

# Intertemporal Smoothing and Intergenerational Risk Sharing: The Impact on Objective Utility and Subjective Attractiveness of Retirement Savings Products

- Ulm Actuarial Day
- March 2025
- Timon Kramer
- Joint work with Alexander Kling, Jochen Ruß, and Stefan Schelling



# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

# Introduction

## Motivation and Objective of this Paper

### ■ Starting point:

- In traditional life insurance, policyholders participate in the same pool of assets.
- Enables the implementation of various **return smoothing mechanisms**
  - Goal: Reduction of the volatility of the policyholders' return.
- Concrete type of smoothing mechanism can have a significant effect on **risk-return-characteristics** cf. Kling et al. (2024).
  - Intertemporal smoothing reduces pathwise volatility.
  - Intergenerational risk sharing reduces uncertainty of terminal value.

### ■ This raises the question which type of smoothing mechanism results in a

- high **objective utility** according to standard economic models of rational decision making.
- high **subjective attractiveness** for policyholders according to decision models of behavioral economics.

### ■ Can we identify so-called “**compromise products**” which perform well in both dimensions?

# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

# Capital Market and Products

The Capital Market Model and the Book Of Business (Kling et al. 2024)

## Capital Market Model:

- On a finite time horizon  $\mathbb{T} = [-T, T]$ , our model consists of
  - a **risky asset**  $(S_t)_{t \in \mathbb{T}}$  following a geometric Brownian motion

$$dS_t = S_t(\mu dt + \sigma dW_t)$$

with constant drift  $\mu$  and volatility  $\sigma$ .

- a **risk-free asset** given by

$$\beta_t = e^{r(t-(-T))}$$

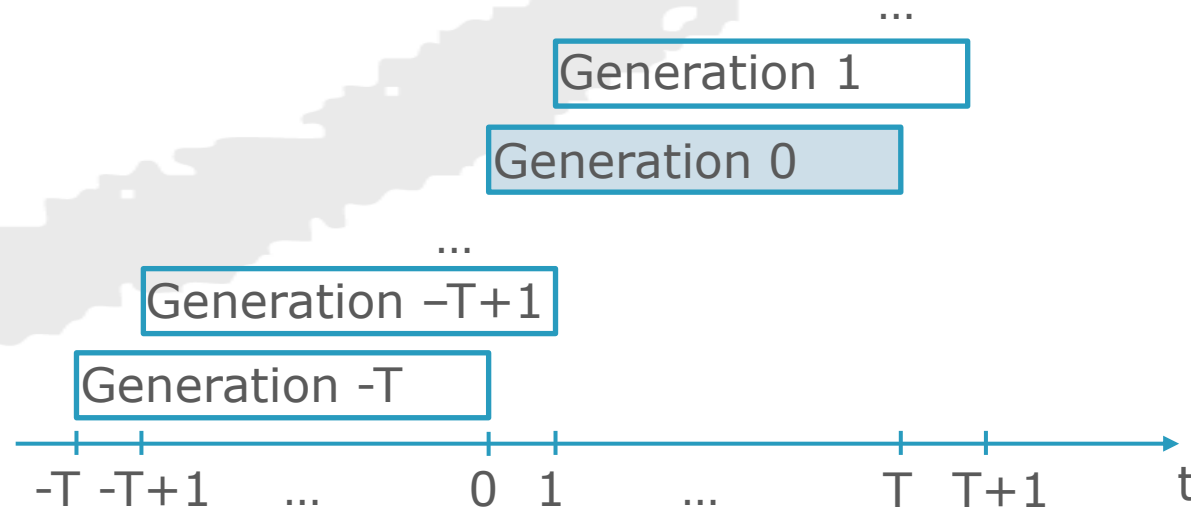
- From these we derive a fund process  $F^\delta$

$$dF_t^\delta = F_t^\delta \left( \delta \frac{dS_t}{S_t} + (1 - \delta) \frac{d\beta_t}{\beta_t} \right)$$

with  $\delta \in [0, 1]$ .

## Book Of Business:

- Policyholder pays a single premium at the start of the contract.
- After  $T$  years policyholder's account is paid out.
- Based on different generations buying the same contract at different points in time.
- The set of all generations is  $\mathcal{H} = \{-T, \dots, T\}$ .
- Analysis focuses on the generation entering at time  $t = 0$  while the company is in a going-concern state.



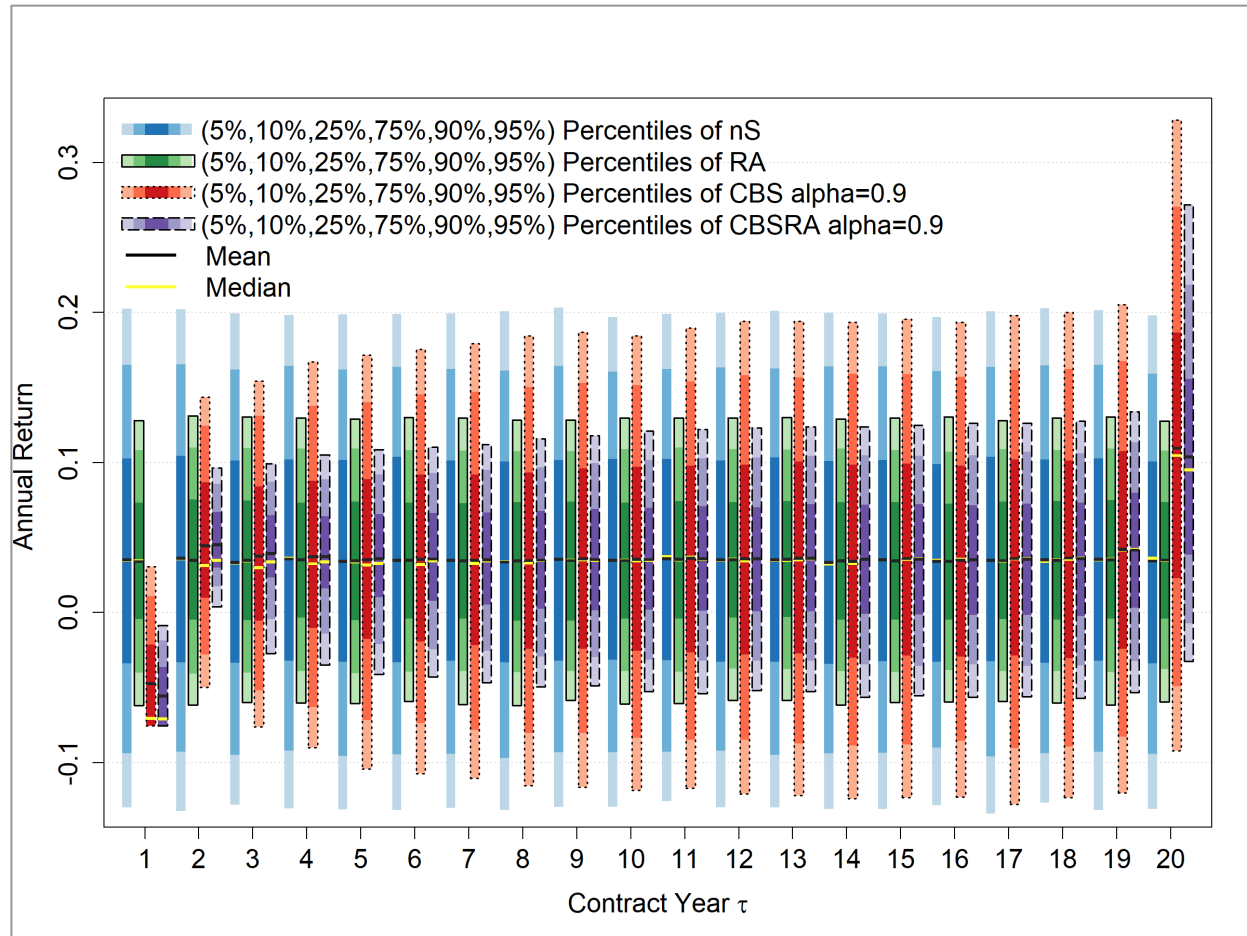
# Capital Market and Products

## Smoothing Mechanisms (Kling et al. 2024)

- Intertemporal Smoothing: **Return Averaging (RA)** - 16 products
  - Return = average return of the past 3 years
- Intergenerational Risk Sharing: **Collective Buffer Smoothing (CBS)** - 64 products
  - Policyholders pay a share  $(1 - \alpha)$  of their premium into a buffer account.
  - The buffer account is used to keep the annualized return of the policyholders' account within some target range.
  - Payment of a terminal bonus.
- Combination of the approaches: **Collective Buffer Smoothing and Return Averaging (CBS RA)** - 64 products
- Additionally: **Product with no smoothing (nS)** - 16 products
- For the model we use the following parameters:  $T = 20$ ,  $P = 10,000$ ,  $r = 2\%$ ,  $\mu = 6\%$ , and  $\sigma = 20\%$ .
- For the fund process, we consider different levels of  $\delta \in \{0.25, 0.3, \dots, 0.95, 1.0\}$ .
- For collective buffer smoothing, we consider four different degrees of intergenerational risk sharing  $\alpha \in \{0.9, 0.95, 0.98, 1.0\}$ .

# Capital Market and Products

## Effects of Return Smoothing on Annual Returns



### Return Averaging:

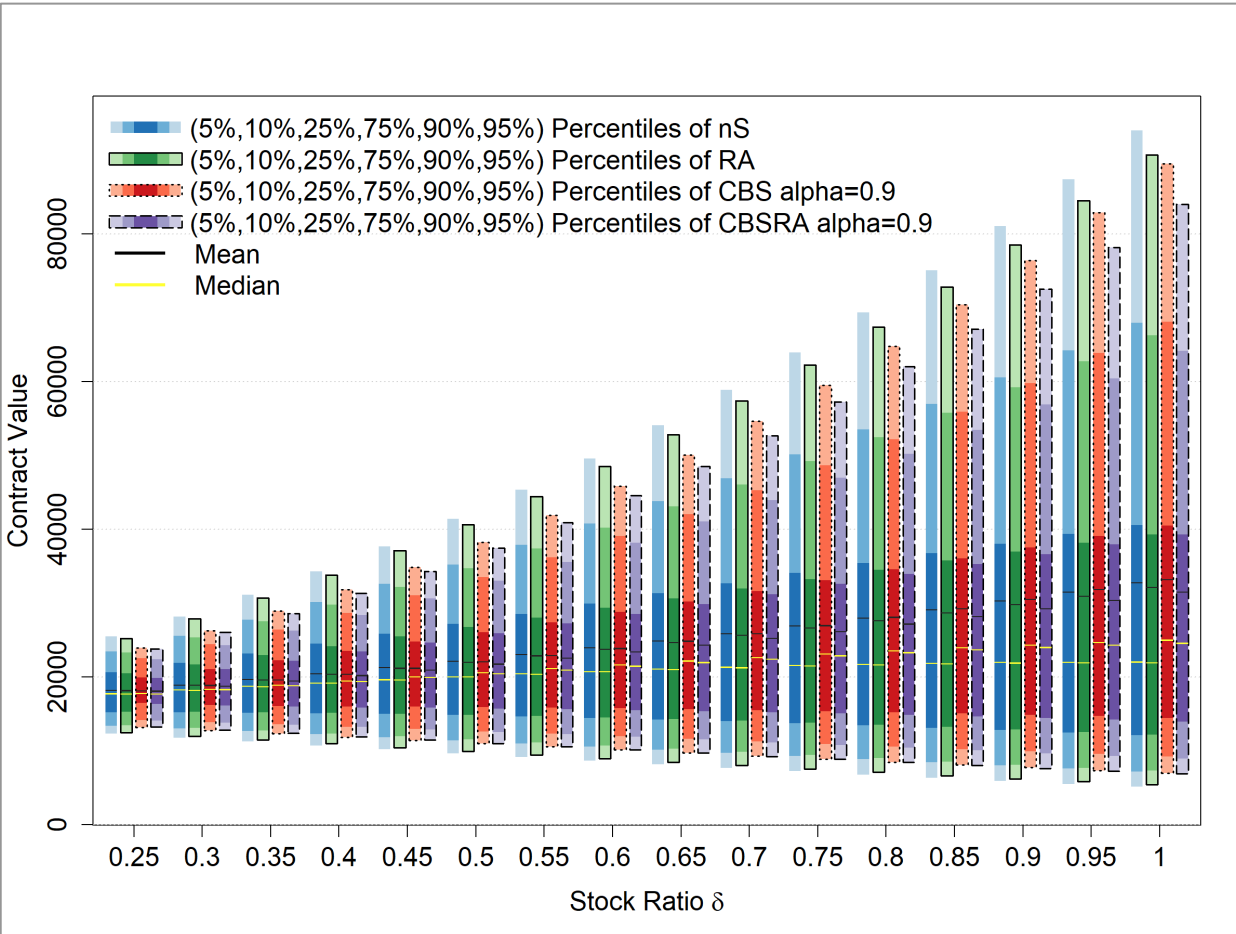
- Strong reduction of standard deviation of annual returns.
- Pathwise volatility is reduced.
- Expected return does not change.
- Effective in reducing risk of annual returns.

### Collective Buffer Smoothing:

- Low annual return in the first year.
- High annual return in the last year.
- Reduction of variance in the first years. Effect diminishes over time.
- Much smaller and erratic effect on annual returns.

# Capital Market and Products

## Effects of Return Smoothing on Terminal Distribution



### Return Averaging:

- Return averaging has almost no effect on risk and return profile of the whole contract.
- Slight effect on upper percentiles for higher stock ratios.
- Almost no effect on risk.

### Collective Buffer Smoothing:

- Distribution of terminal wealth is narrower.
- Expected performance does not change.
- Effective in reducing uncertainty of terminal value.



- Intertemporal smoothing reduces pathwise volatility but has hardly any effect on the terminal value.
- Intergenerational risk sharing can significantly reduce the uncertainty of the terminal value.



# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

# Analysis of Objective Utility and Subjective Attractiveness

## Normative and Descriptive Models

### ■ Normative models (Objective Utility)

- Describe how individuals **should** make decisions.

- E.g., Expected Utility Theory (EUT) with Power utility function  $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$

### ■ Descriptive models (Subjective Attractiveness)

- Describe how individuals **make** decisions.

- E.g., Cumulative Prospect Theory (CPT) (cf. Tversky and Kahneman 1992)

- For long-term investment decisions, CPT fails to explain typical behavior.

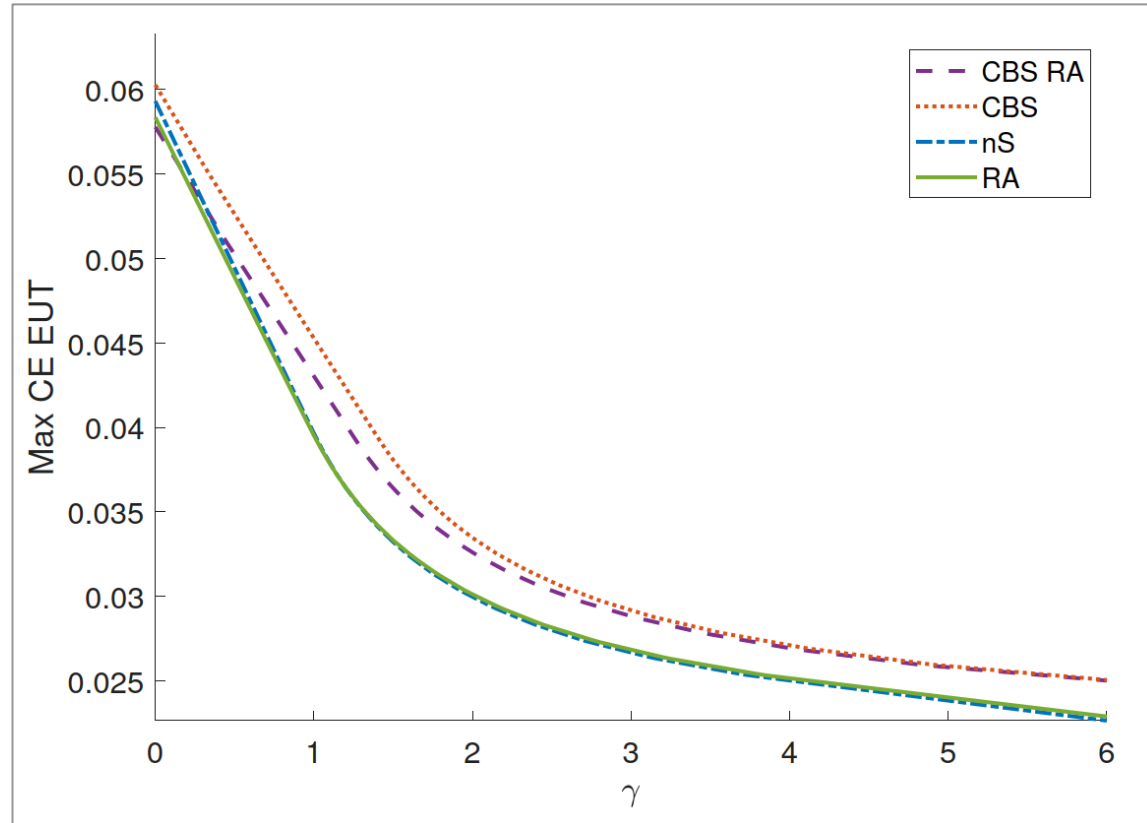
- Ruß and Schelling (2018) propose the Partial Multi Cumulative Prospect Theory (PMCPT):

  - Apply CPT on annual gains and losses and the total value change of the product.

### ■ Under both preference formulations we can derive unique certainty equivalent values (annual CE returns).

# Analysis of Objective Utility and Subjective Attractiveness

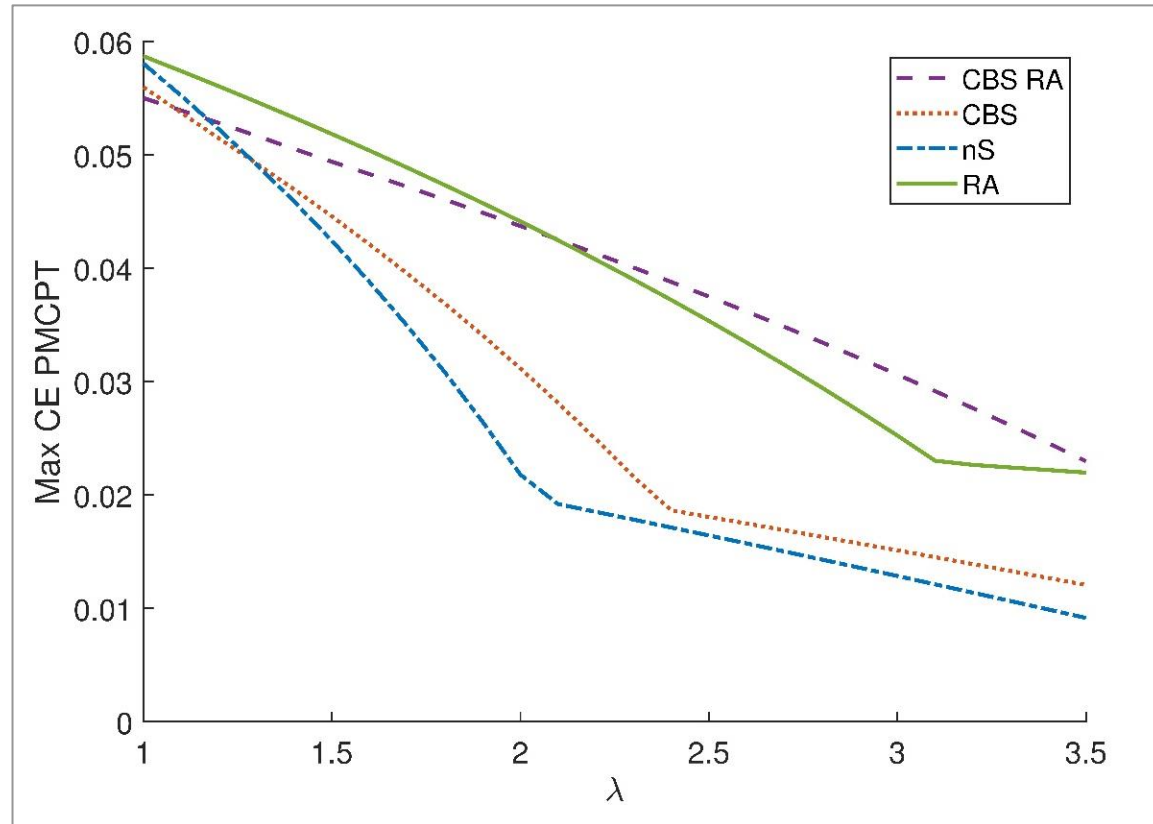
## Objective Utility - EUT



- CBS product dominates.
  - For  $\gamma > 1$ , both products with collective buffer smoothing clearly dominate.
  - For very low risk aversion, nS and RA have similar expected utility to that of CBS products.
  - Optimal stock ratio decreases with increasing risk aversion  $\gamma$  (e.g.  $\gamma = 1, \delta = 100\%$ ;  $\gamma = 3, \delta = 45\%$ ).
- Intergenerational risk sharing leads to higher objective utility.

# Analysis of Objective Utility and Subjective Attractiveness

## Subjective Attractiveness - PMCPT



- Products with return averaging dominate the other products.
  - For  $\lambda < 2.25$  product with RA is dominating.
  - For  $\lambda > 2.25$  product with CBSRA is dominating.
  - The optimal stock ratio is  $\delta = 100\%$ .
- Intertemporal smoothing is subjectively very attractive.



- The objectively optimal product may differ considerably from the subjectively attractive ones.
- CBS RA products perform well in both dimensions. Their specifications may differ significantly.
- Can we find a **"suitable compromise"**?

# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

# Finding Compromises

## Motivation

- Compromises are important when the objectively best choice is perceived subjectively very unattractive and vice versa.
  - Often the case in the context of retirement savings due to complexity of long-term financial decision.
  - Problem: Strongly sub-optimal decisions can have significant negative consequences for the standard of living in retirement.
- Finding a compromise is highly relevant for
  - Policymakers: Can promote suitable product designs that are accepted by many consumers.
  - Financial advisors: Guide clients toward objective better choices while still accounting for their preference.
  - Individuals: Reduce postponement of decision and negative emotions (cf. Luce et al. 1999).
- We use a framework introduced by Ruß et al. (2023) to simultaneously assess objective utility and subjective attractiveness to identify compromise products that create a
  - High (albeit not the maximum possible) objective utility and
  - High (albeit not the maximum possible) subjective attractiveness.

## Finding Compromises

Proposed simultaneous evaluation approach (Ruß et al. 2023)

- Assessing objective utility and subjective attractiveness by preference formulations:

- objective:  $G_1 = EUT$

- subjective:  $G_2 = PMCPT$

- The combined preference function  $K$  for the simultaneous evaluation for a product  $c_i \in C$  with CE returns  $r_i^{G_1}$  and  $r_i^{G_2}$  is

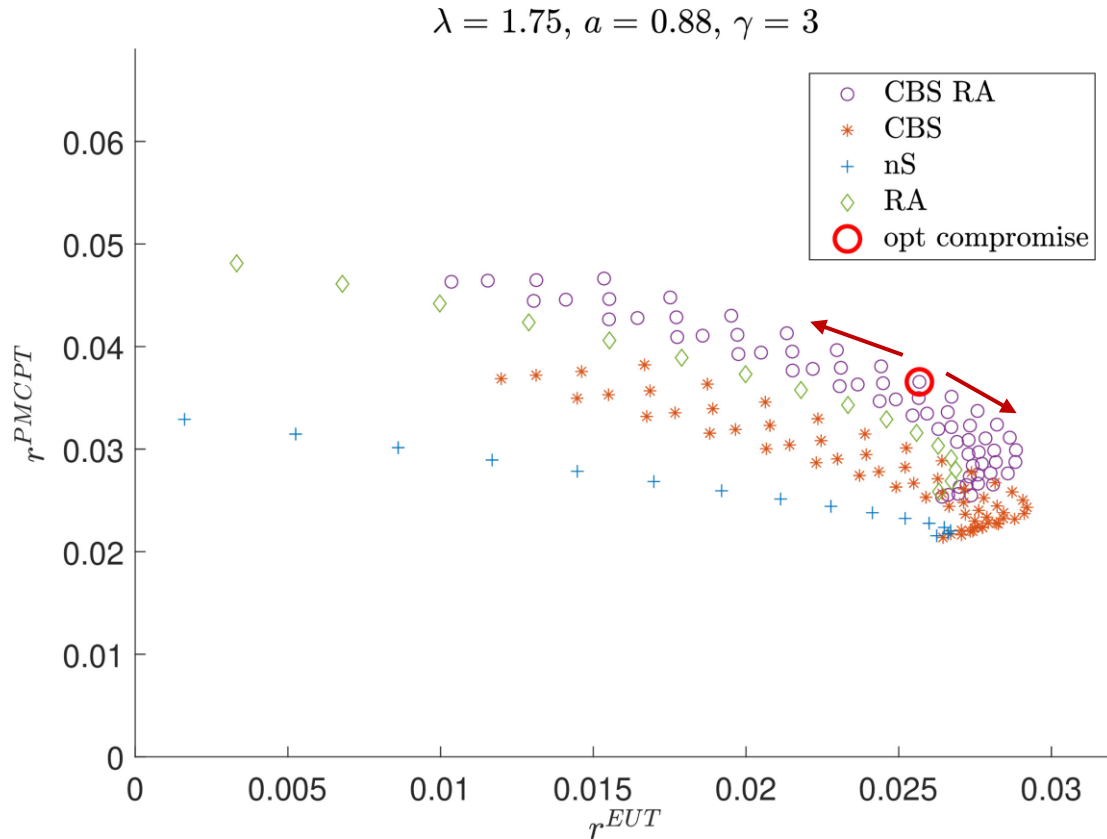
$$K(c_i) := (1 - \omega)(r_i^{G_1})^\xi + \omega(r_i^{G_2})^\xi$$

with preference weight  $\omega \in [0,1]$  and degree of well-balancedness  $\xi \in (0,1)$ .

- If there is a  $c_i$  s.t.  $K(c_i) > K(c_j) \forall c_j \in C \setminus c_i$  then  $c_i$  is called the **favorable compromise product**.

# Finding Compromises

## Numerical Findings

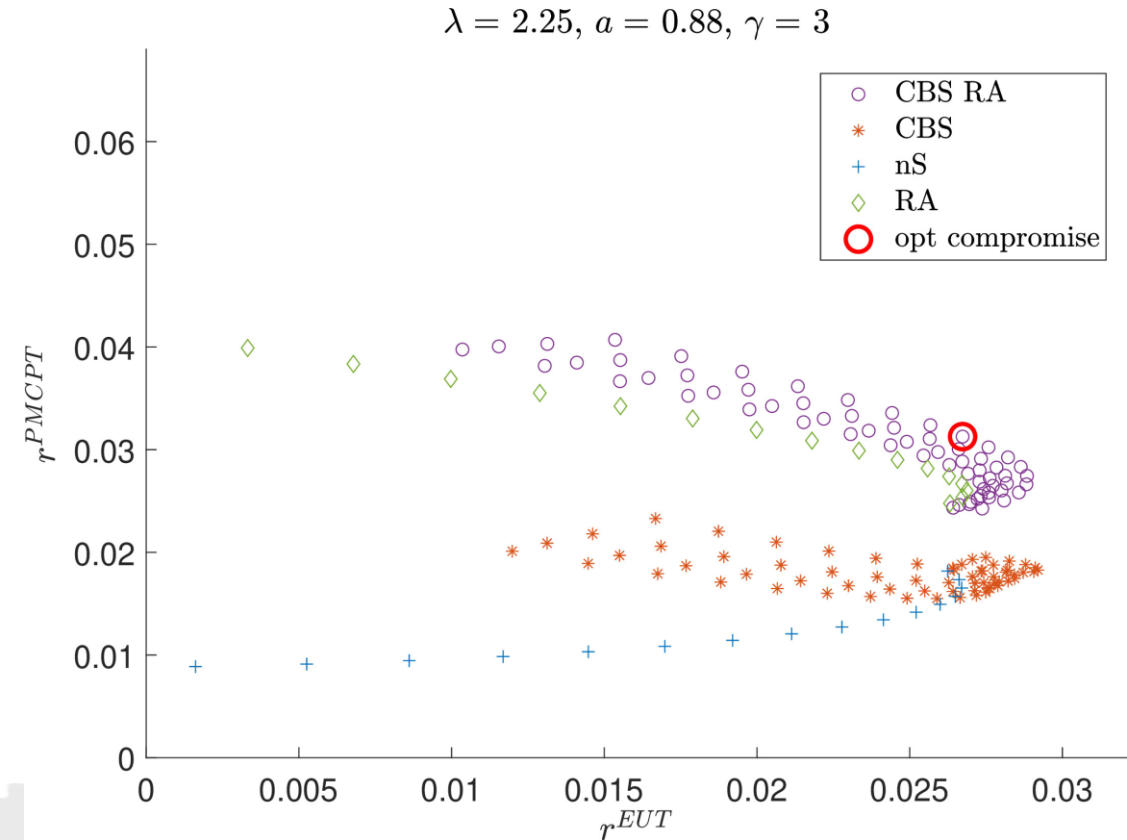
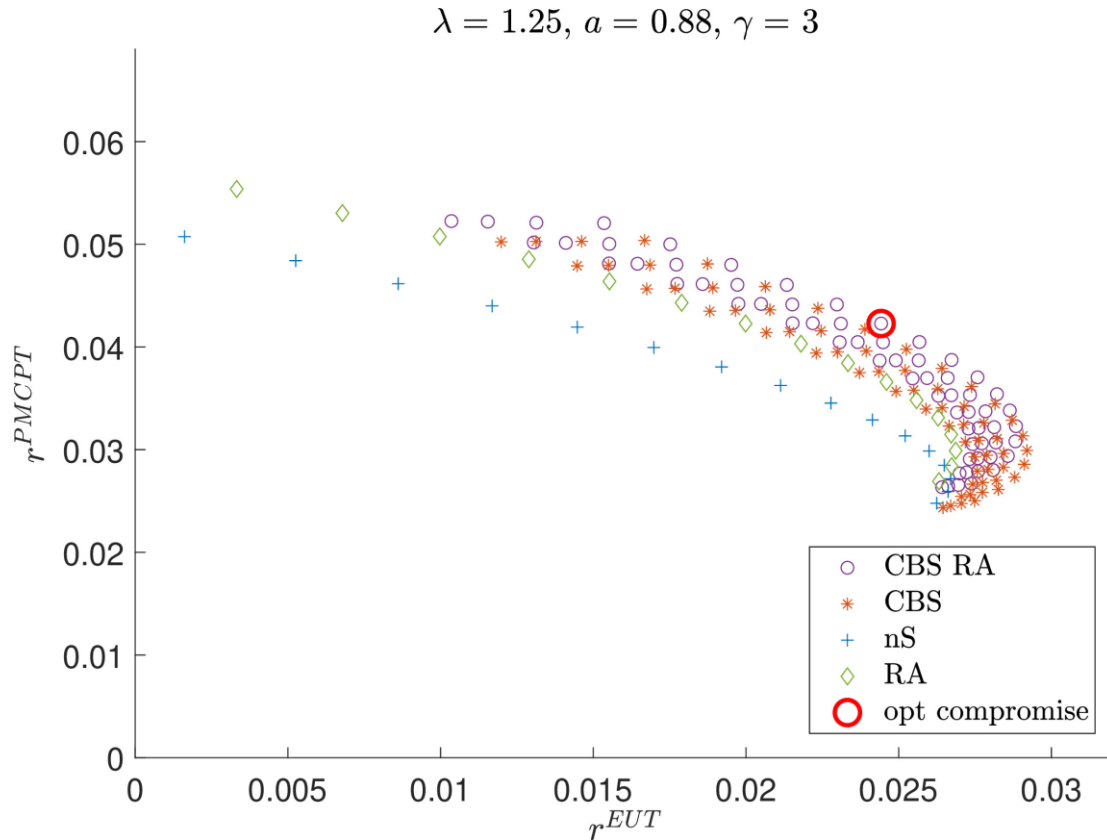


- Subjectively most attractive choice performs very poorly objectively and vice versa.
- CBS RA products are better suited than others to serve as a compromise.
- The favorable compromise is CBS RA with  $\alpha = 0.9$  and  $\delta = 70\%$ .
- Compromise product performs well in both dimensions:
  - Objective price  $\sim 30$ bp, Subjective gain  $\sim 110$ bp
  - Subjective price  $\sim 120$ bp, Objective gain  $\sim 220$ bp
- The optimal compromise moves within the frontier for different preference weight  $\omega$ .



# Finding Compromises

## Numerical Findings



- CBSRA is a very robust candidate for a favorable compromise.
- Specification ( $\alpha$  and  $\delta$ ) of the favorable compromise is very similar in all settings.

# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

# Concluding Notes

## Our main results:

- Intergenerational risk sharing can significantly increase objective utility.
- Intertemporal smoothing can strongly increase subjective attractiveness.
- Smoothing elements can help to design a good compromise.

## Consequences:

- Our results indicate that it could be beneficial to all stakeholders if a larger variety of different smoothed products were offered for long-term retirement savings.
  - Consumers would benefit from better trade-off between objective utility and subjective attractiveness.
  - Product providers could shift business from guaranteed products to smoothed products which are less risky and hence less capital intensive.

## Obstacles:

- Increasing complexity and decreasing transparency which contradicts the targets of upcoming European regulation (e.g., “value for money” or IDD).
- Often lower surrender values in the first years (intergenerational risk sharing).
- Complexity of implementing different collective cover funds with different risk profiles (equity ratios) in practice.

# Contact Details

**Prof. Dr. Alexander Kling**  
a.kling@ifa-ulm.de



**Timon Kramer**  
t.kramer@ifa-ulm.de



**Dr. Stefan Schelling**  
stefan.schelling@uni-ulm.de



**Prof. Dr. Jochen Ruß**  
j.russ@ifa-ulm.de



# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**

## Bibliography

Chiappori, P. A., & Paiella, M. (2011). Relative risk aversion is constant: Evidence from panel data. *Journal of the European Economic Association*, 9(6), 1021-1052.

L'Haridon, O., & Vieider, F. M. (2019). All over the map: a worldwide comparison of risk preferences. *Quantitative Economics*, 10(1), 185-215.

Luce, M. F., Payne, J. W., & Bettman, J. R. (1999). Emotional trade-off difficulty and choice. *Journal of Marketing Research*, 36(2), 143-159.

Kling, A., Kramer, T., & Ruß, J. (2024). From Intertemporal Smoothing to Intergenerational Risk Sharing: The Effects of Different Return Smoothing Mechanisms in Life Insurance. Available at SSRN: <https://ssrn.com/abstract=4744873> or <http://dx.doi.org/10.2139/ssrn.4744873>

Ruß, J., & Schelling, S. (2018). Multi cumulative prospect theory and the demand for cliquet-style guarantees. *Journal of Risk and Insurance*, 85(4), 1103-1125.

Ruß, J., Schelling, S., & Schultze, M. B. (2023). What to offer if consumers do not want what they need? A simultaneous evaluation approach with an application to retirement savings products. *European Actuarial Journal*, 1-29.

## Bibliography

Ruß, J., Schelling, S., & Schultze, M. B. (2024). The benefits of return smoothing in insurer's cover funds – Analyses from a client's perspective. *The European Journal of Finance*, 1-31.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5, 297-323.

# Agenda

**Introduction**

**Capital Market and Products**

**Analysis of Objective Utility and Subjective Attractiveness**

**Finding Compromises**

**Concluding Notes**

**Bibliography**

**Appendix**



## Appendix

### Parameter choice for the subjective attractiveness

- Preference parameters used in the base case are

$\gamma$	$a$	$\lambda$	$\beta^+$	$\beta^-$	$\alpha^+$	$\alpha^-$	$\rho$
$\in [0, 6]$	0.88	$\in [1, 3.5]$	1.052	0.934	0.767	0.863	1

- The range for  $\gamma$  is in line with the literature cf. Chiappori & Paiella (2011).
- The choice of the subjective parameters is based on l'Haridon & Vieider (2019).