetime pension pools

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Outline

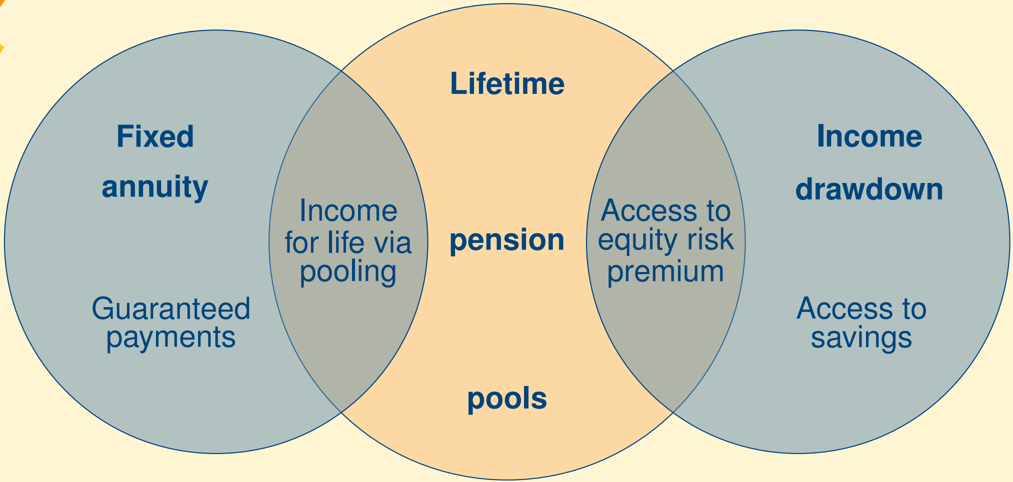
1 Motivation and introduction
2 The assumed data generating process
3 Lifetime pension pool design
4 Benefit volatility targeting
5 Implementation of the strategy
6 Limitations in practical situations
7 Concluding remarks and future research

Motivation and introduction

Lifetime pension pools

- Lifetime pension pools arrangements allow retiring individuals to convert a lump sum into **income for life**.
- It does not guarantee a specific level of income; instead, the pension payable **varies with the investment and mortality experience** of the group.

Lifetime pension pools

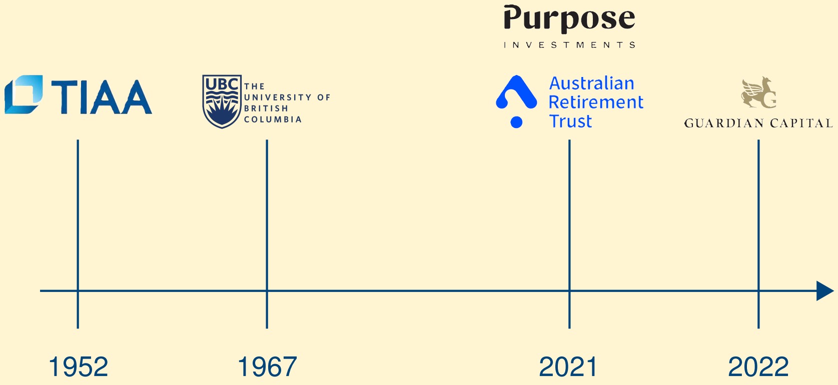


Same same, but different

Very broad definition that matches **different designs**:

- Group self-annuitization schemes (e.g., Piggott et al., 2005; Qiao and Sherris, 2013; Hanewald et al., 2013).
- Retirement tontines (e.g., Milevsky and Salisbury, 2015, 2016; Fullmer, 2019; Chen et al., 2021).
- Pooled annuity funds (e.g., Stamos, 2008; Donnelly et al., 2013).
- Variable annuity (e.g., Balter et al., 2020).
- Variable payment life annuity (e.g., ACPM, 2017).
- Variable payout annuities (e.g., Horneff et al., 2010).
- Decumulation-only collective defined contribution schemes (e.g., Donnelly, 2023).

Real-world examples



Why now?

- **Declining** prevalence of guaranteed pensions.
- Maturation of account-based accumulation schemes.
- Growth of conditional and variable elements in lifetime income provision.

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Investment strategies

- The design of these pools has primarily been examined within the context of elementary investment strategies, like **constant, static allocations** and investment strategies that only involve **risk-free assets**.
- Two notable exceptions rely on **volatility targeting**:
  - Olivieri, Thirurajah, and Ziveyi (2022).
  - Li, Labit Hardy, Sherris, and Villegas (2022).
- Both studies showed that volatility targeting **improves the investment performance** while **reducing volatility and downside risk**.
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How to adjust the asset allocation to **target the total volatility** of the benefit adjustment?

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Contributions

This study offers two contributions:

1. **Theoretical**: We derive an asset allocation strategy that considers both investment and mortality risks at the same time.
2. **Applied**: We investigate the implementation of the volatility-targeting strategy, which requires state-of-the-art data generating process.

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The assumed data generating process

Components of the generating process

1. A model for **financial asset returns**.
2. A **mortality** model.

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Risky asset

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$$\frac{dP_t}{P_t} = r_t\, P_t\, dt,$$

   where r_t represents the time-t risk-free rate and is based on a three-factor Vasicek model.

2. A risky asset modelled using a continuous-time two-factor stochastic volatility model that allows for jumps.

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Mortality model

- We propose using a **stochastic mortality model** that accounts for improvements in the spirit of Lee and Carter (1992) and Cairns et al. (2006).
- We use a continuous-time version of a two-factor APC model; we model the time-t central death rate for age x as

$$\log\left(m_{x,t}\right) = \alpha_{\lfloor x \rfloor} + \kappa_{1,t} + \kappa_{2,t}\left(\lfloor x \rfloor - \bar{x}\right).$$

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Mortality model

- We assume that the **first and second period effects** are modelled by the following SDEs in continuous time:

$$d\kappa_{1,t} = \theta_{\kappa_1}\, dt + \sigma_{\kappa_1} \boxed{dW_{\kappa_1,t}},$$

$$d\kappa_{2,t} = -\zeta_{\kappa_2}\, \kappa_{2,t}\, dt + \sigma_{\kappa_2} \boxed{dW_{\kappa_2,t}},$$

where $[\ W_{\kappa_1} \quad W_{\kappa_2}\ ]^\top$ is a two-dimensional standard Brownian motion with $d\langle W_{\kappa_1}, W_{\kappa_2}\rangle = \rho_{\kappa_1,\kappa_2}\, dt$.

Mortality model

- Once death rates are generated, we can recover **survival probabilities** using the following relationship:

$$ {}_{s}p_{x,t} = \exp\left(- \int_0^s m_{x+u,t+u}\, du \right). $$

Lifetime pension pool design

Basic assumptions

- A total of L_0 **members** joins the pool, each bringing K at inception.
- All members joining have the **same age** x at inception.
- We assume that members receive **m payments each year** of $h = \frac{1}{m}$ as long as they are alive.

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Benefit payments

- The **total benefit** amount paid by the lifetime pension pool at time t is

$$B_t = \frac{1}{m}\,\frac{F_t}{\ddot{a}_{x,t}^{(m)}}\,\mathbf{1}_{\{L_t \geq 1\}},$$

where $\ddot{a}_{x,t}^{(m)}$ denotes the actuarial value of a whole life annuity due making m payments per year (see, e.g., Bégin et al., 2024, for a semi-closed-form solution).

- Each surviving member's benefit amount is given by

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Fund value dynamics

- The investment allocation at time $t - h$ is given by:
  - Proportion ω_t in the **risky asset**.
  - Proportion $1 - \omega_t$ in the **risk-free asset**.
- The fund dynamics can be described as follows:

$$F_t = \left(F_{t-h} - B_{t-h}\right)\left(\omega_t \frac{S_t}{S_{t-h}} + (1 - \omega_t)\frac{P_t}{P_{t-h}}\right).$$

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Benefit adjustment rule

- The **time-*t* benefit** is updated according to

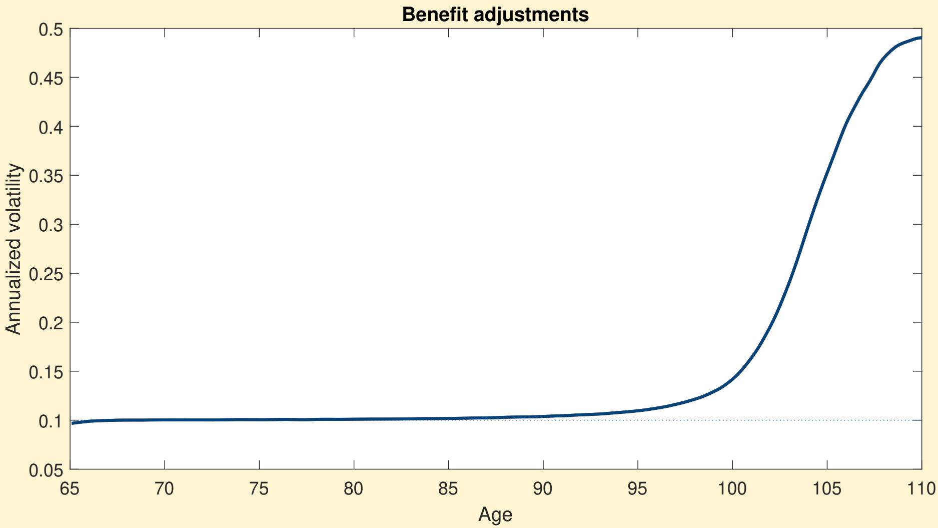
$$b_t = \eta_t \, b_{t-h},$$

where the adjustment is a product of three components:

$$\eta_t = \overbrace{\left(\omega_t \frac{S_t}{S_{t-h}} + (1-\omega_t)\frac{P_t}{P_{t-h}}\right) e^{-yh}}^{I_t \text{ or IEA}} \; \overbrace{\frac{L_{t-h}\, {}_h p_{x-h,t-h}}{L_t} \mathbf{1}_{\{L_t \geq 1\}}}^{M_t \text{ or MEA}}$$
$$\times \underbrace{\frac{\ddot{a}^{(m)}_{x-h,t-h} - h}{\ddot{a}^{(m)}_{x,t}\, e^{-yh}\, {}_h p_{x-h,t-h}}}_{C_t \text{ or CEA}}$$

and y is the **hurdle rate** used to compute the annuity price

Benefit volatility for static allocation



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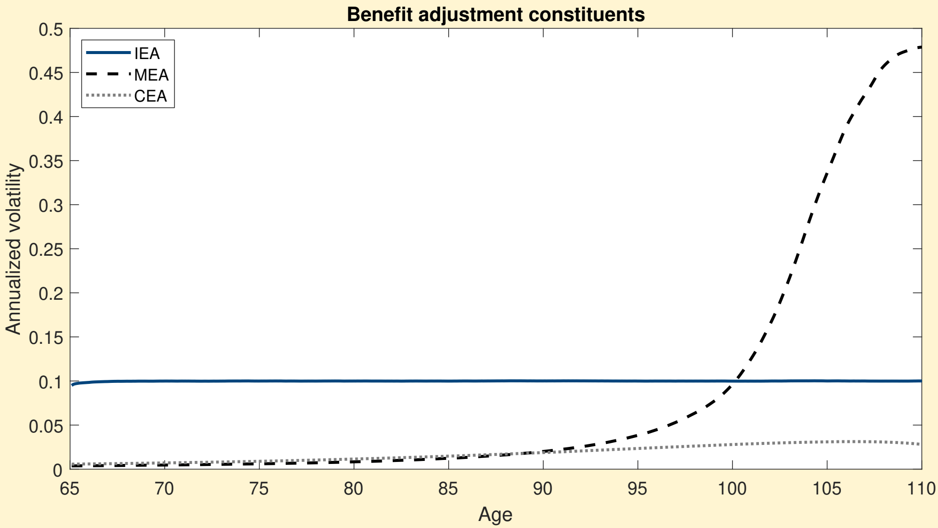

Benefit volatility targeting

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- The proportion ω_t can be controlled by the pool operator such that

$$\operatorname{Var}[\eta_t \mid \mathcal{F}_{t-h}] = \sigma_*^2 h,$$

where σ_* is the (annualized) exogenous **volatility target**.

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Pool operator assumptions

- The **mortality table is static** and based on time $t - h$ information.
- The number of pool members at time t conditional on information at time $t - h$ is given by

$$L_t \sim \mathrm{Bin}(L_{t-h}, {}_h\tilde{p}_{x-h,t-h}).$$

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- The time-t **risky asset price** S_t based on the information at time $t-h$ is given by

$$S_t = S_{t-h}\, e^{\varepsilon_t}, \quad \varepsilon_t \sim \mathcal{N}\left(\left(\hat{r}_t + \xi - \frac{\hat{\sigma}_t^2}{2}\right)h, \hat{\sigma}_t^2 h\right),$$

where $\hat{\sigma}_t^2$ is obtained via nonparametric volatility forecasting methods based on high-frequency returns.

- We rely on a simple implementation of the heterogenous autoregressive (HAR) model similar to that of Corsi (2009) and Corsi and Renò (2012):

$$\hat{\sigma}_t = \chi + \beta^{(d)}\,\mathrm{RVol}_{t-h}^{(1)} + \beta^{(w)}\,\mathrm{RVol}_{t-h}^{(5)} + \beta^{(m)}\,\mathrm{RVol}_{t-h}^{(21)} + \gamma^{(d)}\,r_{t-h}^{(1)} + \gamma^{(w)}\,r_{t-h}^{(5)} + \gamma^{(m)}\,r_{t-h}^{(21)} + \epsilon_t,$$

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Pool operator assumptions

- Let $\tilde{I}_t$, $\tilde{M}_t$, and $\tilde{C}_t$ be the **pool operator's proxy** for I_t, M_t, and C_t, respectively.
- We can thus obtain that

$$
\begin{aligned}
\sigma_*^2 h &= \mathrm{Var}\left[\tilde{I}_t \tilde{M}_t \tilde{C}_t \,\middle|\, L_{t-h},\ {}_h\tilde{p}_{x-h,t-h}, \hat{r}_t, \hat{\sigma}_t\right] \\
&= \mathrm{E}\left[\tilde{I}_t^2 \,\middle|\, \hat{r}_t, \hat{\sigma}_t\right] \mathrm{E}\left[\tilde{M}_t^2 \,\middle|\, L_{t-h},\ {}_h\tilde{p}_{x-h,t-h}\right] \\
&\quad - \mathrm{E}\left[\tilde{I}_t \,\middle|\, \hat{r}_t, \hat{\sigma}_t\right]^2 \mathrm{E}\left[\tilde{M}_t \,\middle|\, L_{t-h},\ {}_h\tilde{p}_{x-h,t-h}\right]^2
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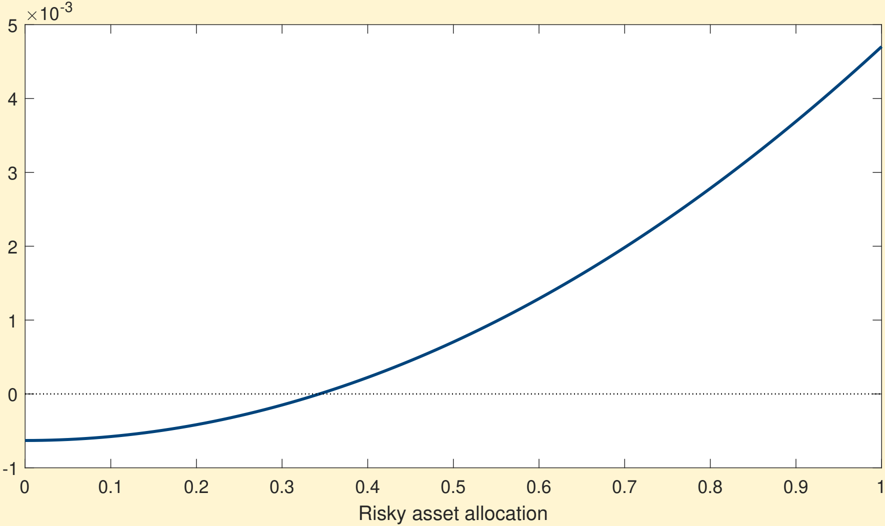
Volatility-targeting-based allocation



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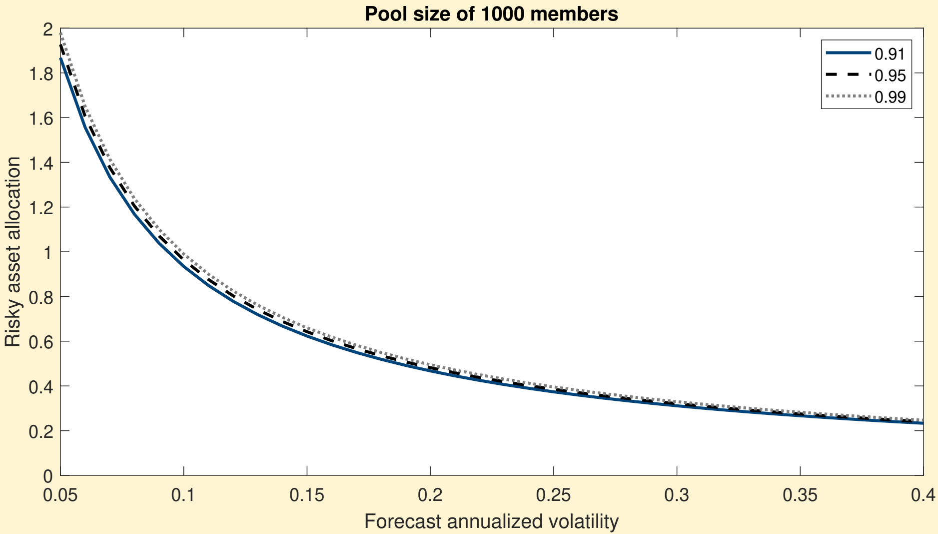

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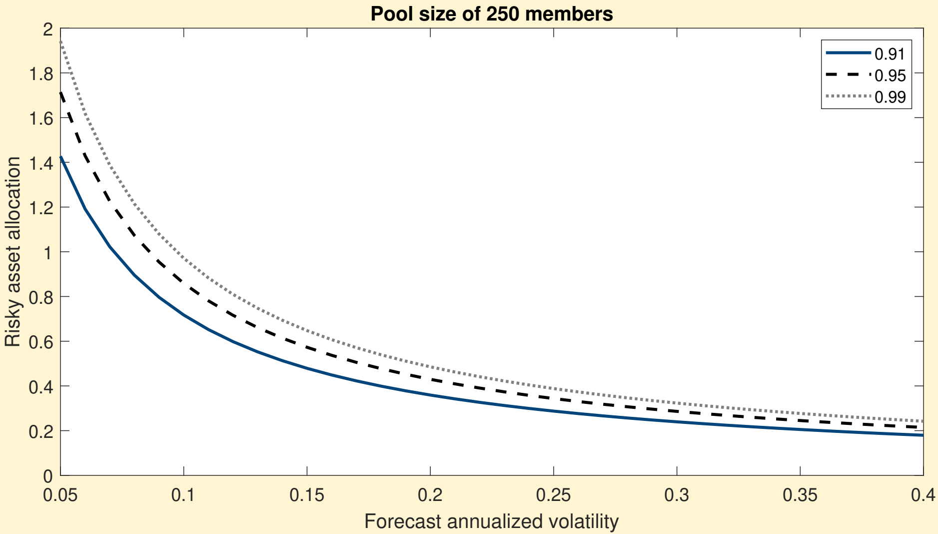
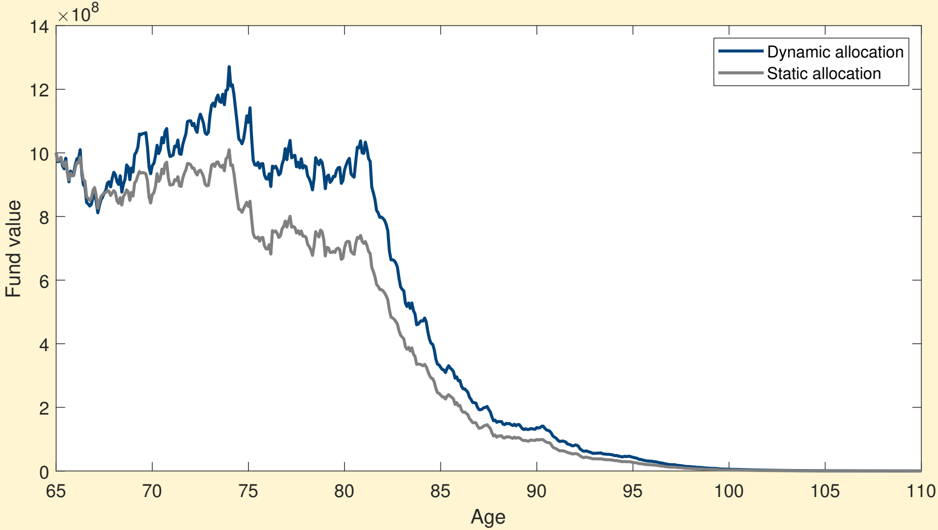


Implementation of the strategy

Base assumptions

- Pool of **1,000 members**.
- Each member bring **$1,000,000** to the pool.
- Benefits are paid **monthly**, at the beginning of each month.
- Volatility target σ^* set to **10%**.

Illustrative example



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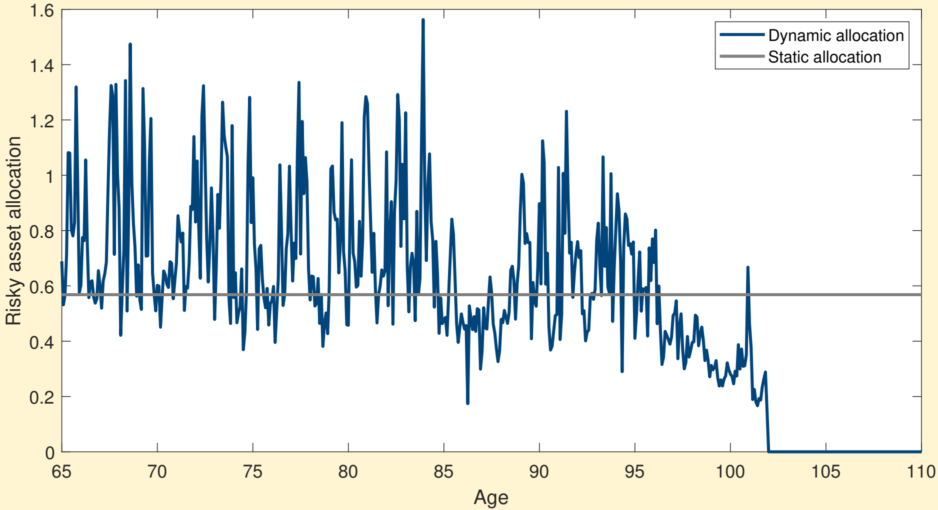

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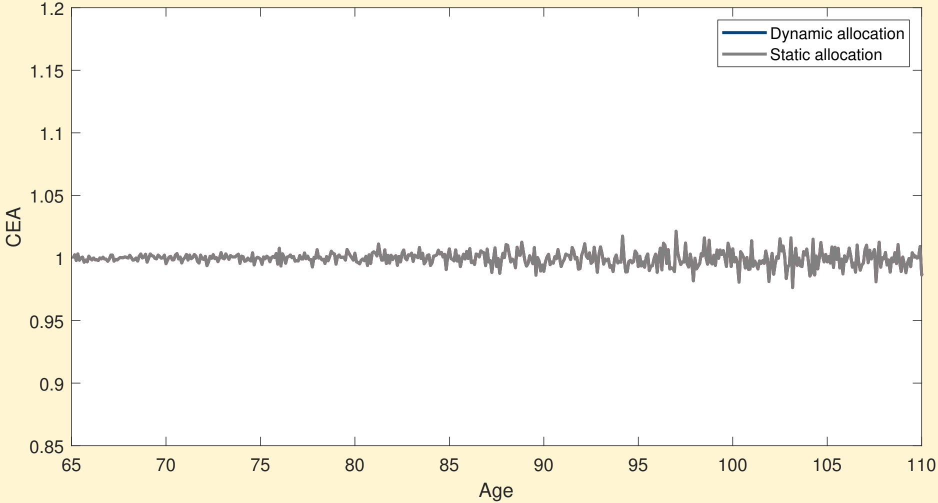

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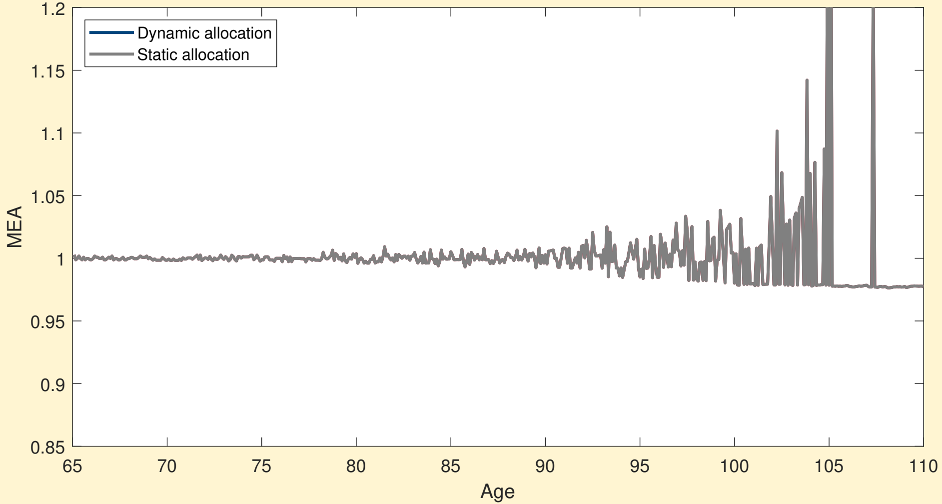

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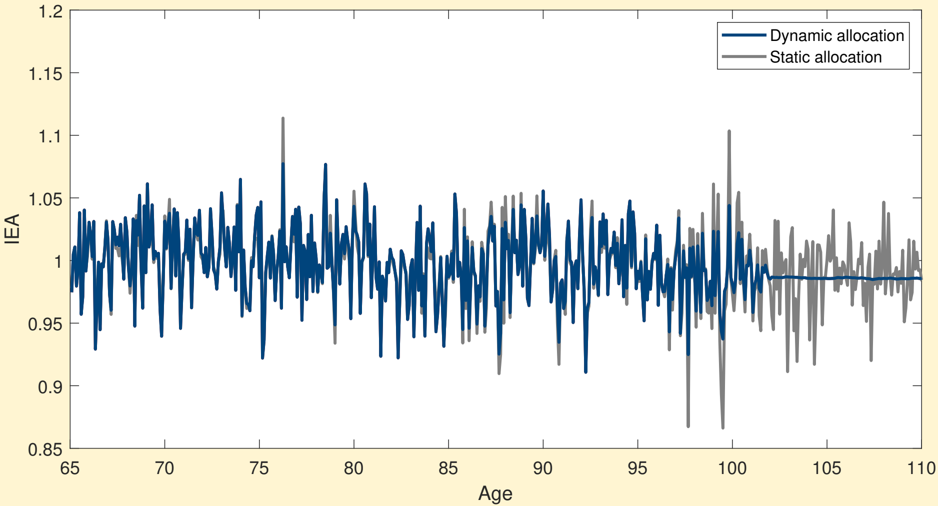
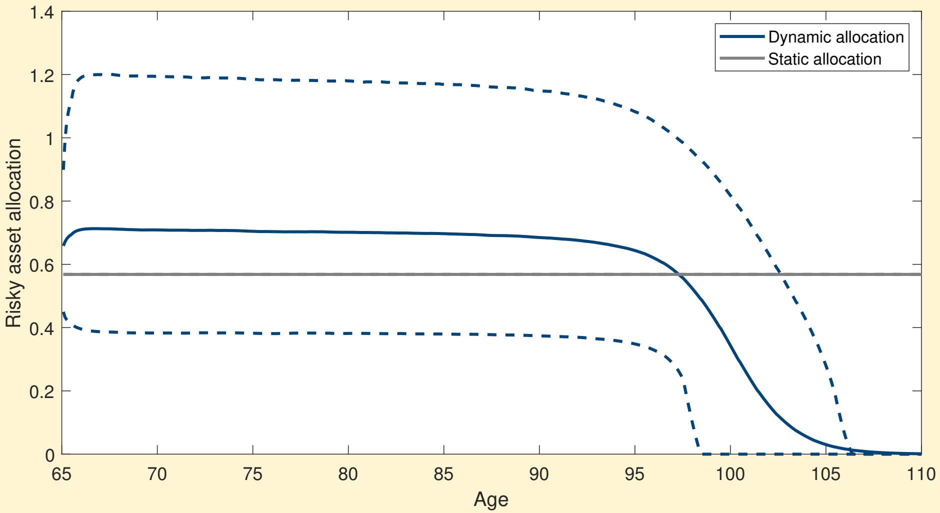
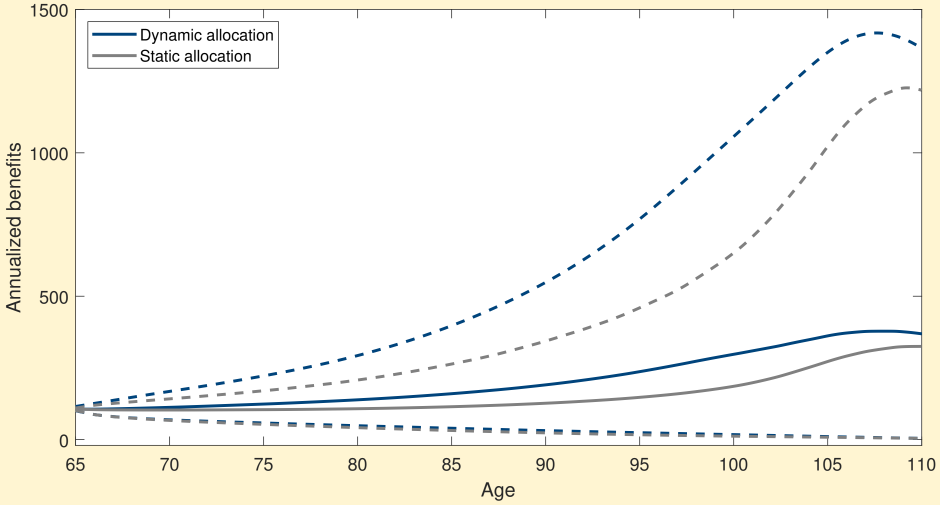


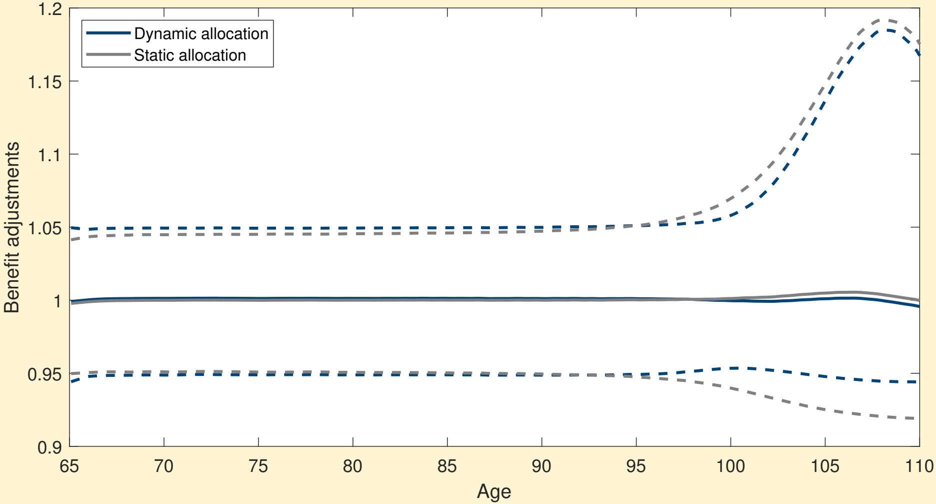
Risky asset allocation



Annualized benefits



Benefit adjustments



Robustness tests

- Exogenous volatility target: 13% and 16%.
- Pool size: 250 and 500 members

All tests lead to very robust results.

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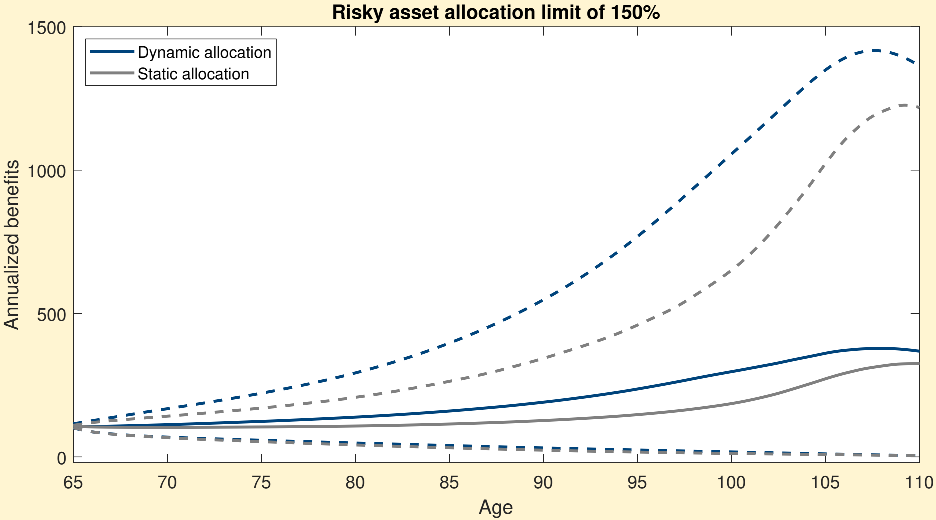
Limitations in practical situations

Leverage constraints



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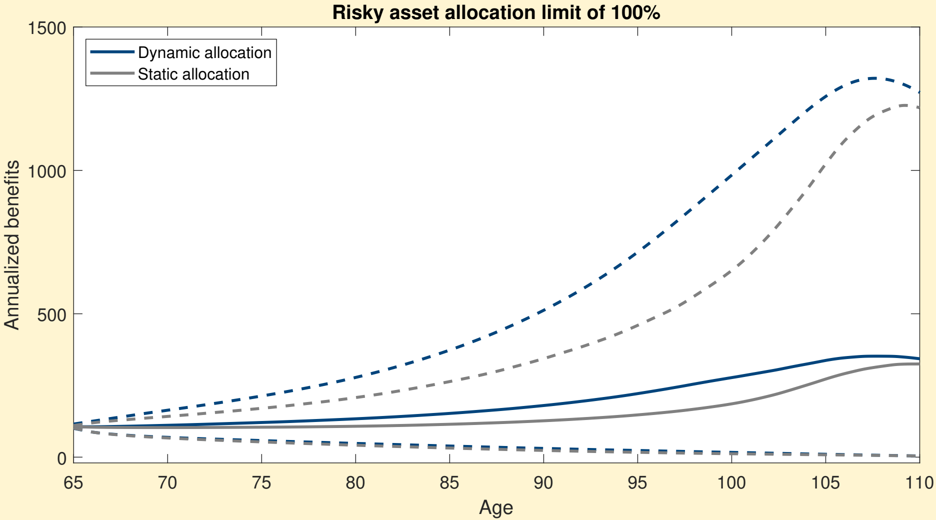
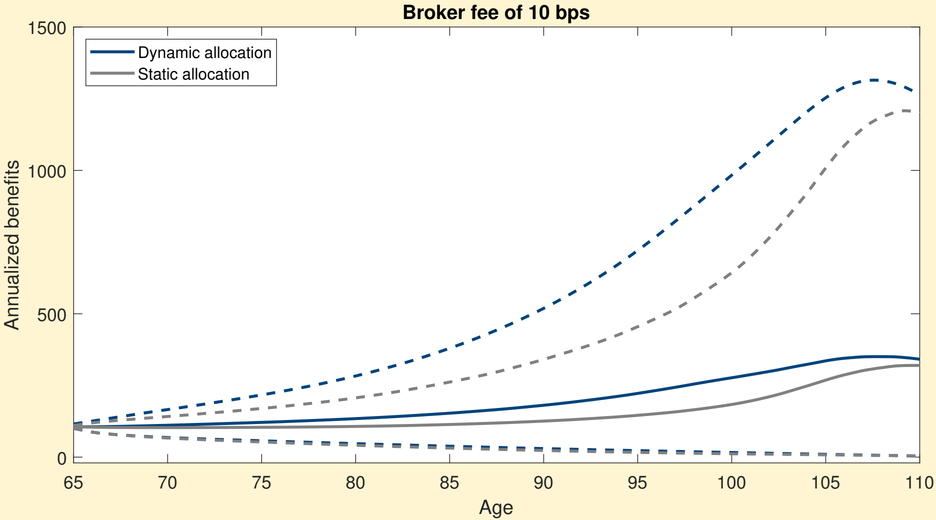


Brokerage fees



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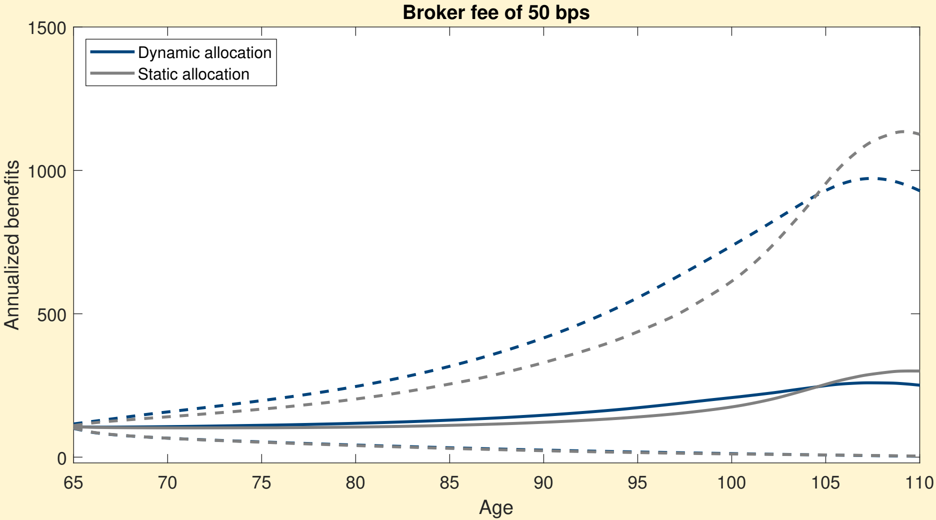

Rebalancing frequency

The average is **higher** for the dynamic allocation; the standard deviation is **higher**, too, but it is driven by the right tail.

	75		85		95		105	
	DA	SA	DA	SA	DA	SA	DA	SA
Annual	108.2 42.0	99.9 32.8	125.6 89.6	109.3 71.6	164.1 204.0	139.2 169.6	226.7 541.0	244.9 663.4
Semiannual	113.0 46.1	102.3 34.6	134.8 101.5	112.2 75.1	182.0 243.2	143.3 178.6	260.0 721.3	256.6 730.2
Bimonthly	118.7 49.6	103.9 35.7	147.2 114.6	114.0 77.6	209.2 293.1	146.5 186.2	316.5 882.5	272.8 757.1
Biweekly	134.3 56.1	104.5 36.2	187.9 147.9	114.3 78.2	303.6 433.0	147.7 189.6	484.2 1339.6	271.9 743.3
Weekly	153.3 63.6	104.6 36.3	243.5 190.7	114.7 78.6	442.4 626.8	148.0 191.1	747.9 2041.1	273.0 755.7

DA stands for dynamic allocation and SA for static allocation.

Rebalancing frequency

The benefits obtained with the dynamic allocation method tend to be **higher** than those obtained with static allocation.

	75		85		95		105	
	DA	SA	DA	SA	DA	SA	DA	SA
Annual	77.1	22.9	82.3	17.7	83.5	16.5	47.9	52.1
Semiannual	78.6	21.4	85.5	14.5	88.3	11.7	54.4	45.6
Bimonthly	82.3	17.7	90.2	9.8	93.6	6.4	67.2	32.8
Biweekly	96.4	3.6	99.4	0.6	99.8	0.2	94.4	5.6
Weekly	99.8	0.2	100.0	0.0	100.0	0.0	99.7	0.3

DA stands for dynamic allocation and SA for static allocation.

Rebalancing frequency and fees

- Combining both realistic broker fees and weekly rebalancing does lead to a **slight reduction** in the benefit streams compared to those of without commissions.
- However, these benefits remain **significantly higher** than those achieved through the static strategy.

Concluding remarks and future research

Concluding remarks

Volatility-targeting approaches that account for investment and mortality risks yields **more steady benefit streams**, reducing benefit risk.

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Future research

1. Optimal **hurdle rate policy**.
   - Could change as a function of investment and mortality experience.
2. **Fees** in lifetime pension pools.
   - Optimal fee structures.
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Thank you! Obrigado!

Questions?

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References

ACPM (2017). Decumulation, the next critical frontier: Improvements for defined contribution and capital accumulation plans. Technical report, Association of Canadian Pension Management.

Balter, A. G., M. Kallestrup-Lamb, and J. Rangvid (2020). Variability in pension products: A comparison study between the Netherlands and Denmark. *Annals of Actuarial Science 14*(2), 338–357.

Bégin, J.-F., N. Kapoor, and B. Sanders (2024). A new approximation of annuity prices for age–period–cohort models. *European Actuarial Journal Forthcoming*.

Cairns, A. J., D. Blake, and K. Dowd (2006). A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *Journal of Risk and Insurance 73*(4), 687–718.

Chen, A., M. Guillen, and M. Rach (2021). Fees in tontines. *Insurance: Mathematics and Economics 100*, 89–106.

Christoffersen, P., S. Heston, and K. Jacobs (2009). The shape and term structure of the index option smirk: Why multifactor stochastic volatility models work so well. *Management Science 55*(12), 1914–1932.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics 7*(2), 174–196.

Corsi, F. and R. Renò (2012). Discrete-time volatility forecasting with persistent leverage effect and the link with continuous-time volatility modeling. *Journal of Business & Economic Statistics 30*(3), 368–380.

Donnelly, C. (2023). Heterogeneous membership in decumulation-only CDC plans. *Working Paper*.

Donnelly, C., M. Guillén, and J. P. Nielsen (2013). Exchanging uncertain mortality for a cost. *Insurance: Mathematics and Economics 52*(1), 65–76.

Fullmer, R. K. (2019). Tontines: A practitioner's guide to mortality-pooled investments. Technical report, CFA Institute Research Foundation: Charlottesville, VA, United States of America.

Hanewald, K., J. Piggott, and M. Sherris (2013). Individual post-retirement longevity risk management under systematic mortality risk. *Insurance: Mathematics and Economics 52*(1), 87–97.

Horneff, W. J., R. H. Maurer, O. S. Mitchell, and M. Z. Stamos (2010). Variable payout annuities and dynamic portfolio choice in retirement. *Journal of Pension Economics & Finance 9*(2), 163–183.

References

Lee, R. D. and L. R. Carter (1992). Modeling and forecasting US mortality. *Journal of the American Statistical Association 87*(419), 659–671.

Li, S., H. Labit Hardy, M. Sherris, and A. M. Villegas (2022). A managed volatility investment strategy for pooled annuity products. *Risks 10*(6), 121.

Milevsky, M. A. and T. S. Salisbury (2015). Optimal retirement income tontines. *Insurance: Mathematics and Economics 64*, 91–105.

Milevsky, M. A. and T. S. Salisbury (2016). Equitable retirement income tontines: Mixing cohorts without discriminating. *ASTIN Bulletin 46*(3), 571–604.

Olivieri, A., S. Thirurajah, and J. Ziveyi (2022). Target volatility strategies for group self-annuity portfolios. *ASTIN Bulletin 52*(2), 591–617.

Piggott, J., E. A. Valdez, and B. Detzel (2005). The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance 72*(3), 497–520.

Qiao, C. and M. Sherris (2013). Managing systematic mortality risk with group self-pooling and annuitization schemes. *Journal of Risk and Insurance 80*(4), 949–974.

Stamos, M. Z. (2008). Optimal consumption and portfolio choice for pooled annuity funds. *Insurance: Mathematics and Economics 43*(1), 56–68.