The impact of carbon risk mitigation on optimal portfolio composition

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September 05, 2024

Motivation

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- Transition towards a low-carbon economy requires major transformations in every industry sector
- A company's large carbon footprint can pose a threat to its share price due to stranded assets (cf. [van der Ploeg and Rezai, 2020], [Curtin et al., 2019])
- Investors want to limit the carbon risk of their portfolio
- According to [BlackRock, 2023] 46% of institutional investors surveyed ranked "navigating the transition to a low-carbon economy" as their top investment priority
- Companies often try to reduce their carbon footprint, and investors also want to tighten the carbon risk limit over time

Outline

Model setup

Notation

- $[0, T]$: Time span considered by the investor
- B_t, S_t^g, S_t^b: Price of risk-free, green and brown asset at time $t \in [0, T]$
- $\pi_t = (\pi_t^g, \pi_t^b)^\top$: Fractions of wealth invested in the green and brown asset
- X_t^π: Wealth of the investor at t when following the strategy π
- $\mathcal{C}_t = (\mathcal{C}_t^g, \mathcal{C}_t^b)^\top$: Carbon risk at time t
- $\tilde{\mathcal{C}}_t$: Maximum carbon risk the investor is willing to take with her portfolio

Model setup

- Financial market with one risk-free, two risky assets ("green" and "brown") which follow the dynamics

$$
\begin{aligned}
dB_t &= rB_t dt \\
dS_t^g &= \mu_g S_t^g dt + \sigma_g S_t^g dW_t^1, \quad S_0^g > 0, \\
dS_t^b &= \mu_b S_t^b dt + \sigma_b S_t^b \left(\rho dW_t^1 + \sqrt{1-\rho^2} dW_t^2 \right), \quad S_0^b > 0.
\end{aligned}
$$

- We assume, that company $i \in \{g, b\}$ reduces its carbon risk according to

$$\mathcal{C}_t^i = \mathcal{C}_0^i \exp\left(-\delta_i t\right).$$

- Similarly, the carbon risk the investor is willing to take decreases by

$$\tilde{\mathcal{C}}_t = \tilde{\mathcal{C}}_0 \exp\left(-\tilde{\delta} t\right).$$

- For $t \in [0, T]$ the investor seeks to
  - (i) maximize the expected terminal utility
  - (ii) while limiting the carbon risk of the portfolio at time t to $\tilde{\mathcal{C}}_t$
- At time $t \in [0, T]$ and with current wealth $x > 0$ the investor tries to find

$$V(t,x) := \sup_{\pi} \quad \mathbb{E}_{(t,x)}\left[U\left(X_T^{\pi}\right)\right] \tag{1}$$

$$=: \sup_{\pi} \quad J(t,x,\pi) \tag{2}$$

  for her utility $U : (0,\infty) \to \mathbb{R}$.
- Results in optimization problem

$$\begin{aligned} &\sup_{\pi} \quad J(t,x,\pi) \\ &s.t. \quad \mathcal{C}_t^{\top}\pi_t \leq \tilde{\mathcal{C}}_t. \end{aligned} \tag{3}$$

Solution to the optimization problem

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Theorem (Solution to optimization problem for power/logarithmic utility)

Let $\tilde{\mathcal{C}}_t$ be the carbon risk level at time t, $t \in [0, T]$ which the investor does not want to exceed with her portfolio. Then the optimal portfolio process is given by

$$\pi_t^{\mathcal{C}} = \begin{cases} \pi_t^* - \frac{\mathcal{C}_t^\top \pi_t^* - \tilde{\mathcal{C}}_t}{\mathcal{C}_t{}^\top \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t} \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t & \text{if } \mathcal{C}_t^\top \pi_t^* > \tilde{\mathcal{C}}_t \\ \pi_t^* & \text{else} \end{cases}$$

where

$$\pi_t^* = \begin{cases} \frac{1}{1-\gamma} \left(\sigma\sigma^\top\right)^{-1} \left(\mu - r\underline{1}\right) & \gamma \geq 0,\ \gamma \neq 1 \\ \left(\sigma\sigma^\top\right)^{-1} \left(\mu - r\underline{1}\right) & \gamma = 1 \end{cases}$$

is the optimal portfolio process from the unconstrained optimization problem for the power utility and the special case of a logarithmic utility ($\gamma = 1$).

Theorem (Solution to optimization problem for power/logarithmic utility continued)
The value function is given by

$$V(t,x) = \frac{1}{1-\gamma} x^{1-\gamma} \exp\left(-(1-\gamma) \int_t^T r + \left(\pi_u^{\mathcal{C}}\right)^\top (\mu - r\underline{1}) - \frac{1}{2}\gamma \left(\pi_u^{\mathcal{C}}\right)^\top \sigma\sigma^\top \pi_u^{\mathcal{C}} du \right)$$

for $\gamma \geq 0$, $\gamma \neq 1$ and by

$$V(t,x) = x + \int_t^T r + \left(\pi_u^{\mathcal{C}}\right)^\top (\mu - r\underline{1}) - \frac{1}{2} \left(\pi_u^{\mathcal{C}}\right)^\top \sigma\sigma^\top \pi_u^{\mathcal{C}} du$$

for $\gamma = 1$.

Optimal portfolio

For $\mathcal{C}_t^\top \pi_t^* > \tilde{\mathcal{C}}_t$ the investments in the green and brown asset shall be adjusted to

$$\left(\pi_t^g\right)^{\mathcal{C}} = \left(\pi_t^g\right)^* - \kappa\left(\sigma_b^2 \mathcal{C}_t^g - \sigma_g \sigma_b \rho \mathcal{C}_t^b\right) \tag{4}$$

$$\left(\pi_t^b\right)^{\mathcal{C}} = \left(\pi_t^b\right)^* - \kappa\left(\sigma_g^2 \mathcal{C}_t^b - \sigma_g \sigma_b \rho \mathcal{C}_t^g\right), \tag{5}$$

where

$$\kappa = \frac{\mathcal{C}_t^g \left(\pi_t^g\right)^* + \mathcal{C}_t^b \left(\pi_t^b\right)^* - \tilde{\mathcal{C}}_t}{\sigma_b^2 \left(\mathcal{C}_t^g\right)^2 - 2\sigma_g \sigma_b \rho \mathcal{C}_t^g \mathcal{C}_t^b + \sigma_g^2 \left(\mathcal{C}_t^b\right)^2}.$$

Observations

(i) Since $-1 \leq \rho \leq 1$ and $\mathcal{C}_t^{\pi^*} = \mathcal{C}_t^g \left(\pi_t^g\right)^* + \mathcal{C}_t^b \left(\pi_t^b\right)^* > \tilde{\mathcal{C}}_t$, it follows that $\kappa > 0$.

(ii) In the case of two *uncorrelated* shares, the term in the parentheses is greater than zero (c.f. (4) and (5)), i.e. the carbon constraint causes a reduction in the fractions invested into both shares compared to the unconstrained case.

(iii) The green investment will be increased compared to the unconstrained case, if

$$\mathcal{C}_t^g < \frac{\sigma_g}{\sigma_b} \rho \mathcal{C}_t^b.$$

(iv) The fraction invested into the brown asset will be increased if

$$\mathcal{C}_t^b < \frac{\sigma_b}{\sigma_g} \rho \mathcal{C}_t^g.$$

Numerical example

Example

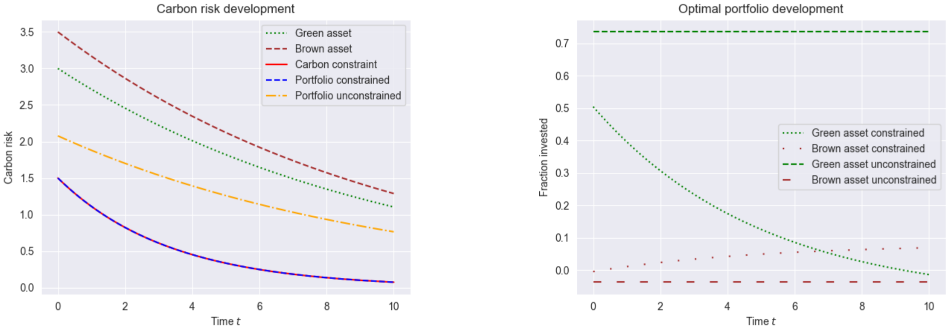
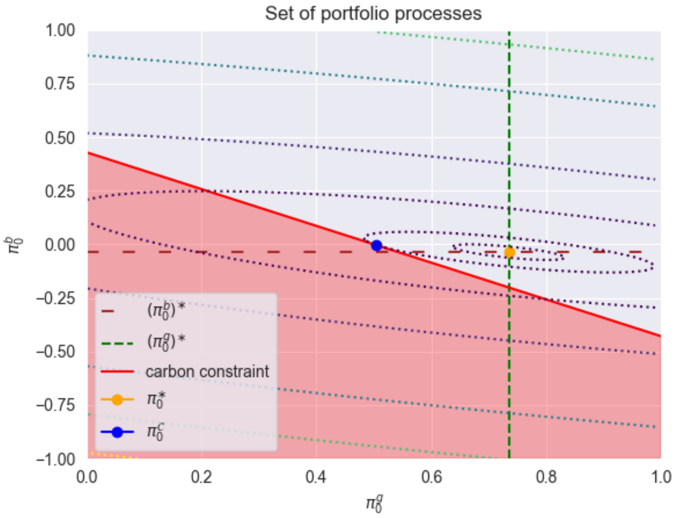


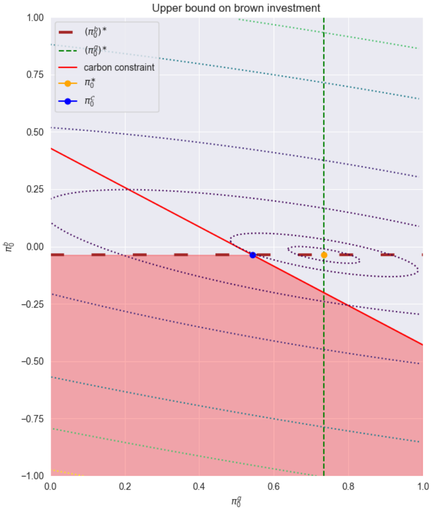
Figure 1: The carbon constraint leads to an undesirable effect!

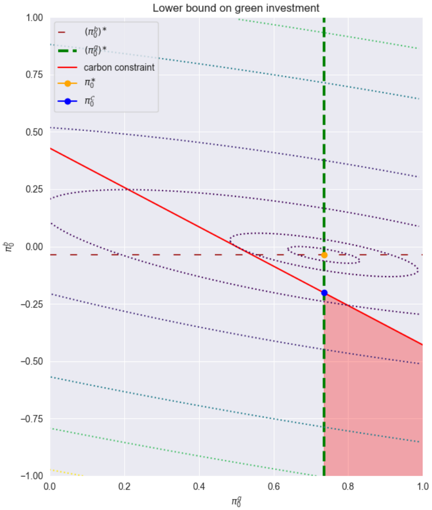
Parameters: $\mu_g = 0.025$, $\mu_b = 0.035$, $\sigma_g = 0.15$, $\sigma_b = 0.4$, $\rho = 0.7$, $r = 0.01$, $T = 10$, $C_0^g = 3$, $C_0^b = 3.5$, $\tilde{C}_0 = 1.5$, $\delta_g = \delta_b = 0.1$, $\tilde{\delta} = 0.3$

Permissible portfolios under carbon constraint



Permissible portfolios under additional constraints





Additional constraints

Optimal portfolio under additional constraints

Since

$$\pi_t^g \geq \left(\pi_t^g\right)^* \quad \Rightarrow \quad \pi_t^b \leq \left(\pi_t^b\right)^*,$$

we include the constraint on the green asset and receive the new optimization problem

$$\begin{aligned} \sup_{\pi} \quad & J(t, x, \pi) \\ s.t. \quad & \mathcal{C}_t^\top \pi_t \leq \tilde{\mathcal{C}}_t, \\ & \pi_t^g \geq \left(\pi_t^g\right)^*. \end{aligned} \tag{6}$$

To simplify the notation in the following proposition, we define

$$\bar{\pi}_t^{\mathcal{C}} := \frac{\mathcal{C}_t^\top \pi_t^* - \tilde{\mathcal{C}}_t}{\mathcal{C}_t{}^\top \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t} \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t.$$

Proposition

The optimal portfolio at time t, $t \in [0, T]$ for problem (6) is given by

$$\left(\pi_t^g\right)^{\mathcal{C}} = \begin{cases} \left(\pi_t^g\right)^* - \left(\bar{\pi}_t^g\right)^{\mathcal{C}} & \text{if } \mathcal{C}_t^\top \pi_t^* > \tilde{\mathcal{C}}_t \text{ and } \left(\bar{\pi}_t^g\right)^{\mathcal{C}} \leq 0, \\ \left(\pi_t^g\right)^* & \text{else}, \end{cases}$$

$$\left(\pi_t^b\right)^{\mathcal{C}} = \begin{cases} \frac{\tilde{\mathcal{C}}_t - \mathcal{C}_t^g \left(\pi_t^g\right)^*}{\mathcal{C}_t^b} & \text{if } \mathcal{C}_t^\top \pi_t^* > \tilde{\mathcal{C}}_t \text{ and } \left(\bar{\pi}_t^g\right)^{\mathcal{C}} > 0, \\ \left(\pi_t^b\right)^* - \left(\bar{\pi}_t^b\right)^{\mathcal{C}} & \text{if } \mathcal{C}_t^\top \pi_t^* > \tilde{\mathcal{C}}_t \text{ and } \left(\bar{\pi}_t^g\right)^{\mathcal{C}} \leq 0, \\ \left(\pi_t^b\right)^* & \text{else}, \end{cases}$$

where π^* again denotes the optimal portfolio process from the unconstrained optimization problem.

Conclusion

Conclusion

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- Aim to limit the carbon risk in an investor's portfolio
- Solve the optimization problem analytically for power/logarithmic utility
- Find conditions, which tell when the investment in the green asset increases compared to the unconstrained case
- Possible, that carbon constraint leads to increased investment in the brown asset
- See preprint for: Comparison of optimal portfolio under carbon intensity constraint and under BGS constraint

Thank you!

References

References 1

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BlackRock (2023).
Global perspectives on investing in the low-carbon transition.

Curtin, J., McInerney, C., Ó Gallachóir, B., Hickey, C., Deane, P., and Deeney, P. (2019).
Quantifying stranding risk for fossil fuel assets and implications for renewable energy investment: A review of the literature.
Renewable and Sustainable Energy Reviews, 116:109402.

Görgen, M., Jacob, A., Nerlinger, M., Riordan, R., Rohleder, M., and Wilkens, M. (2020).
Carbon risk.
SSRN.

References 2

van der Ploeg, F. and Rezai, A. (2020).
The risk of policy tipping and stranded carbon assets.
Journal of Environmental Economics and Management, 100:102258.

Appendix

Carbon metrics

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1. **Carbon emissions**: carbon risk rises with quantity of emissions, but larger companies tend to emit more carbon
2. **Carbon intensity**: set emissions in relation to size of the company by dividing by its revenue (or other appropriate variable)
3. **Brown-Green-Score (BGS)**: introduced by [Görgen et al., 2020], assesses a company's carbon risk based on the three dimensions *value chain*, *public perception* and *adaptability*:

$$\mathcal{BGS}_t = 0.7 \cdot v_t + 0.15 \cdot p_t + 0.15 \cdot a_t$$

Theorem (Solution to optimization problem under BGS constraint)
For

$$\bar{\pi}_t^{\mathcal{CI}} := \frac{\mathcal{CI}_t^\top \pi_t^* - \tilde{\mathcal{CI}}_t}{\mathcal{CI}_t^\top \left(\sigma\sigma^\top\right)^{-1} \mathcal{CI}_t} \left(\sigma\sigma^\top\right)^{-1} \mathcal{CI}_t$$

and

$$\tilde{\mathcal{CI}}_t^{sc} := \frac{\tilde{\mathcal{CI}}_t - \mathcal{CI}_t^g \underline{1}\pi_t^*}{\mathcal{CI}_t^b - \mathcal{CI}_t^g}$$

we can rewrite the optimal portfolios under the carbon intensity constraint $\left(\pi^{\mathcal{CI}}\right)$ *and under the BGS constraint* $\left(\pi^{\mathcal{BGS}}\right)$ *as*

$$\pi_t^{\mathcal{CI}} = \pi_t^* - \bar{\pi}_t^{\mathcal{CI}}$$

and

Theorem (Solution to optimization problem under BGS constraint continued)

$$
\begin{aligned}
\left(\pi_t^g\right)^{\mathcal{BGS}} &= \left(\pi_t^g\right)^* - \kappa_t^1 \left(\bar{\pi}_t^g\right)^{\mathcal{CI}} - \kappa_t^3 \Big(\sigma_b^2 \mathcal{BGS}_t^g - \sigma_g \sigma_b \rho \mathcal{BGS}_t^b\Big) \\
&\quad + \kappa_t^2 \Big(\sigma_b^2 \left(\left(0.7\mathcal{CI}_t^g\right)/\left(\mathcal{CI}_t^b - \mathcal{CI}_t^g\right) - 0.15\left(p_t^g + a_t^g\right)\right) \\
&\quad - \sigma_g \sigma_b \rho \left(\left(0.7\mathcal{CI}_t^g\right)/(\mathcal{CI}_t^b - \mathcal{CI}_t^g - 0.15\left(p_t^b + a_t^b\right)\right)\Big), \\
\left(\pi_t^b\right)^{\mathcal{BGS}} &= \left(\pi_t^b\right)^* - \kappa_t^1 \left(\bar{\pi}_t^b\right)^{\mathcal{CI}} - \kappa_t^3 \Big(\sigma_g^2 \mathcal{BGS}_t^b - \sigma_g \sigma_b \rho \mathcal{BGS}_t^g\Big) \\
&\quad + \kappa_t^2 \Big(\sigma_g^2 \left(\left(0.7\mathcal{CI}_t^g\right)/(\mathcal{CI}_t^b - \mathcal{CI}_t^g - 0.15\left(p_t^b + a_t^b\right)\right) \\
&\quad - \sigma_g \sigma_b \rho \left(\left(0.7\mathcal{CI}_t^g\right)/\left(\mathcal{CI}_t^b - \mathcal{CI}_t^g\right) - 0.15\left(p_t^g + a_t^g\right)\right)\Big)
\end{aligned}
$$

Theorem (Solution to optimization problem under BGS constraint continued)
where

$$\kappa_t^1 = \frac{0.49}{\left(\mathcal{CI}_t^b - \mathcal{CI}_t^g\right)^2} \cdot \frac{\sigma_b^2 \left(\mathcal{CI}_t^g\right)^2 - 2\sigma_g\sigma_b\rho\mathcal{CI}_t^g\mathcal{CI}_t^b + \sigma_g^2\left(\mathcal{CI}_t^b\right)^2}{\sigma_b^2\left(\mathcal{BGS}_t^g\right)^2 - 2\sigma_g\sigma_b\rho\mathcal{BGS}_t^g\mathcal{BGS}_t^b + \sigma_g^2\left(\mathcal{BGS}_t^b\right)^2},$$

$$\kappa_t^2 = \frac{0.7}{\mathcal{CI}_t^b - \mathcal{CI}_t^g} \cdot \frac{\mathcal{CI}_t^g\left(\pi_t^g\right)^* + \mathcal{CI}_t^b\left(\pi_t^b\right)^* - \tilde{\mathcal{CI}}_t}{\sigma_b^2\left(\mathcal{BGS}_t^g\right)^2 - 2\sigma_g\sigma_b\rho\mathcal{BGS}_t^g\mathcal{BGS}_t^b + \sigma_g^2\left(\mathcal{BGS}_t^b\right)^2},$$

$$\kappa_t^3 = 0.15 \cdot \frac{\left(p_t^g + a_t^g\right)^\top\left(\pi_t^g\right)^* + \left(p_t^b + a_t^b\right)^\top\left(\pi_t^b\right)^* - \left(\tilde{p}_t + \tilde{a}_t\right)}{\sigma_b^2\left(\mathcal{BGS}_t^g\right)^2 - 2\sigma_g\sigma_b\rho\mathcal{BGS}_t^g\mathcal{BGS}_t^b + \sigma_g^2\left(\mathcal{BGS}_t^b\right)^2},$$

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Theorem (Solution to optimization problem under BGS constraint continued)

and $\tilde{BGS}_t = 0.7\tilde{v}_t + 0.15\tilde{p}_t + 0.15\tilde{a}_t$ *is the limit the BGS of the Portfolio shall not exceed at time* $t \in [0, T]$*. Further,*

$$\pi_t^* = \begin{cases} \frac{1}{1-\gamma}\left(\sigma\sigma^\top\right)^{-1}(\mu - r\underline{1}) & \gamma \geq 0, \gamma \neq 1 \\ \left(\sigma\sigma^\top\right)^{-1}(\mu - r\underline{1}) & \gamma = 1 \end{cases}$$

is the optimal portfolio process from the unconstrained optimization problem for the power utility and the special case of a logarithmic utility $(\gamma = 1)$*.*

Observations

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(i) It holds $\kappa_t^1, \kappa_t^2 > 0$. Further it is $\kappa_t^3 > 0$ if $(p_t + a_t)^\top \pi_t^* > \tilde{p}_t + \tilde{a}_t$

(ii) Due to the many terms in the optimal portfolio process $\pi^{\mathcal{BGS}}$, it is not possible to specify a simple condition under which the green investment increases. Taken together, the conditions

$$\sigma_b \mathcal{CI}_t^g < \sigma_g \rho \mathcal{CI}_t^b,$$
$$\sigma_b \left(p_t^g + a_t^g\right) < \sigma_g \rho \left(p_t^b + a_t^b\right),$$
$$\sigma_b > \sigma_g \rho$$

are sufficient for an increasing investment in the green asset. Note, that this condition is sufficient but not necessary.

Observations

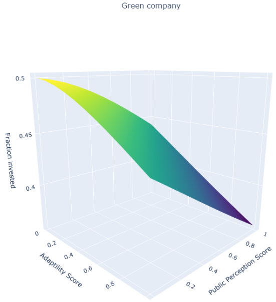
(iii) Similarly, the conditions

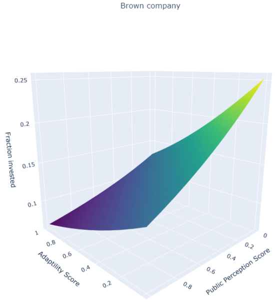
$$\sigma_g \mathcal{CI}_t^b < \sigma_b \rho \mathcal{CI}_t^g,$$
$$\sigma_g \left(p_t^b + a_t^b\right) < \sigma_b \rho \left(p_t^g + a_t^g\right),$$
$$\sigma_g > \sigma_b \rho$$

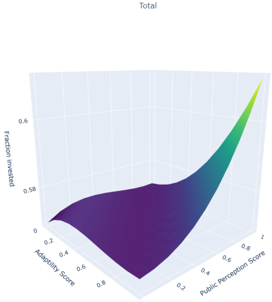
are sufficient for a decreasing brown investment.

(iv) Note that the conditions for the BGS constraint to increase the green investment are not necessarily stricter than the one for the carbon intensity constraint. As the conditions are sufficient but not necessary, overfulfillment of a condition can compensate for not fulfilling the other ones. Since the BGS consists of several components, only the BGS as a whole counts, not the individual components.

Example


Green company
Fraction invested
0.5
0.45
0.4
Adaptility Score
0
0.2
0.4
0.6
0.8
Public Perception Score
0.2
0.4
0.6
0.8
1


Brown company
Fraction invested
0.25
0.2
0.15
0.1
Adaptility Score
1
0.8
0.6
0.4
0.2
Public Perception Score
0
0.2
0.4
0.6
0.8


Total
Fraction invested
0.6
0.58
Adaptility Score
0
0.2
0.4
0.6
0.8
Public Perception Score
0.2
0.4
0.6
0.8
1

If the investor evaluates the utility of her assets according to the log-utility, the expected value to be optimized results in

$$\mathbb{E}\left[\ln\left(X_T^\pi\right)\right] = \mathbb{E}\left[\ln(x) + \int_0^T \left(r + \pi_u^\top\left(\mu - r\underline{1}\right) - \frac{1}{2}\pi_u^\top\sigma\sigma^\top\pi_u\right)du + \int_0^T \pi_u^\top\sigma dW_u\right]$$
$$= \ln(x) + rT + \left[\int_0^T \left(\pi_u^\top\left(\mu - r\underline{1}\right) - \frac{1}{2}\pi_u^\top\sigma\sigma^\top\pi_u\right)du\right].$$

In order to maximize the expected utility of the final wealth, it suffices to maximize the integrand pointwise in t and ω.

As Lagrange function we get

$$\mathcal{L}\left(\pi_t, \lambda_t\right) = \pi_t^\top \left(\mu - r\underline{1}\right) - \frac{1}{2}\pi_t^\top \sigma\sigma^\top \pi_t - \lambda_t \left(\mathcal{C}_t^\top \pi_t - \tilde{\mathcal{C}}_t\right).$$

The stationary condition yields

$$\frac{d}{d\pi_t}\mathcal{L}\left(\pi_t, \lambda_t\right) = \left(\mu - r\underline{1}\right) - \sigma\sigma^\top \pi_t - \lambda_t \mathcal{C}_t \stackrel{!}{=} 0$$

$$\Leftrightarrow \quad +\sigma\sigma^\top \pi_t - \left(\mu - r\underline{1} - \lambda_t \mathcal{C}_t\right) = 0. \tag{7}$$

By the complementary slackness condition have

$$\lambda_t \left(\mathcal{C}_t^\top \pi_t - \tilde{\mathcal{C}}_t\right) \stackrel{!}{=} 0$$

$$\Leftrightarrow \qquad \mathcal{C}_t^\top \pi_t - \tilde{\mathcal{C}}_t = 0. \tag{8}$$

since we do not consider $\lambda_t = 0$. Combining both equations yields

$$\pi_t^{\mathcal{C}} = \pi_t^* - \frac{\mathcal{C}_t^{\pi^*} - \tilde{\mathcal{C}}_t}{\mathcal{C}_t^\top \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t} \left(\sigma\sigma^\top\right)^{-1} \mathcal{C}_t \tag{9}$$

as a candidate for the optimal portfolio process. Due to $\sigma_g, \sigma_b > 0$ it holds that

$$\frac{d}{d^2\pi_t} \mathcal{L}\left(\pi_t, \lambda_t\right) = \sigma\sigma^\top < 0$$

and $\pi_t^{\mathcal{C}}$ indeed maximizes the optimization problem.